# Fourier Analysis of India's Implied Volatility Index

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## Abstract

The objective of this study was to analyze and model periodic behavior observed in India's Nifty Implied Volatility (VIX) Index and to seek the origins of these previously unreported calendar variations. Implied volatility and its variations are important to understand as the pricing of many financial assets and derivatives depend upon this variable. A novel modeling approach to this task uses Fourier analysis for the first time in the literature of implied volatility and calendar effect modeling. For this purpose, daily closing levels for the Nifty VIX Index were gathered covering trading days from 2010 through January 2014. Time series of VIX levels after removing a trend line were tested for normality, stationarity, and autocorrelation. Nifty index and Nifty VIX data were also tested for two-way Granger causality. Detrended Nifty VIX levels were Fourier analyzed to determine primary Fourier frequencies contributing to VIX periodic behavior. A Fourier model of VIX movements was constructed using four primary frequencies from the Fourier power spectrum. This model showed surprising accuracy in identifying the temporal location of VIX peaks and troughs. The probable origin of one recurring VIX calendar frequency was traced to India's earnings release cycle.

*Keywords:* Nifty, Nifty VIX, Fourier analysis, calendar effects, implied volatility

JEL Classification: G12, G 13, G14, G15, G17

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Recurring patterns in financial time series continue to be the subject of extensive studies, many of which seek specifically to understand changes in stock market returns and the volatility of those returns. Periodic behavior in time series for stock market prices includes a list of observed calendar effects :

- Day of the week effect- Rogalski (1984),
- Weekend effect- French (1980),
- End of month effect- Lakonishok & Smidt (1988); Chandra (2011),
- Month of the year effect - Choudhary (2008),
- Expiration day effect - Narang and Vij (2013),
- Holiday effect- Ariel (1990),
- January effect- Haugen and Lakonishok (1988),
- Halloween effect - Jacobsen and Zhang (2012),
- Seasonal patterns - Zhang and Jacobsen (2012).

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McMillan (2012) observed that many of these anomalies typically occur within days or just a few weeks of one another, while some are repeated once per year, or even potentially once every 4 years. The repetition frequencies for quarterly or biannual events appearing in this study do not appear elsewhere in the literature. As pointed out by Malkiel (2003), non - random calendar effect patterns normally do not dependably repeat. When they do repeat, they typically do not allow investors to systematically obtain excess risk adjusted returns.

Separately, there are many studies of time series for stock market returns and volatility, which make common use of regression and GARCH-related models for fitting and forecasting. An extensive literature exists on these topics, reviewed by Bollerslev, Chou, and Kroner (1992) and Bollerslev, Engle, and Nelson (1994). NYU Stern (2014) also models and forecasts almost 40,000 GARCH-related financial time series daily. These V-Lab series forecast volatility, correlations, and risk for most asset classes, including global stock markets. However, despite the theoretical and practical importance of implied volatility in pricing assets, comparatively fewer studies have focused on modeling this important parameter. There appears to be no concurrent identification or modeling of calendar effects by authors who have focused on this parameter (Ahoniemi, 2006 ; Brooks & Oozeer, 2002 ; Harvey & Whaley, 1992 ; Mixon, 2002). This remains true for studies in the Indian stock market where, for example, Rajan (2011) and Kumar and Saravanan (2013) modelled spot volatility but not implied volatility, while Shaikh and Padhi (2014) modelled implied volatility but not in connection with calendar effects. On the other hand, Narang and Vij (2013) and Vipul (2005) studied Indian spot volatility in conjunction with the expiration day effect in India's Nifty futures and options, but not in association with implied volatility. It appears likely that implied volatility and the Nifty implied volatility index (Nifty VIX) have each been studied, but not with reference to calendar effects.

Identifying and forecasting repeating patterns in implied volatility is important as this variable affects asset and derivative pricing and plays an important role in practical hedging and trading strategies. In this study, we identify a previously unreported quarterly calendar effect in India's Nifty VIX time series along with a second biannual calendar effect. Two other previously unobserved VIX calendar variations are also confirmed, but have no presently known origins.

## Analysis Method and Organization

In this study, a model is developed that describes the observed periodic presence of maxima and minima in India's Nifty VIX index. A novel modeling approach to this task uses Fourier analysis. To our knowledge, such an analysis and the resulting Fourier model represent the first application of its kind in the literature of implied volatility modeling. This approach allowed identification of the temporal location of most Nifty VIX extrema within about six trading days of their observed occurrence.

## Data Analysis and the Fourier Model

Daily closing Nifty and Nifty VIX levels were gathered from the website of the National Stock Exchange of India covering the period from December 31, 2009 to January 31, 2014. Each day for which there was a published level of the Nifty or VIX (Trade Day) was assigned a sequential trade date number starting with zero. By taking the natural log of successive daily levels for Nifty and Nifty VIX, two additional return time series (Nifty Returns and VIX Returns) were formed for statistical description and testing. Two further time series were constructed by removing the ordinary least squares linear trend line from each of the Nifty and Nifty VIX time series (Detrended Nifty and DetrendedNifty VIX). The primary purpose of this study is to analyze periodic variations in the Nifty VIX, leaving other time series for future investigation.

➲ **Normality Test :** The Jarque-Bara (JB) test was applied to each series to determine normality. The hypothesis alternatives that were tested are as follows :

Table 1. Statistical $p$ values for JB, LBQ , and ADF Tests

| Time | *P* values | | | Statistics | | |
|---|---|---|---|---|---|---|
| Series | JB test | LBQ test | ADF test | JB test | LBQ test | ADF test |
| Nifty | 0.00 | 0.00 | 0.19 | 38.23 | 6556.67 | -2.81 |
| VIX | 0.00 | 0.00 | 0.01 | 57.69 | 5695.49 | -3.86 |
| Detrended Nifty | 0.00 | 0.00 | 0.00 | 9.14 | 6331.67 | -3.11 |
| Detrended Nifty VIX | 0.00 | 0.00 | 0.00 | 111.95 | 5534.32 | -3.86 |
| Nifty Return | 0.00 | 0.31 | 0.00 | 24.07 | 8.28 | -21.71 |
| VIX Return | 0.00 | 0.02 | 0.00 | 271.91 | 16.70 | -20.05 |

➔ **Null Hypothesis:** The sample is normal distributed.

➔ **Alternative Hypothesis:** The sample is not normal distributed.

The null hypothesis is rejected at the 1% level of significance for all six time series.

➲ **Autocorrelation Test :** The Ljung-Box Q (LBQ) test was applied to determine the number of time-lagged components to include in any autoregressive model of a time series. The hypothesis alternatives that were tested are :

➔ **Null Hypothesis:** The sample is not jointly autocorrelated from lag 1 through lag $h$.

➔ **Alternative Hypothesis:** At least one of the lag 1 to lag $h$ autocorrelations is not zero.

The LBQ tests revealed that VIX, Detrended Nifty VIX, VIX Returns, Nifty and Detrended Nifty were all autocorrelated at the 5% level of significance, but Nifty Returns were not autocorrelated.

➲ **Stationarity Test :** A time series for the Augmented Dickey-Fuller (ADF) test must first be stationary to avoid creating misleading results. For a time series $y(t)$, the following general model for ADF stationarity testing was used.

**Model:** $\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \cdots + \delta_h \Delta y_{t-h}$ (1)

This model can take one of the three separate forms:

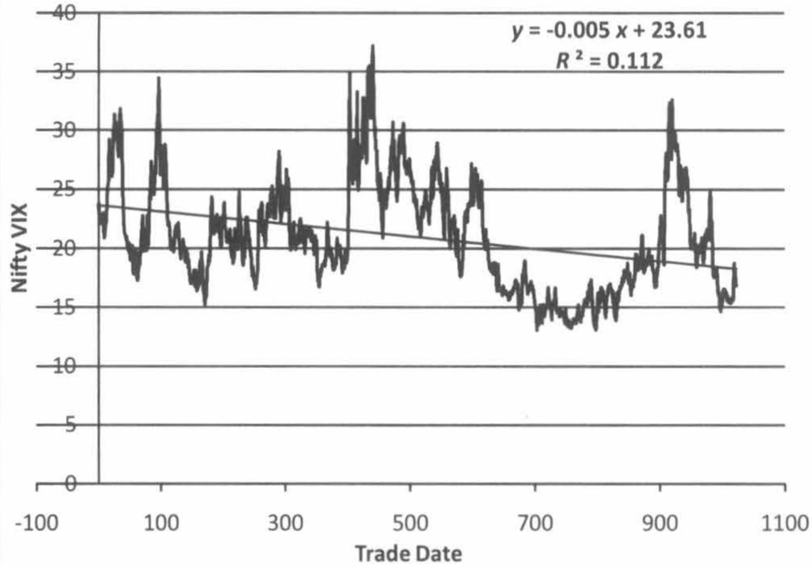
Model 1: $\alpha = 0$ and $\beta = 0$,
Model 2: $\beta = 0$,
Model 3 : No constraint $\alpha$ and $\beta$.

For each time series, the number of lags $h$ that minimized the Bayesian information criterion was chosen where $h = 1,2,..,10$. Using an optimal model, the ADF statistic and its $p$ value were calculated for testing the following hypotheses :

➔ **Null hypothesis :** $\gamma = 0$ (The time series has a unit root and is non-stationary).
➔ **Alternative hypothesis :** $\gamma < 0$ (There is no unit root in the time series and it is stationary).

Model 1 was used for Detrended Nifty, Detrended Nifty VIX, VIX Returns, and Nifty Returns data to test for stationarity, since there is no evidence of a drift or time trend in those time series. In this case, these four time series were found to be stationary at the 1% significance level. For Nifty and VIX data, Model 3 was used since there are significant time trend and drift terms. Nifty testing failed to reject the null hypothesis, and so the Nifty time series

**Figure 1. Nifty VIX With Linear Trend Line**




Source: National Stock Exchange of India. Retrieved from http://www.nseindia.com/products/content/equities/indices/historical_vix.htm

was determined to be non-stationary, but VIX testing rejected the null hypothesis at the 5% significance level, so VIX was determined to be stationary. The *p* values and test statistics for JB, LBQ, and ADF tests are depicted in the Table 1.

➲ **Granger Causality Test :** The Granger causality test was employed to understand if VIX data is useful in the prediction of Nifty or vice versa.

Let *x*, *y* be two stationary time series. Consider the following 2 models :

**Model 1:** $y_t = a_0 + a_1 y_{t-1} + \cdots + a_h y_{t-h}$ (2)

**Model 2:** $y_t = a_0 + a_1 y_{t-1} + \cdots + a_h y_{t-h} + b_1 x_{t-1} + \cdots + b_k x_{t-k}$ (3)

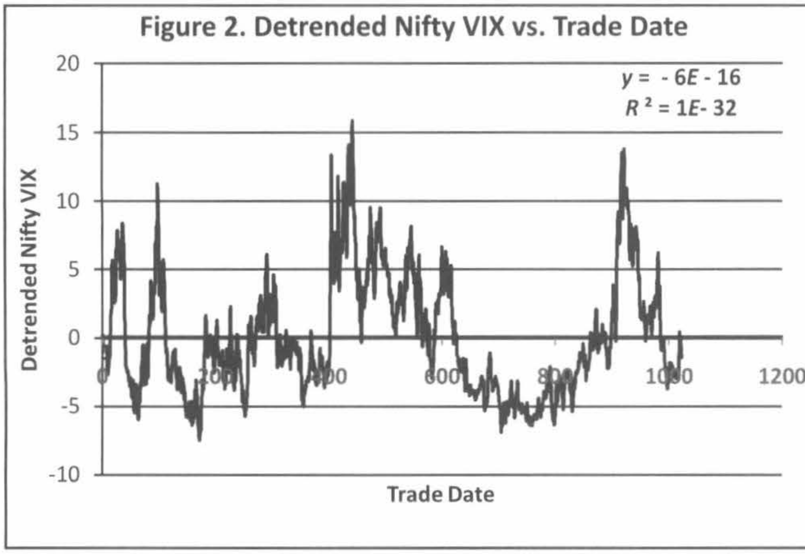
$X$ is said to Granger cause $Y$ if $b_1, \ldots, b_k$ are not all zero. The number of lags, $h$ and $k$ can be chosen by the Akaike information criterion, and an $F$ test can be used to test if $b_1, \ldots, b_k$ are all zero. The hypotheses that were tested are as follows :

➔ **Null Hypothesis:** $X$ does not Granger cause $Y$.
➔ **Alternative Hypothesis:** $X$ Granger causes $Y$.

The *p* value for testing if Nifty returns Granger cause VIX returns is 0.049, so that while at the 5% level of significance, Nifty returns Granger cause VIX returns, this result is barely convincing. On the other hand, the *p* value for testing if VIX returns Granger cause Nifty returns is 0.86, suggesting, at the 5% level of significance, that VIX returns do not Granger cause Nifty returns.

## Discrete Fourier Transforms and the Nifty VIX

The Fourier transform of a general function can be obtained over a continuous or discrete set of frequencies. Such a transform decomposes the function into a combination of trigonometric functions, each with a different



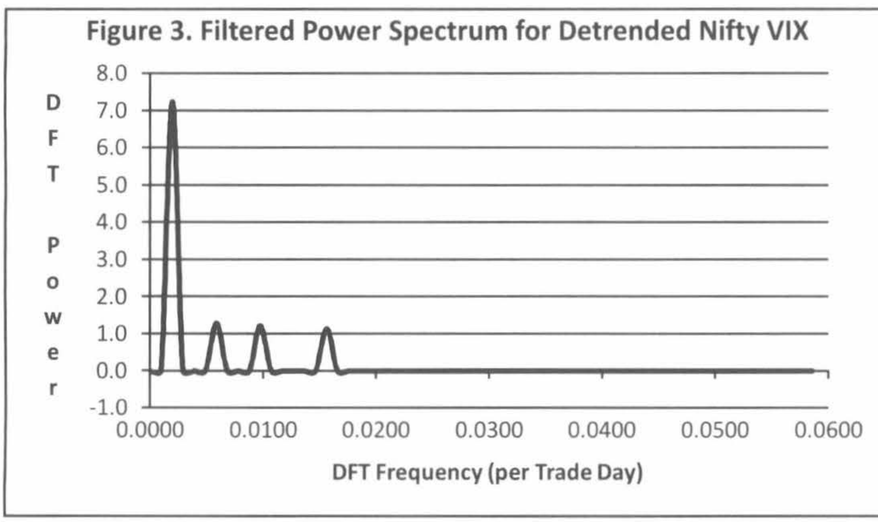




**Table 2. Four Primary Fourier Frequencies and Periods in the Filtered Power Spectrum for Detrended Nifty VIX**

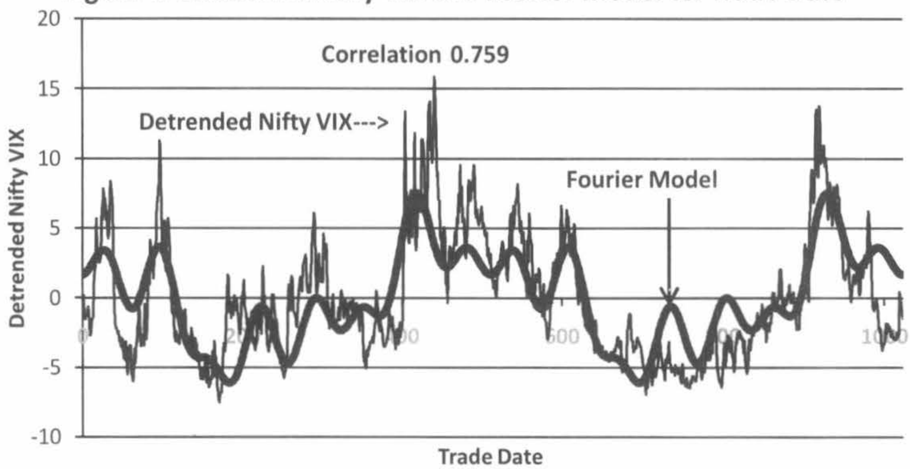
| *k* | DFT Frequency *fk* (per Trade Day) | DFT Period *Tk* = 1/*fk* (Trade Days) |
|---|---|---|
| 1 | 0.001953 | 512 |
| 2 | 0.005859 | 171 |
| 3 | 0.009766 | 102 |
| 4 | 0.015625 | 64 |

frequency. Discrete Fourier transforms (DFTs) can be especially useful when analyzing an economic time series where data points are themselves already discrete being typically daily, weekly, monthly, or annual. However, no known DFT application has been found in the literature analyzing India's financial time series. Accordingly, the application found here is likely the first of its kind. To efficiently calculate DFTs, a computational method called a fast Fourier transform (FFT) algorithm is normally used. FFT algorithms require quite fewer arithmetic operations than needed for a standard DFT computation. The Danielson-Lanczos (1942) lemma used in this study further speeds computation, but requires a discrete data set containing a number of data points equal to a power of 2. For this reason, our data set was chosen to contain 1024 sequentially numbered trade dates. Simple inspection of the

**Table 3. Coefficients for the Fourier Model of Detrended Nifty VIX**

| Fourier Component | Coefficients | *P* - value |
|---|---|---|
| Intercept | -9.1E-16 | 1 |
| $\sin(w_1 t)$ | -1.83886 | 1.95E-44 |
| $\cos(w_1 t)$ | 3.32525 | 1.8E-118 |
| $\sin(w_2 t)$ | -0.20424 | 0.103078 |
| $\cos(w_2 t)$ | -1.58526 | 3.13E-34 |
| $\sin(w_3 t)$ | 0.548809 | 1.29E-05 |
| $\cos(w_3 t)$ | 1.455083 | 2.07E-29 |
| $\sin(w_4 t)$ | 0.015741 | 0.899956 |
| $\cos(w_4 t)$ | -1.50744 | 2.59E-31 |

**Figure 4. Detrended Nifty VIX and Fourier Model vs. Trade Date**




graph of India's Nifty VIX Index in Figure 1 suggests that the index may exhibit periodic behavior. Assuming periodicity can be confirmed using a DFT. It then remains to determine the causal origin of recurring behavior. Prior to subjecting data to a DFT, any systematic bias needs to be removed so that the resulting data has no trend and zero mean. Failure to perform this step reduces the accuracy in identifying important Fourier frequencies. In Figure 1, a trend line was fitted to the data, and the trend was removed to create a Detrended Nifty VIX for further study. In Figure 2, a new line was fitted to the detrended data. A linear regression of Detrended Nifty VIX vs. Trade Date contains a slope and intercept, each statistically found to be zero at the 1% level of significance. The Detrended Nifty VIX data was next Fourier transformed, and the Gaussian filters were applied to the power spectrum to facilitate more accurate identification of the primary Fourier frequencies. The lower frequency portion of the filtered power spectrum appears in Figure 3, and the four primary frequencies and periods appear in Table 1.

Each of the four primary frequencies in Table 2 relates to a Fourier component that can be represented as a weighted linear combination of a sine and cosine. The first component can be written as :

$$A_{11} \sin(w_1 t) + A_{12} \cos(w_1 t) \qquad (4)$$

$A_{11}, A_{12}$ are constants,

$w_1 = 2\,\mathrm{pi} f1,$

$\mathrm{pi} = 3.14159,$

**Table 4. Predicted & Observed Maxima and Minima for Detrended Nifty VIX**

| Predicted Trade Date | Observed Trade Date | Trade Date Difference |
|---|---|---|
| 26 | 26 | 0 |
| 62 | 63 | -1 |
| 96 | 97 | -1 |
| 184 | 171 | 13 |
| 223 | 226 | -3 |
| 257 | 252 | 5 |
| 293 | 290 | 3 |
| 323 | 317 | 6 |
| 352 | 348 | 4 |
| 372 | 378 | -6 |
| 418 | 416 | 2 |
| 456 | 457 | -1 |
| 481 | 484 | -3 |
| 511 | 518 | -7 |
| 538 | 545 | -7 |
| 574 | 582 | -8 |
| 608 | 606 | 2 |
| 696 | 704 | -8 |
| 735 | 733 | 2 |
| 769 | 759 | 10 |
| 805 | 806 | -1 |
| 835 | 830 | 5 |
| 864 | 864 | 0 |
| 884 | 894 | -10 |
| 930 | 925 | 5 |
| 968 | 959 | 9 |
| 993 | 991 | 2 |
| | Average | 0.44 |
| | Std Dev | 5.82 |

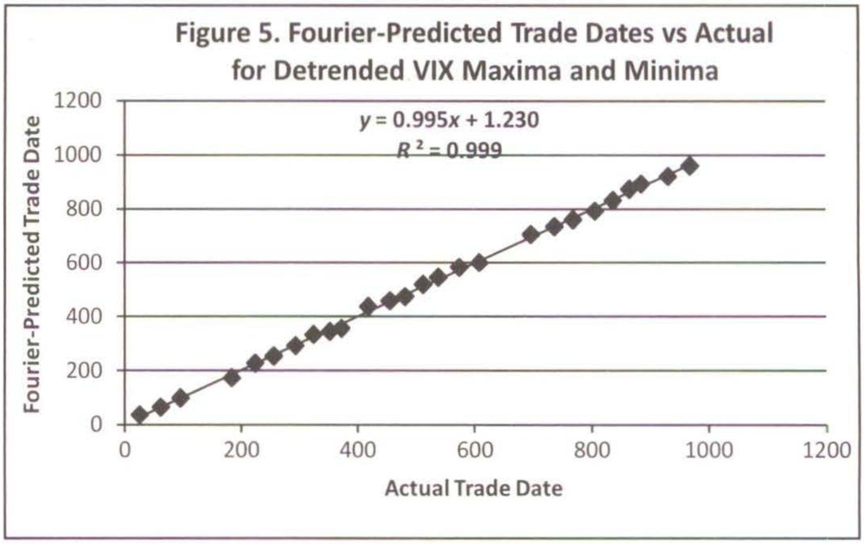
$f_1$ = DFT frequency #1,
$t$ = time in Trade Days,

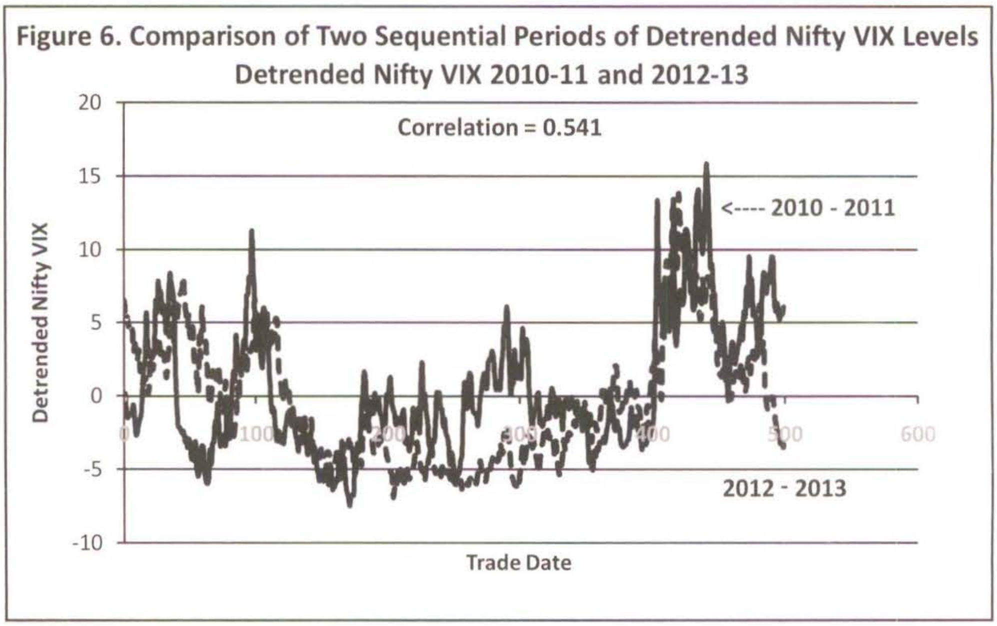
This formulation enables a simplified Fourier model to be constructed of the Detrended Nifty VIX *(DNVIX)* as :

$$DNVIX(t) = A_0 + \Sigma^4_{k=1}\,[A_{k1}\sin(w_k t) + A_{k2}\cos(w_k t)] \qquad (5)$$

This Fourier model can be regressed against the original Detrended Nifty VIX data to determine the coefficients which appear in the Table 3. It can be seen that for alpha at the 5% intercept, *A*21 and *A*41 are not significantly different from zero. The Fourier model of Detrended Nifty VIX then consists of only six terms containing sines and cosines covering four frequencies listed in Table 1.

The resulting equation for the Fourier model is plotted together in Figure 4 with the Detrended Nifty VIX. The correlation of 0.759 can be improved substantially by widening the spread of the Gaussian filters or, in the




extreme, not filtering the Fourier transform at all. In this way, more frequencies contribute to the model, improving its accuracy, but at the cost of increasing complexity in the number of fitted coefficients. Since the objective of this study is to find a reasonably simple model of periodic Nifty VIX behavior, we chose to retain only the four primary frequencies, leaving the more complex models for future research.

## Results and Discussion

As a measure of performance for the Fourier model, Trade Dates for predicted maxima and minima were located and compared with the observed temporal maxima and minima in the Detrended Nifty data. To locate the Trade Date of observed maxima, the following procedure was used. For any three consecutive sequentially observed Fourier model maxima with Trade Dates $t1 < t2 < t3$, the observed maximum corresponding to the Fourier maximum at $t2$ was identified by locating the maximum Detrended Nifty VIX value between $t2 - (t2 - t1)/2$ and $t2 + (t3 - t2)/2$. The Trade Date for this observed companion of $t2$ was noted, and the procedure repeated for all possible Fourier model maxima, and again repeated separately for Fourier model minima. The number of Trade

**Table 5. Trade Days for 2010 - 2013 and Implied Quarterly Frequency**

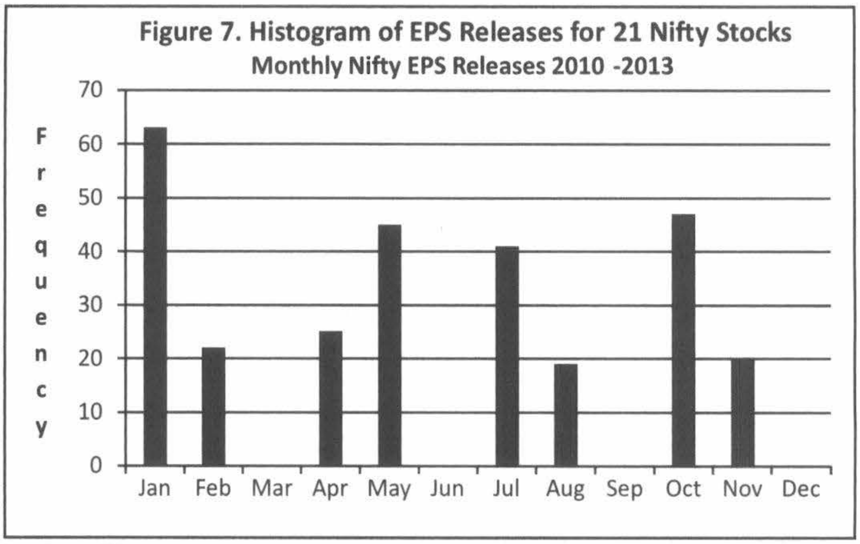
| Year | # Trade Days |
|---|---|
| 2010 | 252 |
| 2011 | 247 |
| 2012 | 251 |
| 2013 | 250 |
| Average | 250 |
| Per Quarter | 62.50 |
| Implied Quarterly Frequency | 0.0160 |

Days by which the Fourier model predicted dates and observed dates differed was noted and is depicted in the Table 4. The Fourier model appears to be able to locate 63% (96%) of the observed peaks within about one (two) standard deviation(s), or about six (12) trading days.

In Figure 5, the predicted vs. observed extrema Trade Dates are plotted along with a linear trend line fit. A regression of predicted against observed maxima and minima with alpha at 5% resulted statistically in an intercept of zero and slope of one. The ability of the Fourier model to identify the Trade Dates where Detrended Nifty VIX maxima and minima occur is impressive, but requires an explanation relating to market behavior. The search for a probable origin of these periodic occurrences begins with an inspection of the periods in Table 1. A period of 512 days is approximately two years of Trade Date data and suggests a possible two-year cycle governing VIX variations. Comparing the first 512 Trade Dates data (2010-11) with that for the following 512 Trade Dates (2012-13) reveals a surprisingly high correlation of 0.541 between the two sub-periods. The graphical comparison of sub-periods appears in the Figure 6. Nifty VIX data began publication on March 2, 2009 ; hence, convincingly establishing that a 2-year cycle exists is impractical with little historical data. Despite this data limitation, the slope and intercept of a linear trend line fitted to 2010-11 data, when regressed against 2012-13 data proved significant at the 5% level. We leave the subject of the biannual origin for future study.

The two intermediate frequencies, $f2$ and $f3$, have periods respectively of 171 and 102 trading days. No known correspondence of these periods with market events has yet been established, and this again remains a matter for future research. However, inclusion of these two intermediate frequencies appears beneficial as it slightly shifts the model extrema locations closer to their observed occurrence in actual time. The fourth frequency in Table 1 may have a testable explanation in market events. One possibility that was investigated was that VIX peaks occur around expiration dates for options and futures. One prior study of Indian spot market volatility identified a derivatives expiration effect lasting only few days (Vipul, 2005). However, derivative expirations are monthly, and the dates do not seem to correlate with VIX extrema dates. Furthermore, the frequency associated with a monthly period of 250/12 = 20.83 days is 0.0480, and there appears only a minor peak in the power spectrum at or near this frequency. Whether the peak can be associated with option and futures expirations is left for future research.

A second interesting possibility is that investor uncertainty near periodic earnings announcement dates might cause cyclical peaks in market volatility. Subtracting non-trading weekend days from a calendar year of 365 days leaves 250 Trade Days on average for the full study period as in Table 5. The number of Trade Days in a quarter of 1 year is then found to be 62.5 days, on an average implying a frequency of 0.0160 per Trade Day. This frequency is quite close to frequency 4 in Table 1 and suggests that we might be observing a quarterly calendar event as the cause of VIX periodic variation. To investigate this possible cause, the dates of earnings per share (EPS) released by 21 of the 50 Nifty stocks having the highest market capitalization were gathered for 2010 through 2013, and a frequency histogram of EPS releases was constructed by calendar month. These 21 members of the Nifty index contained 75% of the index market weight by capitalization (Figure 7). Earnings releases for a single company typically followed a quarterly cycle beginning either in January or February. Even for a company having a January cycle, however, the EPS release date was typically not consistent from year to year, varying by several



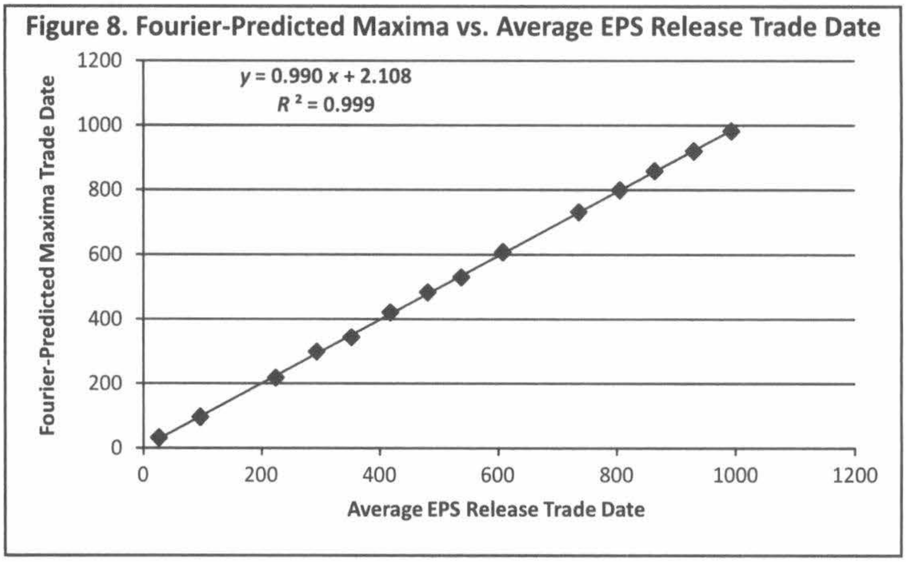




**Table 6. Trade Dates for Fourier-Predicted Maxima and Average EPS Releases**

| Fourier-Predicted Maxima Trade Date | Average EPS Release Trade Date |
|---|---|
| 26 | 31 |
| 96 | 95 |
| 224 | 217 |
| 293 | 298 |
| 352 | 343 |
| 418 | 421 |
| 481 | 483 |
| 538 | 530 |
| 608 | 608 |
| 736 | 732 |
| 805 | 800 |
| 864 | 859 |
| 930 | 921 |
| 993 | 983 |

days and sometimes by a few weeks. Added to the variability of the number of days in a given month, it is understandable that there should arise a Trade Date difference observed in Table 4 since neither of these variations is strictly periodic as would befit a simple Fourier model. Nevertheless, an average date of EPS release was computed for each calendar month for comparison with the Trade Dates predicted by the Fourier model for VIX maxima (Table 6). The result of this comparison appears in Figure 8. The pattern and timing of the VIX maxima suggests that the causal factor could very likely be the Nifty EPS release cycles.

## Research Implications

The current literature contains many separate studies of calendar effects, spot, and implied volatility. There are also studies of the relationships between calendar effects and spot volatility as well as studies between spot and implied volatility. However, there is a notable gap in investigations of the connection between calendar effects and implied volatility. The present paper addresses that gap and suggests that there is room for future fruitful study.

Most calendar effects occur within days, weeks, or months, and have received the greatest attention in the literature. The historic availability of large amounts of data with these repeating frequencies has influenced the regularity of publishing historical studies. Such availability has also made regression modeling a preferred way of examining these events. Less available, however, and so less studied for statistical testing and modeling, are analyses of calendar events with lower frequencies. In such a case, Fourier analysis can be especially helpful. As best, we can discern that the present paper utilizes for the first time this distinct analytical tool for demonstrating its usefulness in the analysis of Nifty VIX data and in the construction of a model to track VIX behavior.

## Conclusion

Implied volatility and its variations are important to understand since the pricing of many financial assets and especially derivatives depend upon this variable. This study of daily closing Nifty VIX levels throughout 2010-2013 reveals previously unreported periodic behavior that has been confirmed by a novel application of Fourier analysis. Such analysis appears to be employed for the first time in the literature of implied volatility analysis and calendar effects in India. The analysis also connects the separate topics of implied volatility and calendar effects, which otherwise appear to have a gap in the literature of these subjects.

Certain repeating peaks in Nifty VIX values during the study period corresponded to quarterly and biannual occurrences. These two particular frequencies have not been previously recounted in the literature of calendar effects in global stock markets, a literature which normally deals with higher frequency behavior, typically daily, weekly, or monthly. The highest of the four prominent Fourier frequencies has a likely origin in the annual earnings release cycle for Nifty stocks. The lowest frequency has no known origin in biannual market events. The two intermediate frequencies also require future explanations.

Using only the four primary frequencies identified in the Fourier power spectrum, a simple model of Nifty VIX performance was fitted. The Fourier model successfully identified the temporal location of VIX maxima, typically within six trading days of their observed occurrence. This result demonstrates the usefulness of Fourier analysis for the identification of lower frequency VIX calendar events. The fitted Fourier model of VIX performance may also be promising for use in Nifty VIX forecasting; however, satisfactory testing of such a model will have to await the availability of future out-of-sample VIX data.

## Limitations of the Study and Scope for Further Research

Two primary limitations in this study are the following. First, the standard algorithm used in calculating fast Fourier transforms requires a data set with a length equal to a power of two. We chose a power of 10 and selected 1024 daily VIX levels covering the time period from 2010-2013. This amount of data was judged to be the minimum necessary to confirm with confidence the presence of variations having a period in excess of 60 trading days ; as Nifty VIX data was available only from March 2, 2009, the relatively small size of data remaining outside the study period severely limits out of sample analysis. To establish the repeatable presence of the four confirmed VIX frequencies outside the chosen study period then will have to await the publication of future Nifty VIX data. A second limitation of this study is uncertainty about the behavioral origins of the observed VIX frequencies. If the frequencies detected are to be made useful for asset pricing and strategy development, a connection must be established with the behavior of market participants or with influential market events, whether political or economic. For one of the four frequencies, a plausible connection has been suggested in the annual Nifty earnings release cycle. Origins for the remaining three frequencies remain under review for future study.

Certain future research directions depend upon the availability of additional Nifty VIX data. For example, the fitted Fourier model of VIX performance may be promising for use in Nifty VIX forecasting; however, satisfactory testing of such a model for forecasting will have to await the availability of a meaningful supply of future out-of-sample VIX data. Furthermore, additional years of VIX data will be required to verify repetition of a biannual occurrence. Other research directions have no such Nifty VIX data limitation. For example, the confirmed presence of periodic VIX behavior suggests that the same fluctuations may be present in other time series tested in the first section of this study. Future research on these related time series would form a natural extension of this paper. In addition, it would be helpful to know how closely peaks in the Nifty VIX are related to dips in a Fourier model of the Nifty index itself. Separately, it may be possible to associate the two intermediate Fourier frequencies with investor behavior or with recurring political or economic events.

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