

## Application of Markowitz Model to BSE

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### **Abstract**

*In this paper an attempt has been made to construct portfolio using Markowitz model. For this purpose, BSE SENSEX and its 30 blue chip companies have been considered. To construct the portfolio six years of data i.e. from January 2007 to December 2012 have been considered. Besides this an attempt has been made to test whether or not this models provides better portfolio selection decision to BSE SENSEX investors. From the 30 blue chip companies only qualified securities are considered for selection. Markowitz two-stock model has been applied to this. An efficient frontier has been developed to find out efficient portfolios out of the possible set of portfolios. In this model we identified six efficient portfolios on its frontier*

**Keywords:** *Optimum portfolio, Markowitz model, NSE blue chip companies*

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INVESTMENT in a single security is always risky. A rational investor always expects highest returns at least risk. It is possible through diversifying their investment in creating a portfolio. The idea behind creating a portfolio is, "keeping all eggs in different baskets is safer than keeping all eggs in a single basket". Creating a portfolio is a strategy to escape risk and achieving investor's objective. We have many models to guide us to construct an efficient portfolio. Markowitz model is concerned with creating an optimal portfolio using efficient frontier by risk averse investors. Markowitz used dominance principle to select optimum portfolio. According to this principle dominant portfolio is either the portfolio which maximises the returns at the given level of risk or minimises the risk at a given level of return.

The present study mainly focuses on developing an optimal portfolio using Modern Technique namely, Markowitz model by considering 30 blue chip companies of the BSE.

### **Theoretical Framework**

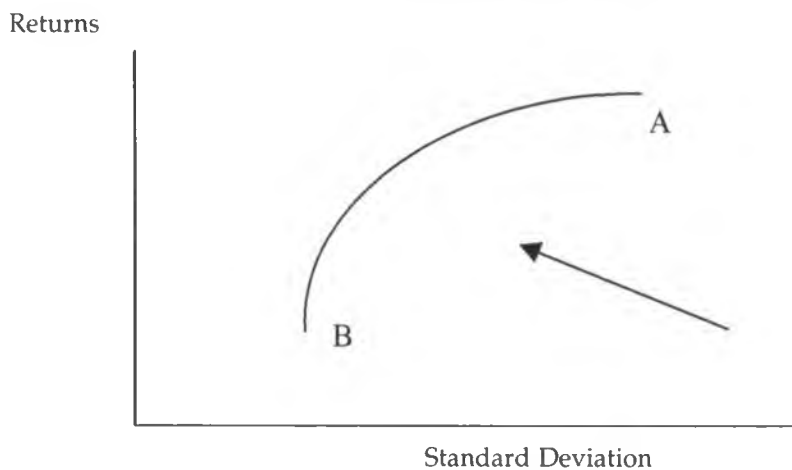
#### **Harry M. Markowitz portfolio Model**

Markowitz came with a model to construct optimal portfolio using Mean return, Standard Deviation and Correlation with each security. In his two security model of portfolio construction he explained the process of choosing an optimum portfolio. Simply the investor should consider overall risk-reward characteristics of the whole portfolio than risk-reward characteristics of any individual security. So Markowitz gave up a single stock portfolio and introduced diversification.

To choose the feasible portfolio from the available number Markowitz used dominance principle. Portfolios that are dominated by others are known as efficient portfolios. The criterion

for dominance is: choose a portfolio with highest risk when two portfolios are offering same level of risk or choose a portfolio with less risk when two portfolios are offering same returns. This principle is based on the assumption that, investors are rational and also risk averse. When all these efficient sets of portfolios are plotted on the graph considering standard deviation along X axis and returns along Y axis it will give us efficient frontier. Fig.1 shows graphical presentation of efficient frontier. The selection of optimal portfolio depends on the risk perception of the investor.

Fig 1: The Efficient Frontier



### Selection of Optimal Portfolio

After constructing efficient frontier the immediate question is what is the optimal portfolio. The selection of a portfolio from the efficient portfolios depends on the investor's degree of aversion to risk. This can be graphically shown through risk return utility curves. Different investors have different risk return utility curves. The tangency point between risk return utility curve (indifference curves) and efficient frontier represent optimal portfolio. This point of tangency represents the highest level of utility the investor can reach.

### Review of Literature

Many researchers made attempts to apply modern portfolio theories to construct an optimal portfolio, and succeeded. There are a few master degree dissertations (Bhatta 1995, Adhiari 2002, Joshi 2002, Poudyal 2002, Shresha 2004 and Shrestha A 2004) on this topic. These studies have a number of limitations including: unrealistic assumptions, small number of samples, samples from one industry only etc.

**Markowitz (1952, 1959)** proposed a mean-variance model for constructing the portfolio. He assumed that all investors are rational and risk averse. He formulated an efficient portfolio using Standard deviation and return of the portfolio. He followed the rule of dominance to find out the eligible portfolio to construct the frontier. Portfolios on this frontier are optimum.

**j. Elton, Martin j. Gruber (1997)**, they reviewed the issues including history and future of portfolio theory, the key inputs necessary to perform portfolio optimisation, special problems

in applying portfolio theory to financial institutions and methods of evaluating how well portfolios are managed.

**Yusen Xia, Baoding Liu and KK Lai (2000)** proposed a new model for constructing portfolio using expected returns of the securities instead of arithmetic means. They also considered the transaction cost, which was not considered in original model of Markowitz. They concluded that the performance of the new model is better than that of the original model.

**Anton Abdulbash Kamil and Kawan Mei Wan (2004)** extended Markowitz model for portfolio construction. They compared Excess return with standard deviation with cut off rate to arrive at the eligibility of security traded in Kuala Lumpur Stock exchange (KLSE). They also considered the value of expected returns of the security.

**Rajan Bahadur Paudel and Sujan Koirala (2006)** constructed a portfolio by applying Markowitz model and Sharpe single index model to the securities of Nepalese stock market. They tested the validity of the two models. They concluded that, investor in Nepalese stock market can reduce risk in his/her investment by applying these models.

**Chin W. Yang and Ken Hug (2010)** added the measures of Skewness and Peakedness to the Basic Markowitz model. They found that the magnitude of risk and shape of the efficient frontier are different from the basic model.

**Bruna Skarica and Zrinka Lukac (2012)** compared the Basic and extended Markowitz model on Croatian Capital Market. They used transaction costs when rebalancing a portfolio in their new model. They concluded that inclusion of transaction costs increased the returns for every level of risk for each portfolio on the efficient frontier.

### Sample and Methodology

This study is based on the risk and return data of the 30 Blue chip companies of BSE. Selection criteria for constructing portfolio using Blue chip companies are:

- (a) Since the study is conducted for the period from January 2007 to December 2012, only those securities whose values are available are considered for analysis. Here DLF data are not available for the above period so they are eliminated.
- (b) The securities with negative returns and negative beta were also eliminated from the sample. Finally 24 securities with positive returns and beta are considered for construction.

Source of the data: the data for the analysis purpose have been collected from official website of Bombay Stock Exchange (BSE) i.e. [www.bseindia.com](http://www.bseindia.com). The following procedure has been used for construction of optimal portfolio using Markowitz two-stock portfolio model:

- (a) Mean returns, standard Deviations, Beta of each stock have been calculated based on past 6 years' returns (Jan 2007-Dec 2012).
- (b) Two hundred and seventy six two-stock portfolios have been calculated from 24 qualified blue chip companies and correlations of these 276 sets has been calculated
- (c) From the above 276 sets 50 sets with least correlations have been considered.
- (d) For the above 50 two-stock portfolios weights, portfolio returns, portfolio standard deviations have been calculated. Formula are as follows:

$$(i) \text{ Weight of Stock 1 } (W_1) = (\sigma_2^2 - r_{12}\sigma_1\sigma_2) / (\sigma_1^2 + \sigma_2^2 - 2r_{12}\sigma_1\sigma_2)$$

- (ii) Weight of Stock 2 ( $W_2$ ) =  $1 - W_1$
- (iii) Portfolio Return ( $R_p$ ) =  $W_1 \cdot E(R_1) + W_2 \cdot E(R_2)$
- (iv) Portfolio Standard Deviation ( $\sigma_p$ ) =  $\text{SQRT}((\sigma_1^2 \cdot W_1^2) + (\sigma_2^2 \cdot W_2^2) + (2 \cdot r_{12} \cdot \sigma_1 \cdot \sigma_2 \cdot W_1 \cdot W_2))$
- (e) An efficient frontier has been developed from the above 50 sets of portfolios. Then portfolios on efficient frontier have been identified as optimum.
- (f) We can rank the performance of the selected efficient portfolios using Sharpe's and Treynor's ratios.

#### Analysis and Discussion:

##### Risk and Return of Individual Securities and Market

The Mean return and Risk in terms of Standard Deviation and Coefficient of variation (CV) and BETA have been calculated for 24 qualified securities and shown in table1. Out of 24 qualified securities TATA Motors offers highest Mean Return of 59.30% followed by Reliance Infrastructure (50.42%) and L&T (42.34%). Similarly, ONGC offered least return of 1.04% followed by Sun Pharma (6.03%) and Reliance Industries (6.96%). Risk in terms of CV of HERO Motor Corp is very little 2.09 followed by SBI 2.33 and L&T 2.41. Risk is highest to ONGC (57.01) because of less return than risk free rate. TATA Motors is more sensitive to Stock market returns measured by Beta (3.02) followed by Sterilite (2.17) and M&M (2.10). Sun Pharma and ITC recorded low BETA (0.4). On the other hand Mean return of the Stock market is 15.7%, Risk in terms of CV is 3.07

**Table 1: Mean Returns, Standard Deviation, CV and BETA of the 24 qualified stocks and market**

S.No	Company	Return	S.D	CV	$\beta$
1	ACC Ltd	13.04	44.47	3.41	0.77
2	Grasim	15.90	55.39	3.48	1.05
3	Hero Motor Corp.	21.90	45.85	2.09	0.54
4	HDFC	8.75	63.79	7.29	0.99
5	HDFC Bank	17.47	63.74	3.65	1.15
6	HINDALCO	28.60	101.79	3.56	1.90
7	ICICI Bank	21.13	61.42	2.91	1.21
8	Infosys	11.92	63.78	5.35	0.97
9	ITC	12.46	31.31	2.51	0.40
10	L&T	42.34	101.84	2.41	1.87
11	M&M	36.02	130.46	3.62	2.10
12	MARUTI	29.48	91.85	3.12	1.59
13	NTPC	8.09	42.08	5.20	0.62
14	ONGC	1.04	57.01	54.84	1.10
15	Reliance Industries	6.96	64.57	9.28	0.75
16	Reliance Infrastructure	50.42	143.68	2.85	2.06
17	SBI	24.91	58.16	2.33	1.13

<i>S.No</i>	<i>Company</i>	<i>Return</i>	<i>S.D</i>	<i>CV</i>	$\beta$
18	Sterilite	24.32	120.90	4.97	2.17
19	Sun Pharma	6.03	42.90	7.11	0.40
20	TCS	8.88	42.75	4.81	0.63
21	TATA Motors	59.30	179.64	3.03	3.02
22	Tata Power	21.62	92.34	4.27	1.55
23	TATA Steel	31.48	96.27	3.06	1.94
24	Wipro	12.44	89.33	7.18	1.42
25	MARKET	15.70	48.23	3.07	

(Source: Self Constructed)

### Selection of Stocks for Two-stock Portfolio

According to Markowitz model two stock portfolios whose correlation is less than positive are qualified. For that purpose correlation of 286 possible pairs has been calculated from 24 qualified sample stocks. These 276 calculated correlations have been presented in annexure 2. The summary of the correlation has been tabulated in the Table 2:

**Table2: Summary of 276 possibles pair of correlations**

<i>Correlation (<math>r_{12}</math>) range</i>	<i>Pairs</i>	<i>Remarks</i>
0.750-1.000	110	Highest: 0.997174 (M&M and WIPRO)
0.500-0.749	87	
0.250-0.499	53	
0.000-0.249	20	
Less than 0.000	6	Lowest: -0.2757 (HERO and Reliance Industries)

(Source: Self Constructed)

### Calculation of Weights, Expected Returns and Standard Deviations of 50 Portfolios

By applying Markowitz minimum variance portfolio model, the weights of each stock of 50 portfolios have been calculated and shown in annexure 3. The expected returns of the portfolios ( $R_p$ ) and Standard Deviation of the portfolios ( $\sigma_p$ ) also shown in annexure 3. Out of 50 combinations the set 21 i.e., combination of stocks of Reliance Infrastructure (64.03% investment) and TATA Motors (35.97% investment) recorded highest return of 53.61% followed by set-28 i.e. the combination of stocks of Mahindra and Mahindra (56.61% Investment) and Reliance Infrastructure (43.49% Investment) recorded the mean return of 42.28%. The least return of 4.55% recorded by the set-39 i.e. the combination of stocks of ONGC (29.59% Investment) and SUN Pharma (70.41% Investment). At the same time set-21 i.e. combination of stocks of Reliance Infrastructure (64.03% Investment) and TATA Motors (35.97% Investment) provides highest risk (with standard deviation of returns of 123.63%) followed by the set-28 i.e. M&M (56.51% Investment) and Reliance Infrastructure (43.49% investment) with 108.44% of Standard deviation of returns. Set-20 i.e. ITC (69% investment) and TCS (31% investment)

provides least risk (standard deviation of 27.68% of the returns). In simple words the range of mean returns of the 50 sets is 4.55%-53.61% whereas portfolio risk (Standard deviation) ranges from 27.68-123.63.

### Identification of Efficient Sets

Markowitz had developed an efficient frontier to identify efficient portfolios by using Mean returns and Standard deviations. This efficient frontier for 50 qualified sets has been shown in the figure 3.

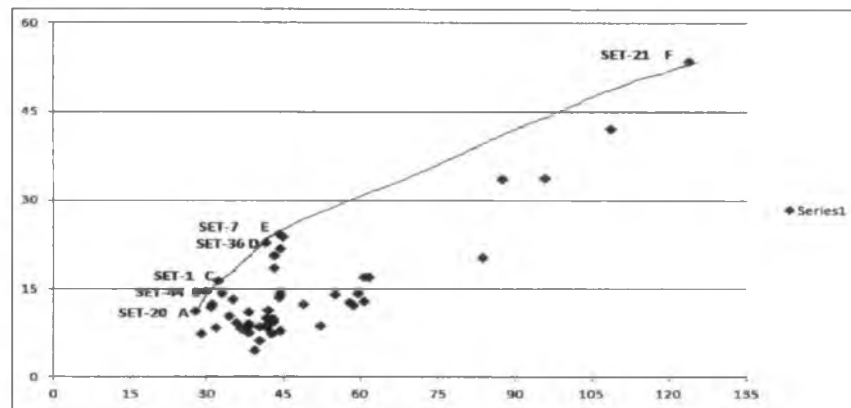


Fig 3: Efficient frontier

Figure 3 shows the risk and return pattern of 50 feasible sets. In the above figure Point A is the minimum risk level. According Markowitz this point is called Global minimum variance. At the same time portfolio Point F offers the highest mean return and is called Maximum return point. The set of portfolios between this global minimum variance (A) and maximum return (F) on the efficient frontier represents the efficient set of portfolios. Simply, the curve AF is known as efficient frontier. All the portfolios that lie on this curve are called efficient portfolios. The efficient portfolios are: set-20 i.e. ITC (69% investment) and TCS (31% investment), set-44 i.e. HERO (23% investment) and ITC (77% investment), Set-1 i.e. HERO (63% investment) and Reliance Industries (37% investment), set-36 i.e. HERO (67% investment) and SBI (33% investment), set-7 i.e. HERO (91% investment) and Reliance infrastructure (9% investment), set-21 i.e. Reliance Infrastructure (64% investment) and TATA Motors (36% investment).

The optimal portfolio will be determined on the basis of risk and return perception of the investors. An investor who wants to take more risk is advised to choose set-21, on the other hand he who wants to take less risk is advised to choose set-20.

### Ranking of Efficient Sets

To rank or to evaluate the performance of any portfolio we can use Sharpe's ratio and Treynor's ratios. Sharpe index measures the risk premium of the portfolio relative to the total amount of risk in the portfolio. Higher the ratio better the performance. On the other hand Treynor's ratio is also called Reward to volatility. Summary of these two measures has been shown in Table 3. From this table it is observed that, as per Sharpe's and Treynor's ratio Set 7 is

performing well compared with other efficient sets followed by set-36. The detailed information about the Sharpe's and Treynor's measures has been shown in annexure 4.1, 4.2 and 4.3.

**Table 3: Ranking of efficient portfolios under Sharpe's and Treynor's ratios**

<i>Two-Stock Portfolios</i>	<i>Sharpe's Ratio</i>	<i>Rank</i>	<i>Treynor's Ratio</i>	<i>Rank</i>
Set-1 (HERO and R.Industies)	0.29	4	15.19	5
Set-7 (HERO and R.Infra)	0.40	1	25.75	1
Set-20 (ITC and TCS)	0.16	6	9.23	6
Set-21 (R.Infra and TATA Motors)	0.38	3	19.38	3
Set-36 (HERO and SBI)	0.39	2	21.63	2
Set-44 (HERO and ITC)	0.26	5	17.56	4

(Source: Self constructed)

### Conclusion

Markowitz gave us a road map to construct the optimal portfolio. If we follow the rules for constructing optimal portfolio we can easily construct the portfolio. But the investor should not get relaxed with this point. Through portfolio evaluation the investor tries to find out how well the portfolio has performed. He/she should evaluate the portfolio from time to time to earn higher returns, because of volatile nature of market and economy. Otherwise risk of the constructed portfolio will increase and return may decrease.

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**Annexure 1: fifty sets of portfolios having least correlation coefficients**

<i>S.No</i>	<i>Two Stocks</i>	<i>Correlation Coefficients</i>	<i>S.No</i>	<i>Two Stocks</i>	<i>Correlation Coefficients</i>
1	HERO and R. Indus	-0.2757	26	ACC Ltd and NTPC	0.2494
2	INFOSYS and R. Indus	-0.1135	27	R.Infra and WIPRO	0.2603
3	SUN Pharma and TCS	-0.0955	28	M&M and R.Infra	0.2625
4	Reliance Industries and WIPRO	-0.0165	29	NTPC and TATA Motors	0.2760
5	M&M and R. Industries	-0.0078	30	HINDALCO and Sun Pharma	0.2829
6	R. Industries and TATA Motors	-0.0066	31	ITC and R.Indus	0.2905
7	HERO and R.Infra	0.0152	32	R.Indus and SUN Pharma	0.3088
8	R.Industries and TCS	0.0581	33	INFOSYS and TATA Power	0.3198
9	ACC Ltd and R.Industries	0.0793	34	HDFC Bank and SUN Pharma	0.3226
10	R.Infra and TCS	0.0809	35	MARUTI and R.Infra	0.3306
11	MARUTI and R.Industries	0.0878	36	HERO and SBI	0.3307
12	INFOSYS and R.Infra	0.1094	37	ACC and Sun Pharma	0.3311
13	HERO and NTPC	0.1114	38	M&M and NTPC	0.3332
14	NTPC and TCS	0.1183	39	ONGC and SUN Pharma	0.3349
15	INFOSYS and Sun Pharma	0.1484	40	NTPC and WIPRO	0.3406
16	HDFC and TCS	0.1556	41	ACC and TATA Power	0.3447
17	HINDALCO and R.Industries	0.1721	42	Grasim and R.Indus	0.3473
18	INFOSYS and NTPC	0.1981	43	INFOSYS and ITC	0.3574
19	ACC Ltd and R.Infra	0.2085	44	HERO and ITC	0.3621
20	ITC and TCS	0.2163	45	SUN Pharma and TATA Motors	0.3622
21	R.Infra and TATA Motors	0.2225	46	HERO and HDFC	0.3695
22	HERO and L&T	0.2256	47	MARUTI and NTPC	0.3706
23	HERO and TATA Power	0.2316	48	HERO and HDFC Bank	0.3706
24	TCS and TATA Power	0.2416	49	INFOSYS and L&T	0.3725
25	HERO and SUN Pharma	0.2445	50	HINDALCO and R.Infra	0.3727



Annexure 2: Correlation Coefficients of 286 pairs of portfolios

	ACC Ltd	Grasim	Hero	HDFC Bank	HDFC Bank	HINDA- LCO	ICICI Bank	Infosys	ITC	L&T	M&M	MARUTI	NTPC	ONGC	R. Indus	R. infra	SBI	STER ILITE	S. Pharma	TCS	T. Motors	T. Power	T. Steel	Wipro	
ACC Ltd	1.000																								
Grasim	0.9131	1.000																							
Hero	0.7040	0.646	1.000																						
HDFC	0.4930	0.797	0.370	1.000																					
HDFC Bank	0.5850	0.628	0.371	0.552	1.000																				
HINDA- LCO	0.8990	0.871	0.850	0.552	0.729	1.000																			
ICICI Bank	0.8480	0.855	0.521	0.641	0.915	0.859	1.000																		
Infosys	0.8700	0.775	0.937	0.374	0.505	0.952	0.685	1.000																	
ITC	0.6010	0.787	0.362	0.867	0.404	0.478	0.626	0.357	1.000																
L&T	0.5000	0.726	0.226	0.816	0.839	0.625	0.794	0.372	0.548	1.000															
M&M	0.8690	0.894	0.905	0.664	0.517	0.910	0.737	0.909	0.698	0.482	1.000														
MARUTI	0.8730	0.890	0.852	0.682	0.670	0.922	0.845	0.871	0.725	0.565	0.974	1.000													
NTPC	0.2490	0.570	0.111	0.816	0.633	0.433	0.541	0.198	0.451	0.936	0.333	0.371	1.000												
ONGC	0.6550	0.738	0.806	0.647	0.934	0.870	0.884	0.700	0.423	0.854	0.697	0.784	0.707	1.000											
R Indust- ries	0.0790	0.347	-0.276	0.601	0.614	0.172	0.466	-0.113	0.290	0.867	-0.008	0.088	0.894	0.537	1.000										
R infrast- ructure	0.2090	0.525	0.015	0.799	0.672	0.373	0.557	0.109	0.459	0.944	0.262	0.331	0.988	0.692	0.940	1.000									
SBI	0.6020	0.753	0.331	0.752	0.939	0.727	0.898	0.484	0.538	0.973	0.558	0.666	0.837	0.928	0.775	0.854	1.000								
STSERI- LITE	0.7150	0.900	0.720	0.903	0.661	0.834	0.776	0.721	0.786	0.763	0.899	0.906	0.688	0.819	0.378	0.639	0.775	1.000							
Sun Pharma	0.3310	0.588	0.245	0.860	0.323	0.283	0.452	0.148	0.944	0.489	0.544	0.583	0.468	0.335	0.309	0.489	0.451	0.714	1.000						
TCS	0.8980	0.703	0.548	0.156	0.541	0.805	0.738	0.797	0.216	0.384	0.622	0.632	0.118	0.574	0.058	0.081	0.505	0.422	-0.096	1.000					
TATA Motors	0.8870	0.828	0.913	0.514	0.664	0.968	0.819	0.953	0.534	0.488	0.950	0.968	0.276	0.797	-0.007	0.222	0.617	0.821	0.362	0.737	1.000				
Tata Power	0.3450	0.585	0.232	0.756	0.847	0.571	0.710	0.320	0.420	0.965	0.409	0.504	0.938	0.874	0.851	0.947	0.943	0.723	0.428	0.242	0.444	1.000			
TATA Steel	0.7940	0.917	0.687	0.802	0.811	0.923	0.889	0.784	0.634	0.857	0.852	0.884	0.730	0.935	0.488	0.687	0.895	0.944	0.503	0.625	0.855	0.809	1.000		
Wipro	0.8560	0.883	0.921	0.649	0.495	0.915	0.710	0.924	0.658	0.474	0.997	0.959	0.341	0.896	-0.017	0.260	0.543	0.894	0.504	0.623	0.946	0.407	0.851	1.000	

**Annexure 3: Weights, portfolio returns, portfolio standard deviation of two-stock portfolios.**

S.No	Two stock Portfolios	$\delta_1$	$\delta_2$	$r_{12}$	$w_1$	$w_2$	$E(R_1)$	$E(R_2)$	$R_p$	$\sigma_p$	$\sigma_p^2$
1	HERO and R. Indus	45.846	64.568	-0.276	0.631	0.369	21.898	6.960	16.383	32.007	1024.431
2	INFOSYS and R. Indus	63.778	64.568	-0.113	0.506	0.494	11.924	6.960	9.470	42.723	1825.215
3	SUN Pharma and TCS	42.902	42.750	-0.096	0.498	0.502	6.033	8.883	7.462	28.800	829.431
4	R. Indust and WIPRO	64.568	89.327	-0.017	0.654	0.346	6.960	12.444	8.856	51.915	2695.197
5	M&M and R. Indus	130.461	64.568	-0.008	0.199	0.801	36.022	6.960	12.733	57.688	3327.861
6	R. Indus and TATA Motors	64.568	179.640	-0.007	0.884	0.116	6.960	59.297	13.032	60.634	3676.455
7	HERO and R.Infra	45.846	143.683	0.015	0.911	0.089	21.898	50.418	24.430	43.865	1924.163
8	R.Industries and TCS	64.568	42.750	0.058	0.294	0.706	6.960	8.883	8.318	36.577	1337.889
9	ACC Ltd and R.Indus	44.467	64.568	0.079	0.693	0.307	13.040	6.960	11.171	37.940	1439.426
10	R.Infrastructure and TCS	143.683	42.750	0.081	0.062	0.938	50.418	8.883	11.455	41.775	1745.188
11	Maruti and R.Indus	91.848	64.568	0.088	0.315	0.685	29.475	6.960	14.063	54.937	3018.057
12	INFOSYS and R.Infra	63.778	143.683	0.109	0.865	0.135	11.924	50.418	17.120	60.448	3653.941
13	HERO and NTPC	45.846	42.076	0.111	0.452	0.548	21.898	8.085	14.327	32.673	1067.520
14	NTPC and TCS	42.076	42.750	0.118	0.509	0.491	8.085	8.883	8.477	31.712	1005.665
15	INFOSYS and Sun Pharma	63.778	42.902	0.148	0.281	0.719	11.924	6.033	7.691	37.905	1436.796
16	HDFC and TCS	63.786	42.750	0.156	0.278	0.722	8.755	8.883	8.847	37.914	1437.466
17	Hindalco and R.Indus	101.793	64.568	0.172	0.248	0.752	28.598	6.960	12.318	58.452	3416.687
18	INFOSYS and NTPC	63.778	42.076	0.198	0.259	0.741	11.924	8.085	9.081	38.066	1449.028
19	ACC Ltd and R.Infra	44.467	143.683	0.209	0.968	0.032	13.040	50.418	14.248	44.232	1956.470
20	ITC and TCS	31.311	42.750	0.216	0.690	0.310	12.459	8.883	11.351	27.681	766.261
21	R.Infra and TATA Motors	143.683	179.640	0.222	0.640	0.360	50.418	59.297	53.611	123.630	15284.454
22	HERO and L&T	45.846	101.839	0.226	0.899	0.101	21.898	42.341	23.966	44.674	1995.786
23	HERO and TATA Power	45.846	92.343	0.232	0.871	0.129	21.898	21.620	21.862	44.235	1956.774
24	TCS and TATA Power	42.750	92.343	0.242	0.897	0.103	8.883	21.620	10.200	41.680	1737.222
25	HERO and SUN Pharma	45.846	42.902	0.245	0.456	0.544	21.898	6.033	13.270	34.934	1220.386
26	ACC Ltd and NTPC	44.467	42.076	0.249	0.463	0.537	13.040	8.085	10.380	34.153	1166.444
27	R.Infra and WIPRO	143.683	89.327	0.260	0.211	0.789	50.418	12.444	20.472	83.658	6998.684
28	M&M and R.Infras	130.461	143.683	0.262	0.565	0.435	36.022	50.418	42.282	108.436	11758.305
29	NTPC and TATA Motors	42.076	179.640	0.276	0.965	0.036	8.085	59.297	9.903	42.784	1830.466
30	Hindalco and Sun Pharma	101.793	42.902	0.283	0.062	0.938	28.598	6.033	7.436	42.462	1802.981
31	ITC and R.Indus	31.311	64.568	0.290	0.901	0.099	12.459	6.960	11.915	30.684	941.515
32	R.Indus and SUN Pharma	64.568	42.902	0.309	0.229	0.771	6.960	6.033	6.245	40.185	1614.864

S.No	Two stock Portfolios	$\delta_1$	$\delta_2$	$r_{12}$	$w_1$	$w_2$	$E(R_1)$	$E(R_2)$	$R_p$	$\sigma_p$	$\sigma_p^2$
33	INFOSYS and TATA Power	63.778	92.343	0.320	0.753	0.247	11.924	21.620	14.323	59.390	3527.221
34	HDFC Bank and SUN Pharma	63.736	42.902	0.323	0.232	0.768	17.475	6.033	8.683	40.232	1618.635
35	MARUTI and R.Infra	91.848	143.683	0.331	0.800	0.200	29.475	50.418	33.666	87.298	7621.002
36	HERo and SBI	45.846	58.157	0.331	0.672	0.328	21.898	24.912	22.886	41.253	1701.770
37	ACC and Sun Pharma	44.467	42.902	0.331	0.473	0.527	13.040	6.033	9.349	35.615	1268.433
38	M&M and NTPC	130.461	42.076	0.333	0.052	0.948	36.022	8.085	9.546	42.638	1817.967
39	ONGC and SUN Pharma	57.011	42.902	0.335	0.296	0.704	1.040	6.033	4.556	39.222	1538.386
40	NTPC and WIPRO	42.076	89.327	0.341	0.932	0.068	8.085	12.444	8.382	41.677	1737.000
41	ACC and TATA Power	44.467	92.343	0.345	0.927	0.073	13.040	21.620	13.668	44.002	1936.152
42	Grasim and R.Indus	55.390	64.568	0.347	0.616	0.384	15.903	6.960	12.467	48.646	2366.473
43	INFOSYS and ITC	63.778	31.311	0.357	0.074	0.926	11.924	12.459	12.420	30.996	960.761
44	HERO and ITC	45.846	31.311	0.362	0.225	0.775	21.898	12.459	14.588	29.606	876.537
45	SUN Pharma and TATA Motors	42.902	179.640	0.362	0.962	0.038	6.033	59.297	8.041	44.191	1952.878
46	HERO and HDFC	45.846	63.786	0.370	0.745	0.255	21.898	8.755	18.550	42.915	1841.707
47	MARUTI and NTPC	91.848	42.076	0.371	0.046	0.954	29.475	8.085	9.071	41.891	1754.827
48	HERO and HDFC Bank	45.846	63.736	0.371	0.745	0.255	21.898	17.475	20.771	42.920	1842.144
49	INFOSYS and L&T	63.778	101.839	0.372	0.828	0.172	11.924	42.341	17.147	61.519	3784.636
50	Hindalco and R.Infra	101.793	143.683	0.373	0.756	0.244	28.598	50.418	33.928	95.719	9162.067

#### Annexure 4.1: Ranking of Two-stock efficient securities under Sharpe's performance measure

Two-Stock portfolio	Portfolio ( $R_p$ ) return ( $R_f$ )	Portfolio S.D ( $\sigma_p$ )	Risk Free rate ( $R_r$ )	$S_p = \frac{R_p - R_f}{\sigma_p}$	Rank
Set-1 (HERO and R.Industries)	16.38	32.01	7	0.29	4
Set-7 ( HERO and R.Infra)	24.43	43.87	7	0.40	1
SET-20 (ITC and TCS)	11.35	27.68	7	0.16	6
SET-21 (R.Infra and TATA Motors)	53.61	123.63	7	0.38	3
SET-36 (HERO and SBI)	22.89	41.25	7	0.39	2
SET-44 (HERO and ITC)	14.59	29.61	7	0.26	5

**Annexure 4.2: Ranking of Two-stock efficient securities under Treynor's performance measure**

<i>Two-Stock portfolio</i>	<i>Portfolio return (<math>R_p</math>)</i>	<i>Portfolio Beta (<math>\beta_p</math>)</i>	<i>Risk Free rate (<math>R_f</math>)</i>	$T_p = (R_p - R_f) / \beta_p$	<i>Rank</i>
Set-1 (HERO and R.Industries)	16.38	0.62	7	15.19	5
Set-7 ( HERO and R.Infra)	24.43	0.68	7	25.75	1
SET-20 (ITC and TCS)	11.35	0.47	7	9.23	6
SET-21 (R.Infra and TATA Motors)	53.61	2.41	7	19.38	3
SET-36 (HERO and SBI)	22.89	0.73	7	21.63	2
SET-44 (HERO and ITC)	14.59	0.43	7	17.56	4

**Annexure 4.3: Calculation Beta of efficient Two-Stock Portfolios**

<i>Two-Stock portfolio</i>	$W_1$	$W_2$	$\beta_1$	$\beta_2$	$\beta_p$
Set-1 (HERO and R.Industries)	0.63	0.37	0.54	0.75	0.62
Set-7 ( HERO and R.Infra)	0.91	0.09	0.54	2.06	0.68
SET-20 (ITC and TCS)	0.69	0.31	0.4	0.63	0.47
SET-21 (R.Infra and TATA Motors)	0.64	0.36	2.06	3.02	2.41
SET-36 (HERO and SBI)	0.67	0.33	0.54	1.13	0.73
SET-49 (INFOSYS and L&T)	0.23	0.77	0.54	0.4	0.43