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Econometric Model Using Arbitrage Pricing Theory and Quantile Regression to Estimate the Risk Factors Driving Crude Oil Returns

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Abstract*— This work presents a novel approach to determining the risk and return of crude oil stocks by employing Arbitrage Pricing Theory and Quantile Regression. Arbitrage Pricing Theory identifies the risk factors likely to impact crude oil returns. Subsequently, Quantile Regression estimates the relationship between the selected factors and the returns across different distribution quantiles. The West Texas Intermediate (WTI) crude oil price is used in this study as a benchmark for crude oil prices. WTI's price fluctuations can significantly impact the performance of global crude oil stocks and, subsequently, the global economy. Various statistical measures are used in this study to determine the proposed model's stability. The results show that changes in WTI returns can have varying effects depending on market conditions and levels of volatility. This study emphasizes the influence of structural discontinuities on returns. These are likely generated by changes in the global economy and the unpredictable demand for crude oil during the pandemic. The inclusion of pandemic, geopolitical, and inflation-related explanatory variables adds uniqueness to the study as it considers current global events that can affect crude oil returns. Findings show that the key factors that pose significant risks to returns are industrial production, inflation, the global price of energy, the shape of the yield curve, and global economic policy uncertainty. This implies that while making investment decisions in WTI futures, investors should pay particular attention to these elements.

***Keywords*— Arbitrage pricing theory; crude oil; econometric model; quantile-regression; risk management; statistical methods.**

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I. INTRODUCTION

International crude oil prices have significantly fluctuated in recent years, mainly due to global economic conditions, technological advancements, political instability, and natural disasters. Despite hundreds of oil production locations, only a few crude oil benchmarks are used for oil pricing: WTI and Brent. The prices of these benchmarks have played a significant role in price variations. This study aims to provide a new way of analyzing the risk and return of crude oil stocks in an uncertain market by combining market fundamentals and economic factors. The literature on multifactor analysis for oil returns is limited, as most research has focused on univariate correlations between oil prices and a single factor. This study uses a multi-factor Quantile Regression (QR) combined with Arbitrage Pricing Theory (APT) to provide a comprehensive understanding of oil prices [1].

APT provides a systematic and quantitative approach to understanding and predicting asset returns by considering the influence of multiple risk factors. By considering various factors simultaneously, it aims to uncover hidden relationships and interactions among them. The fundamental tenet is that an asset's expected return is a linear function of its exposure to different risk factors, plus a particular risk premium unique to that asset. According to the market hypothesis, investors constantly try to take advantage of arbitrage possibilities to correct market inaccuracies and create equilibrium situations. Even the Efficient Market Hypothesis acknowledges transient and short-lived arbitrage opportunities in financial markets. The occurrence of arbitrage opportunities is a critical mechanism that contributes to market efficiency. QR estimates the conditional distribution of data at different quantiles. While the individual methods are not new, their combination and application to crude oil stocks in conjunction with each other is a novel approach.

We find the studies on the relationship between multiple factors and the ROC (return on crude oil) are limited, which indicates a potential gap in the literature. However, there is a growing trend of academic research on macroeconomic factors impacting ROC (e.g., [2], [3], [4], [5], [6], etc.). Researchers have also investigated other factors, such as the link between the epidemic and crude oil ([7], [8], etc.) and geopolitical unpredictability ([9], [10]). In recent academic work, researchers have studied numerous risk factors to evaluate WTI returns, such as:

- Macroeconomic indicators: GDP, inflation, interest rates, and consumer sentiment are significant predictors of WTI returns ([9], [11], [12]).
- Geopolitical events: Political events like wars, terrorist attacks, and geopolitical conflicts significantly influence oil prices ([13], [14], [15], [16]).
- Financial market indicators: It has also been found that factors including stock market indices, volatility, and credit spreads can accurately predict oil prices ([11], [17]).
- Energy policies: Government policies related to energy production, consumption, and conservation can also impact oil prices [11].

These studies provide significant insights into the various factors that have been considered to estimate WTI returns over the last decade. Some of the most researched factors have been:

- US Treasury Spread: Several studies have indicated a considerable effect of the US Treasury yield spread on crude oil prices (e.g., [18], [19], etc.). While another point of view (e.g.,[20]) is that a larger spread causes higher oil prices, the impact is time-varying.
- Global economic policy uncertainty: Empirical investigations show a correlation between the volatility of the oil price and the unpredictability of global economic policy (e.g., [21], [22], [23], etc.]).
- Inflation: Inflation impacts oil prices by affecting the demand for oil as well as the cost of production (such as[24], [25], etc.).
- Industrial production: Changes in industrial production affect the oil demand, as industrial processes often rely on oil as an input ([21], [26]).
- Currency fluctuation against the euro: Exchange rate volatility cannot explain Fluctuations in oil prices [27]. However, a strong link between the volatility of currency rates is evident. A direct linkage between fluctuations in the US dollar's value and oil price changes has been established [28].
- Narrow money supply: As changes in the money supply influence total economic activity, they also influence the oil demand. There is evidence of a correlation between the volatility of the oil price and the unpredictability of economic policy [25].
- Unemployment rate: Empirical results indicate a dynamic causal link between unemployment and ROC [29]. Researchers found that an unanticipated rise in oil price volatility causes the jobless rate to rise persistently [10].
- VIX: The investment horizon determines how investors perceive and react to factors like EPU, the VIX, and GPR when it comes to oil stock movements [11]. VIX is the most significant uncertainty measure in developed markets, while Brazil, India, GPR, and EPU are vulnerable in emerging markets.

The COVID-19 pandemic had a big impact on WTI returns, with prices dropping as demand plummeted due to travel prohibitions and lockdowns [22]. A growing body of research ([30]) points to a non-linear connection between oil prices and economies, despite the studies' primary emphasis being on linear models. Significant external shocks to the oil price [8], discrete regime changes, or the essentially nonlinear nature of the data production process [6] can all lead to nonlinearities.

Despite numerous studies on the relationship between oil prices and macroeconomic factors, the literature on the risk and return characteristics of crude oil assets concerning these variables is still lacking. While some researchers have focused on the relationship between oil prices and macroeconomic indicators (e.g., [19], [31], [32] etc.), they have not adequately investigated the implications of these findings on the risk and return characteristics of crude oil stocks.

Moreover, while some research is available on individual factors such as macroeconomic indicators, geopolitical events, financial market indicators, energy policies, and the impact of COVID-19 on oil prices, there is a gap in the literature regarding the comprehensive analysis of all these factors in a single ROC model.

Table 1 analyzes the citation patterns and impact of scholarly articles in the field. The citation analysis indicates the influence and popularity of the studies in the field.

TABLE I
BIBLIOMETRIC REPORT

Sl. No	Authors	Journals	No. of citations
1	[1]	Elsevier (Energy Economics)	350
2	[2]	Elsevier (Resources Policy)	232
3	[3]	Energy Research Letters, 1(2)	227
3	[4]	Elsevier (Journal of Empirical Finance)	161
4	[5]	Elsevier (International Review of Economics & Finance)	156
5	[6]	Elsevier (Energy Economics)	153
6	[7]	Elsevier (Finance Research Letters)	145
7	[8]	Elsevier (Energy Policy)	131
8	[9]	Elsevier (Energy Economics)	126
9	[10]	Elsevier (Energy Economics)	123
10	[11]	Elsevier (Economic Modelling)	103
11	[12]	Elsevier (International Review of Financial Analysis)	86
12	[13]	Elsevier (Economic Analysis and Policy)	53
13	[14]	Elsevier (Resources Policy)	46
13	[15]	Elsevier (Energy)	31
14	[16]	Taylor & Francis (Journal of Applied Economics)	30
15	[17]	Elsevier (Energy)	29

Sl. No	Authors	Journals	No. of citations
16	[18]	Elsevier (Resources Policy)	24
17	[19]	Elsevier (Energy Economics)	21
18	[20]	Elsevier (Energy)	16
19	[21]	Emerald Publishing Limited (Studies in Economics and Finance)	15
20	[22]	Elsevier (Energy Economics)	12
21	[23]	Elsevier (Economic Modelling)	11
22	[24]	Elsevier (Energy Economics)	9
23	[25]	Emerald (International Journal of Energy Sector Management	6
24	[26]	Elsevier (Resources Policy)	2
25	[25]	Taylor & Francis (Applied Economics)	1

The two main contributions of this paper are:

- Implementation of APT to identify and quantify the impact of various economic and financial variables on WTI returns,
- Application of QR to estimate the effect of these risk factors on different segments of the distribution of the returns.

II. Materials and Method

The APT model provides a framework for understanding asset pricing based on the systematic risk factors that influence asset returns. The equilibrium asset pricing equation according to the APT model is given in Eq. (1):

$$E(R_i) = R_f + \beta_{1i} * f_1 + \beta_{2i} * f_2 + \cdots + \beta_{ki} * f_k \tag{1}$$

where $E(R_i)$ is the expected return of asset i, R_f is the risk-free rate of return, $\beta_{1i}, \beta_{2i}, \ldots, \beta_{ki}$ is the sensitivity coefficients of WTI i to the k systematic risk factors $(f_1, f_2, \ldots, f_k)$, which represent different sources of risk in the economy.

The βs' are estimated by using linear regression. This is calculated using QR with an estimate of the conditional median (0.5 quantiles), and the model's adequacy was checked using various diagnostic tests. The coefficients of the regression represent the sensitivities of the asset to each factor, while the intercept term represents the risk-free rate of return.

TABLE II
VARIABLE SELECTION & DATA SOURCE.

Factors	Characterization variable	Abbreviation
U.S. Treasury Securities at 3-Month Constant Maturity	DGS3MO index	DGS3MO
U.S. Treasury Securities at 5-year Constant Maturity	DGS5 index	DGS5
Industrial Production: Total Index	INDPRO index	PROD
Consumer Price Index for All Urban Consumers	CPIAUCSL	INFLATION
Unemployment rate	UNRATE index	UNRATE
Narrow money supply	M1SL index	M1SL
Change in the exchange rate	CCUSMA02EZM618N index	CCU
S&P 500 index	SP index	SP
CBOE Market Volatility Index	VIX index	VIX
Geopolitical Risk Index	GPR data	GPR
Global price of Energy index	PNRGINDEXM index	GPE
World Pandemic Uncertainty Index	WUPI index	WUPI
Global economic policy uncertainty	GEPU index	GEPU
International crude oil price	WTI crude oil spot price	WTI

Note: Given S&P's reduction of the nation's credit rating in 2011, the common perception that U.S. treasury securities are devoid of credit risk may be debatable; nonetheless, that subject is outside the purview of this study.

A. *Model and Econometric Approach*

We used five years of monthly data from the FRED Economic Database to estimate betas, using yield data for crude oil, precisely the WTI crude oil spot price, to provide insights. As a benchmark for the US equity market and a representation of both financial stability and the country's economic health, the S&P 500 index, symbolized by the SPY, was selected. Table 2 presents selected variables, macroeconomic indicators, and other relevant data sources for the research question.

The US Treasury bill rate is viewed as a risk-free interest rate due to its empirical relevance and theoretical basis for explaining crude oil returns. Here are some reasons why these factors have been chosen:

- SPREAD: The yield spread is a measure of risk and investor sentiment, reflecting market expectations for future economic conditions. It can be used to capture market sentiment and its impact on crude oil returns, as oil prices are sensitive to economic changes. The US Treasury spread is particularly useful in this context.
- GEPU: The study explores the correlation between oil price volatility, economic policy uncertainty, and crude oil returns, highlighting the significant impact of these factors on investment decisions, global economic growth, and geopolitical stability.
- INFLATION: Inflation affects the purchasing power of consumers and can impact the oil demand. Higher inflation may increase production costs and reduce consumer spending, potentially affecting oil prices. We have explored the impact of inflation on crude oil returns by including it as a factor.
- PROD: Changes in industrial production can reflect overall economic activity and oil demand. Industries

heavily reliant on oil as an input may experience fluctuations in production levels, which can, in turn, influence oil prices. Understanding the relationship between industrial production and crude oil returns requires taking it into account as a component.

- CCU: Exchange rate fluctuations, particularly with major currencies like the euro, can affect oil prices. This can impact the affordability of oil for different countries and influence demand. Including currency fluctuation as a factor allowed us to explore the relationship between exchange rates and crude oil returns.
- M1SL: Changes in the money supply can have an impact on overall economic activity, which, in turn, can affect the oil demand. Considering the narrow money supply, we have examined its relationship with crude oil returns and assessed its influence on oil market dynamics.
- UNRATE: This reflects labor market conditions and can indicate overall economic health. High unemployment may affect consumer spending and oil demand. Incorporating the unemployment rate as a consideration helped comprehend the relationship between crude oil returns.
- VIX: The VIX index measures market volatility and investor sentiment. High levels of market volatility can impact oil prices as they affect investor risk appetite and investment decisions. We have investigated VIX's impact on crude oil returns by including it as a factor in the analysis.

We classify the variables into two categories: market fundamentals and economic indicators where, INDPROD, M1SL, CCU, SP, GPR, GPE, WUPI, and GEPU; and DGS3MO, DGS5, CPIAUCSL, UNRATE, and VIX.

1) *Econometric approach*: This work is divided into two stages: during the first stage, we examine the excess return over time, and in the second stage, we analyze the excess return's cross-section components. Our study's central presumptions are that markets are efficient, events cannot be predicted, and time is affected exogenously.

Eq. (1) can be extended to Eq. (2), which formulates the excess return on WTI.

$$\begin{aligned} R_{wti} = \alpha_1 + \beta_M R_{Mt} + \beta_1 SPREAD_t t \\ + \beta_2 INDPRO_t \\ + \beta_3 INFLATION_t \\ + \beta_4 UNRATE_t + \beta_5 M1SL_t \\ + \beta_6 CCUS_t + \beta_7 VIX_t \\ + \beta_8 GPR_t + \beta_9 WUPI_t \\ + \beta_{10} GPE_t + \beta_{11} GEPU_t + \varepsilon_t \end{aligned} \tag{2}$$

Here,

- R_{wti} = the excess ROC; the risk-free rate which is the 3-month US Treasury bill rate here, was deducted from the continuously compounded returns to transform the WTI returns into excess returns,
- R_{Mt} = the excess market return on the stock; thus, excess ROC is affected by the excess market return R_Mt, and the coefficient β_M. We have used SP500 as an excess market return. 〖SP〗_t = the excess market return,
- $SPREAD_t$ = 5-year minus 3-month treasury yield curve,
- β = coefficient,
- ε_t = error term.

All the other important risk factors that have been identified in the literature as having an impact on WTI returns.

B. Model and Econometric Approach

APT focuses on the unanticipated changes in macroeconomic factors rather than their levels. In line with this principle, we started with the naive assumption that investors' expectations for the future value of the variables would remain unchanged. Thus, the change is the variation in the variable from one period to the next. Eq. (3) displays the monthly logarithmic excess returns for WTI, where the 3-month U.S. Treasury rate is the risk-free rate.

$$ER_{wti\,(t)} = Log\,(p_{wti\,(t)} \;/\; p_{wti\,(t-1)}) - r_f \tag{3}$$

In Eq. (3),

- subscriptions $ER_{wti\,(t)}$ is the excess return of WTI at time t,
- subscriptions $p_{wti\,(t)}$ is the price of WTI at time t,
- subscriptions the price of WTI at time t − 1,
- r_f is the 3-month U.S. Treasury rate.

The monthly excess returns are calculated by subtracting the monthly yield on a three-month US Treasury bill from the continuously compounded daily returns on the WTI Index. The log changes of the data are used to express the macroeconomic elements that function as predictors. The VIX and SP are both expressed in levels.

We have used Eq. (4) to calculate the daily log changes:

$$\begin{aligned} VIX_t = log\,(VIXt_t \;/\; VIX_{t-1})\; and\; SP_t \\ = log\,(SP_t \;/\; SP_{t-1}) - r_f \end{aligned} \tag{4}$$

Where VIX_t and SP_t are the daily log change at time t, VIX_t and SP_t are the values at time t, and VIX_{t-1} and SP_{t-1} are the values at time $t-1$.

Rest all the factors are standardized using $\{log\ Difference\ (X_t) - log\ Difference\ (X_{t-1})\}$, where X is the respective factor and X_{t-1} is the value at the previous time. Appendix 1 presents the descriptive statistics for all the variables. Rest all the factors are standardized using, where X is the respective factor and is the value at the previous time. Appendix 1 includes the variance inflation factor (VIF), the Augmented Dickey-Fuller (ADF) unit root test empirical statistics, and the Jarque-Bera (JB) test for normality. The VIF values for the independent variables are all < 3.0, indicating that multi-collinearity is not present [49]. The ADF tests demonstrate that all series are stationary. Many of the variables show skewness and a high amount of kurtosis. Large excess kurtosis coefficients, which is leptokurtosis, are a sign that outliers are present and indicate that there have been numerous price changes in the past (either positive or negative) away from the average returns for the investment. Despite a positive mean reflecting favorable results on the average return for investors, the negative skewness (Fig. 1) shows that more negative data is concentrated on the mean value.

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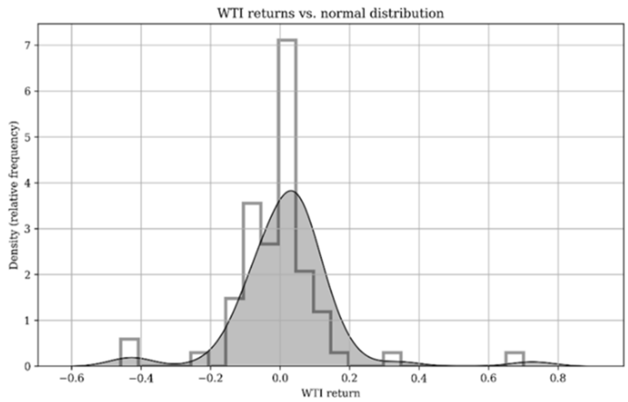

Fig. 1 Skewed target distribution

According to the large standard deviation (σ) values associated with various variables, the pandemic crisis of 2020–21 and the post–crisis period makes up over half of the data set. At the 5% significance level, the JB test statistics reject the null hypothesis (H_0) of a normal distribution for all series.

Considering the minimum values, the lowest in this range is $UNRATE$, with a minimum value of -131.38. $GEPU$ is much more dispersed than other variables, with a standard deviation of 43.40; closely following this are the GPR at 30.93, $UNRATE$ at21.29, and $MONEY$ at 20.32. Negative values for skewness are common (SP, $CURRENCY$, $MONEY$, $UNRATE$, $INFLATION$, GPR, and $SPREAD$) but are positive for the $INDPRO$, $PANDEMIC$, GPE, VIX, and $GEPU$. Most of these factors show excess kurtosis. To develop a new coordinate system and align it with the largest variation in the data, Principal Component Analysis (PCA) was carried out. The results are displayed in the next section.

The Value at Risk (VaR) is determined (Table 3) using simple returns, which represent the worst-case loss associated with probability, and the CVaR is estimated by averaging the severe losses in the tail of the WTI distribution.

TABLE III
WTI VALUE AT RISK.

	VaR	Conditional VaR
90%	-0.10	-0.22
95%	-0.13	-0.30
99%	-0.43	-0.43

The original data was altered using the quantile normalization process to retain the desired variance and eliminate any unintended variation brought on by technical flaws.

Fig. 3 displays the percentage of eigenvalues assigned to each component. This shows the importance of every element to the analysis.

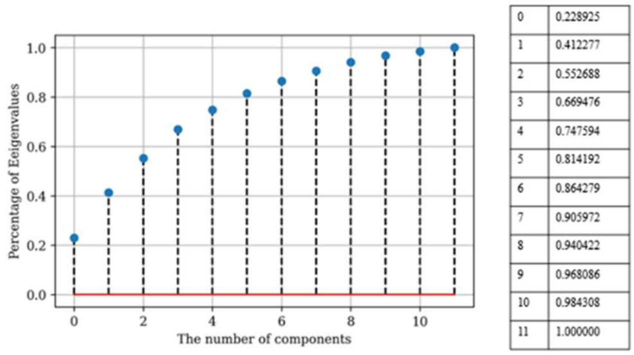

Fig. 2 Boxplot of data after Quantile Normalization.

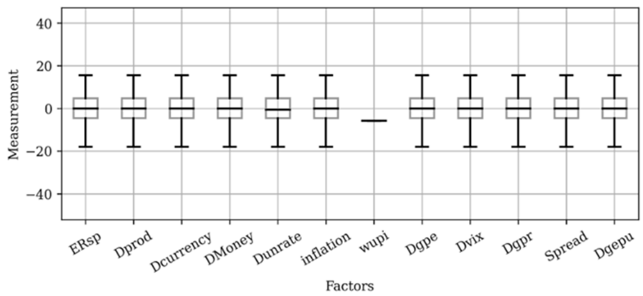

Fig. 3 Percentage of Eigenvalues attributable to each factor.

III. RESULTS AND DISCUSSIONS

Table 4 reports the regression estimation ($Qn0.5$) based on Eq. 4. The diagnostic tests were performed on the conditional median quantile, which has been treated here as the estimation results for the baseline regression.

TABLE IV
REGRESSION ESTIMATION

Variable	Coef	Std err	$P > \lvert t \rvert$
$erSP$	-0.17	0.07	0.01
$dPROD$	0.14	0.06	0.03
$dCURRENCY$	0.04	0.04	0.36
$dMONEY$	-0.10	0.06	0.09
$dUNRATE$	0.30	0.04	0.00
$dINFLATION$	0.12	0.07	0.07
$dWUPI$	-0.04	0.04	0.32
$dGPE$	0.34	0.06	0.00
$dVIX$	-0.47	0.08	0.00
$dGPR$	0.06	0.05	0.20
$dSPREAD$	0.06	0.05	0.26
$dGEPU$	0.08	0.04	0.04
Intercept	-0.01	0.28	0.96
Pseudo R^2	0.5185		

The asymmetry in the model can be seen by comparing the coefficients of various quantiles. A few parameter estimations, e.g., "$Dcurrency$", "$DMoney$", "$inflation$",

“ $wupi$ ”, “ $Dgpr$ ”, and “ $Spread$ ” variables, are not statistically distinct from zero.

$$hypotheses = "Dcurrency = DMoney = inflation = wupi = Dgpr = Spread = 0"$$

The 'D' prefix is added to the relevant dataset after differencing to stabilize the series. The F-value is a measure of the overall fit of the model. A higher F-value suggests a better fit. F = 3.74 and p = 0.003. The low p-value indicates that at least one of the coefficients in the hypothesis is not equal to zero, meaning that the set of variables jointly has a significant effect on the dependent variable.

Heteroscedasticity was assessed using the Breusch-Pagan test. Table 5 reports the test results.

TABLE V
BREUSCH-PAGAN HETEROSKEDASTICITY TEST.

Test statistic	p-value	f-value	f(p-value)
46.71	0.000	10.6969	0.000

p-values < 0.05, indicating a fundamental problem with heteroscedastic errors. Fig. 6 displays the residual vs. prediction error plot, though no clear pattern is visible; however, the Jarque-Bera normality assumption test was performed to ensure the correctness of our assumption. According to Fig. 6, the pandemic produced an early drop in prices in 2020-21, followed by a sharp surge as producers lowered supply and demand increased. The assumption is satisfied because the Durbin-Watson test result of 1.98 indicates that there is no autocorrelation. However, the Breusch-Godfrey (BG) test was employed too, which identifies the autocorrelation up to any predetermined order p. The null hypothesis of BG shows no serial correlation of any order up to p.

TABLE VI
BREUSCH-GODFREY TEST FOR AUTOCORRELATION.

Statistic	p-value	f-value	p-value	lag
36.52	0.00	1.124	0.362	6
39.48	0.00	0.926	0.530	12
52.48	0.00	1.806	0.064	24
65.68	0.00	11.104	0.006	48
65.90	0.065	22.323	0.012	50

Table 6 displays the test statistic $x^2 = 36.52$ and $pvalue = 0.000$, indicating we can reject H_0 and conclude that autocorrelation exists among the residuals at some order less than or equal to 6 lags. We have tested 12, 24, 48, and 50 lags and found a $p - value > 0.05$ at lag 50, where H_0 cannot be rejected. However, considering the seasonal correlation, we have considered adding seasonal dummy variables to the model.

Following that, a normality test was run on the residuals.

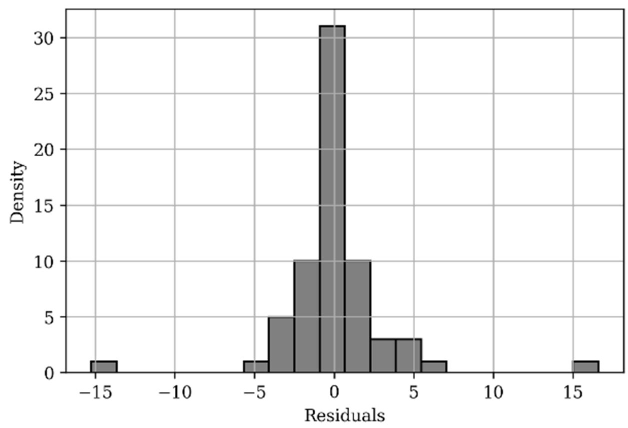

Fig. 4 Distribution of residuals.

We observe from Fig. 4 that the distribution of the residuals roughly resembles a bell shape, although there are a few large outliers that could lead to a significant skewness. However, to ensure the normality assumption, we further checked the QQ plot displayed in Fig. 5, followed by statistical tests displayed in Table 7. The QQ plot indicates a non-normal residual distribution.

TABLE VII
NORMALITY TEST.

Description	Statistics	p-value	output
Shapiro-Wilk Test	0.727	0.00***	data do not look normal (reject H0)
D’Agostino’s K2 Test	30.99	0.00***	data do not look normal (reject H0)
Jarque Bera test	485.86	0.00***	data do not look normal (reject H0)
Anderson Darling test	4.764	critical values = array ([0.546, 0.622, 0.746, 0.870, 1.035]); significance level=array ([15., 10., 5., 2, 1.]). The test results are significant at every significant level, which means H0 can be rejected. Thus, data are not normally distributed.	

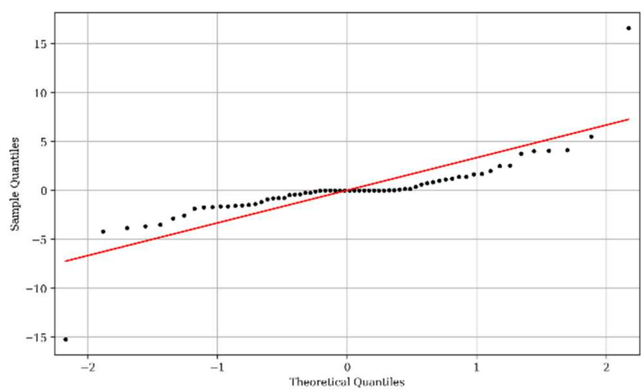

Fig. 5 QQ plot

Fig. 6 displays the regression residuals and fitted series. Numerous significant outliers can be spotted in the graph, but the largest one is in 2020.

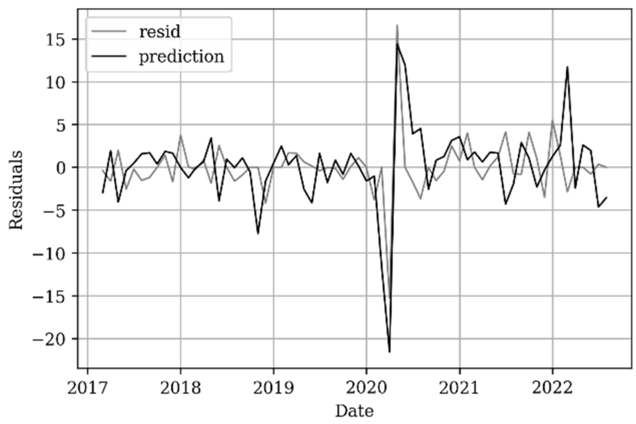

Fig. 6 Regression residuals and fitted series.

Table 8 displays the values for the residuals studied to determine the precise dates when the largest outliers were realized. The two most extreme residuals were in April'20 and May'20. These residuals represent unique or critical events, outliers, or anomalies in the data that have a big impact on WTI returns. The inclusion of dummy variables for these residuals allows the model to adjust and account for these influential observations properly.

TABLE VII
DUMMY VARIABLES CONSTRUCTION.

Date	**Smallest residuals**
Dummy exogenous variables	
2020-04-01	−15.243
2020-05-01	16.589

Due to the perfect fit of the dummy variables to the two extremely outlying observations, the rerun of the regression along with the dummy variables significantly increased the pseudo$-R^2$ value from 0.58 to 0.72. Appendix II reports the estimates of the QR. The distributions were divided into four different quantiles (i.e., $\tau = 0.25, 0.50, 0.75, \& \ 0.90$) to get a mixed variety of low, medium, and high return conditions. Fig. 7 displays the diagnostic plot, where it can be observed that the errors follow a normal distribution. This has effectively established a baseline model to estimate the effect of the event on our target variable.

Furthermore, both missing variables and an inappropriate functional form were discovered using the RESET (Ramsey Regression Equation Specification Error Test). An F-value of 0.248 and a corresponding p-value of 0.620 from the data show that we cannot rule out H_0 that the model contains no omitted variables. To ascertain whether there is a structural break in the data at any given moment, the CUSUM test [42] for parameter stability based on OLS residuals was carried out.

Table 9 displays the cumulative total and cumulative sum of squares of recursive residuals to test the structural stability of the models. The absence of any structural breaks is the null hypothesis. The test statistic and associated p-value (0.90) suggest that H_0 cannot be rejected, and the coefficients are stable over time; this confirms that the model does not have a structural break for any possible break date in the sample.

TABLE IX
PARAMETER STABILITY TEST.

Test statistic	0.56
p-value	0.905
Critical values	$[(1, 1.63), (5, 1.36), (10, 1.22)])]$

A. Causality Analysis

Next, causal impact analysis reduces the noise and provides real statistical insight which leads to the confidence to move forward with. The average value of the response variable is 1.36. If the intervention had not occurred, it was expected that the average response would have been 3.21. The response variable had an overall value of 43.6 when the post-intervention period's data points were added together. But if the intervention had not happened, we would have anticipated a total of 116.77 in absolute terms, with a confidence interval of [80.29, 154.44].

With an upper and lower bound of [-94.96, -31.46], the response variable showed a relative decline of -62.7%. This demonstrates that the detrimental impact seen during the intervention period is statistically significant. Fig. 8 displays the causal impact analysis plot. The Bayesian one-sided tail-area probability of getting this result by chance is exceedingly low ($p = 0.0$). This indicates that the causal effect is statistically significant.

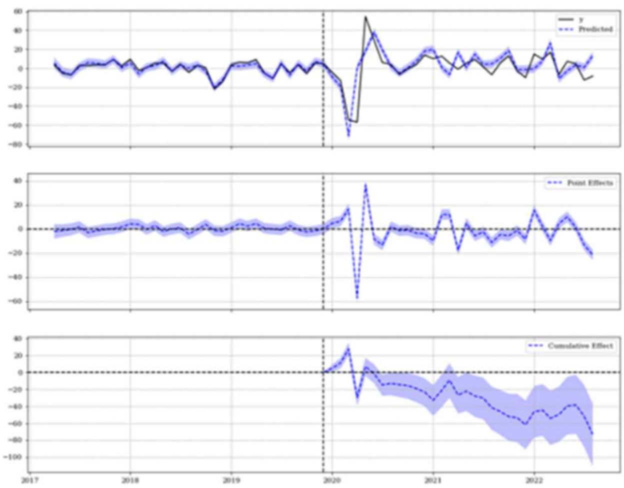
Fig. 7 Causal impact plot

B. Discussions

The quantile analysis found the following intriguing trends: PROD, INFLATION, GPE, and GEPU have a positive and significant impact on the ROC at both the 25% and 50% levels which suggests robust relationships and not just limited to a particular quantile level. This implies that when the market is bullish, these variables have a substantial impact on the return on the asset, and investors need to consider these factors when making investment decisions. The intercept term appeared negative for the lower and median quantiles, which suggests that, on average, WTI returns are negative or below zero at these quantiles, even when the predictor variables are set to zero. This is primarily because of pandemic panic and supply chain disruption during the pandemic phase.

Table 10 presents a complete discussion of each factor based on the QR analysis displayed in Table 9.

TABLE X
EMPIRICAL ESTIMATION.

Variables	Causality analysis
SP	The negative estimate of the coefficient implies that at the 50th quantile, the SP return has a negative effect on the WTI return, whereas at other quantiles, there is no meaningful effect. This seems logical considering the specific combination of conditions that led to this link between the SP return and the WTI return at the 50th quantile during and after the pandemic. Statistically significant positive connections at lower quantiles have been reported by researchers [14]; however, a strong negative relationship between SP and WTI returns during the pandemic was reported [27] which is in line with our findings.
PROD	The coefficient estimates for INDPRO are significant across all quantiles of the WTI return distribution, suggesting the relationship with WTI returns is consistent across different levels of returns. This implies that a strong industrial sector is associated with a higher ROC. This is in line with the researchers who found a positive cointegrated relationship between the INDPRO and oil prices ([28], [29]).
CURRENCY	Positive and statistically significant estimates at the median and 3rd quantile show that when the value of the US dollar goes up, WTI returns go up at the median and 3rd quantile of the distribution. Similar findings were reported by researchers in the recent past ([[30], [2]). The median and 3rd quantiles of our data set correspond to the height of a devastating pandemic supply chain disruption. One possible inference from this relationship is that changes in the value of the US dollar can affect the price of WTI, which in turn can have implications for the wider economy.
M1SL	The effect of M1SL on WTI's return is strongest at the median level but weaker at other levels. This finding may have important implications for investment strategies. The investors may want to adjust their investment strategies, accordingly, depending on whether they expect WTI return to be below or above the median level. However, no evidence of a long-run relationship can be drawn from this. This finding supports a recently concluded study where cointegration was tested on US assets and the money supply [31].
UNRATE	Several studies (e.g., [32], [20],[33]) looked at the link between the unemployment rate and the WTI return, and they have come to different conclusions about the size and direction of the link. During the period of our investigation, we found no statistically significant effect. However, additional research is required to completely comprehend the nature of this link and its operating processes, which is outside the scope of this work.
INFLATION	There is a strong and positive link between inflation and WTI return in the 1st and middle quantiles. Even though a complete analysis of the elements that contribute to oil price volatility, including inflation, implied that the link between these variables can be influenced by a variety of supply and demand factors in the oil market [34], the relationship between the oil price and inflation was further evaluated by researchers and reported a positive and statistically significant relationship between both variables at specific quantiles [35].
WUPI	The epidemic had no meaningful effect on the WTI return across all quantiles of our data. This suggests that, while the pandemic caused some volatility in the WTI price, it did not produce a continuous trend in either direction that would have had a significant impact on the WTI return. Our findings are consistent with the significant works ([36], [37], and [1]), which found that the pandemic did not affect WTI returns across all quantiles.
GPE	All the quantiles show favorable and significant outcomes of GPE. This demonstrates the relationship between the global energy price index and the WTI return, both of which are impacted by the dynamics of supply and demand, geopolitical events, and global economic conditions. Given the close relationship between the WTI return and the global price of energy index, this positive relationship is not unexpected
GEPU	The study reveals that the influence of GEPU on WTI return is stronger in the lower and middle ranges of the WTI return distribution, but weaker in the upper range. This suggests that economic policy uncertainty can significantly affect the oil market during market volatility or stress, while its impact may be less pronounced during market stability or good performance. This is consistent with the previous findings [39].
VIX	VIX indicates a significant negative correlation between WTI return and volatility, indicating an inverse relationship between volatility and returns in financial markets. Increased volatility leads to risk-averse investors selling off riskier assets, potentially resulting in lower returns.
GPR	The study reveals that geopolitical risk's impact on WTI return may be stronger at certain levels of the WTI return distribution. This finding is consistent with the idea that GPR can have a more pronounced impact on the oil market during times of market uncertainty or instability when investors are more sensitive to political and economic events. At the same time, the impact of GPR may be less pronounced during periods of market stability or when the market is performing well.
SPREAD	SPREAD is significant and positive in the 50th and 90th quantiles. Similar reporting was found where researchers specifically examined the relationship between oil and stock market returns through QR and reported similar findings [40]. It suggests that SPREAD yields have a stronger effect on WTI returns when the returns are in the upper half or top decile of their distribution.

The critical findings are summarized as:

- the market return ($erSP$) has a negative effect on crude oil returns at the median and 90th quantiles, but not at the lower or higher quantiles. Production ($dPROD$), global economic policy uncertainty ($dGPE$), and the treasury yield curve ($dSPREAD$) all have positive effects on crude oil returns across all quantiles.
- the money supply ($dMONEY$) has a large negative effect on crude oil returns at the 25^{th} quantile.
- $dUNRATRE$ has a positive effect on WTI returns but is not significant at any of the quantiles (Qn 0.25, Qn 0.50, Qn 0.75, and Qn 0.90). This indicates that the unemployment rate may have some influence on WTI returns but does not reach statistical significance in this model.
- $dINFLATION$ has a considerable positive effect on crude oil returns at the 25th and 50th quantiles, but not at higher quantiles. The correlation between the inflation rate and WTI returns may be nonlinear. The inflation rate may have a greater effect on returns while they are lower (e.g., during recessions), but as returns rise (e.g., during expansions of the economy), its effect may become less pronounced or level out.
- VIX volatility index ($dVIX$) has a major negative impact on crude oil returns at the 25th, 50th, and 90th quantile. Crude oil returns typically suffer negative effects at various levels of the WTI return distribution when market volatility and fear are high (as evidenced by a higher VIX).
- Other factors, such as currency exchange rates ($dCURRENCY$), the pandemic index ($dWUPI$), and the geopolitical risk ($dGPR$), have mixed or minor effects on crude oil returns across quantiles.
- the returns on crude oil at all quantiles are significantly impacted by the month dummies $D\_Apr'20$ and $D\_May'20$. The significance of variables suggests that these extreme deviations have a significant effect on the overall relationship between the predictors and WTI returns.

The pseudo-R^2 values are high, indicating that the model fits the data well. Since economic theory does not say which parts or how many should be used in the study, many possible variables could be considered. Our empirical findings have implications for portfolio design and risk management for investors. It also has significant implications for risk management decisions involving hedging and downside risk, given that the financial utility of oil varies depending on market conditions. Finally, our findings have implications for the forecasting of COP across quantiles based on macroeconomic and financial variables. Furthermore, changes in the several parameters considered for this study account for almost 2/3 of the monthly fluctuation in the excess returns.

IV. Conclusion

This study used an asset pricing model that combined Arbitrage Pricing Theory (APT) and Quantile Regression (QR) to assess the risk-return relationship of WTI crude oil. To evaluate the risk-return connection of WTI crude oil, the model used multivariate risk components and market returns (SP 500). The report finds that market returns, industrial production, global economic policy uncertainty, and the Treasury yield curve have significant positive effects on crude oil returns across all quantiles. The study reveals that the money supply, unemployment rate, inflation rate, and VIX volatility index have significant negative and positive effects on the returns of WTI at different quantiles. The combination of APT and QR provides a comprehensive understanding of the risk-return relationship of the WTI, capturing both linear and nonlinear relationships. The study found that the SP 500 market return is not a significant predictor of WTI returns, suggesting a weak or non-linear relationship. Other key factors, such as PROD, inflation, GPE, and GEPU, have a more significant impact on WTI returns. Since longer-term economic or geopolitical events affect the relationship between the SP 500 return and the WTI return, the analysis's time horizon may be too short to find a meaningful relationship. The results can help identify profitable investment opportunities and make strategic investment decisions. However, developing a robust empirical model requires iteration and is not a precise science. Thus, regular iteration, refining, and testing the model considering the real-life challenges are quite important. The process of developing a model must include sensitivity studies, back testing, and validation against out-of-sample data.

References

[1] L.T. Zhao, Y.Y. Xing, Q.-R. Zhao, and X.-H. Chen, "Dynamic impacts of online investor sentiment on international crude oil prices," Resources Policy, vol. 82, p. 103506, May 2023, doi:10.1016/j.resourpol.2023.103506.

[2] W. Zhang and S. Hamori, "Crude oil market and stock markets during the COVID-19 pandemic: Evidence from the US, Japan, and Germany," International Review of Financial Analysis, vol. 74, p. 101702, Mar. 2021, doi: 10.1016/j.irfa.2021.101702.

[3] X. Zhang, Z. Zhang, and H. Zhou, "Oil price uncertainty and cash holdings: Evidence from China," Energy Economics, vol. 87, p. 104732, Mar. 2020, doi: 10.1016/j.eneco.2020.104732.

[4] M. Yu, M. Umair, Y. Oskenbayev, and Z. Karabayeva, "Exploring the nexus between monetary uncertainty and volatility in global crude oil: A contemporary approach of regime-switching," Resources Policy, vol. 85, p. 103886, Aug. 2023, doi: 10.1016/j.resourpol.2023.103886.

[5] Z. Xiao, H. Guo, and M. S. Lam, "Quantile Regression and Value at Risk," Handbook of Financial Econometrics and Statistics, pp. 1143–1167, Aug. 2014, doi: 10.1007/978-1-4614-7750-1_41.

[6] Y. Wei, S. Qin, X. Li, S. Zhu, and G. Wei, "Oil price fluctuation, stock market and macroeconomic fundamentals: Evidence from China before and after the financial crisis," Finance Research Letters, vol. 30, pp. 23–29, Sep. 2019, doi: 10.1016/j.frl.2019.03.028.

[7] K.H. Wang, C.W. Su, Y. Xiao, and L. Liu, "Is the oil price a barometer of China's automobile market? From a wavelet-based quantile-on-quantile regression perspective," Energy, vol. 240, p. 122501, Feb. 2022, doi: 10.1016/j.energy.2021.122501.

[8] K.H. Wang, C.W. Su, and M. Umar, "Geopolitical risk and crude oil security: A Chinese perspective," Energy, vol. 219, p. 119555, Mar. 2021, doi: 10.1016/j.energy.2020.119555.

[9] K.-H. Wang, L. Liu, X. Li, and L. Oana-Ramona, "Do oil price shocks drive unemployment? Evidence from Russia and Canada," Energy, vol. 253, p. 124107, Aug. 2022, doi: 10.1016/j.energy.2022.124107.

[10] Y. Sun, P. Gao, S. A. Raza, N. Shah, and A. Sharif, "The asymmetric effects of oil price shocks on the world food prices: Fresh evidence from quantile-on-quantile regression approach," Energy, vol. 270, p. 126812, May 2023, doi: 10.1016/j.energy.2023.126812.

[11] J. Sørensen & V. Johansen, "Relationship Between Money Supply and Asset Prices in Developed Countries ," 2021.

[12] S. Singhal, S. Choudhary, and P. C. Biswal, "Return and volatility linkages among International crude oil price, gold price, exchange rate and stock markets: Evidence from Mexico," Resources Policy, vol. 60, pp. 255–261, Mar. 2019, doi: 10.1016/j.resourpol.2019.01.004.

[13] S. J. H. Shahzad, M. A. Naeem, Z. Peng, and E. Bouri, “Asymmetric volatility spillover among Chinese sectors during COVID-19,” International Review of Financial Analysis, vol. 75, p. 101754, May 2021, doi: 10.1016/j.irfa.2021.101754.

[14] A. A. Salisu, R. Swaray, and T. F. Oloko, “Improving the predictability of the oil–US stock nexus: The role of macroeconomic variables,” Economic Modelling, vol. 76, pp. 153–171, Jan. 2019, doi:10.1016/j.econmod.2018.07.029.

[15] A. A. Salisu, A. Olaniran, and J. P. Tchankam, “Oil tail risk and the tail risk of the US Dollar exchange rates,” Energy Economics, vol. 109, p. 105960, May 2022, doi: 10.1016/j.eneco.2022.105960.

[16] R. A. Ratti and J. L. Vespignani, “Oil prices and global factor macroeconomic variables,” Energy Economics, vol. 59, pp. 198–212, Sep. 2016, doi: 10.1016/j.eneco.2016.06.002.

[17] K. P. Prabheesh, R. Padhan, and B. Garg, “COVID-19 and the Oil Price – Stock Market Nexus: Evidence From Net Oil-Importing Countries,” Energy Research Letters, vol. 1, no. 2, Jul. 2020, doi: 10.46557/001c.13745.

[18] W. Ploberger and W. Kramer, “The Cusum Test with Ols Residuals,” Econometrica, vol. 60, no. 2, p. 271, Mar. 1992, doi: 10.2307/2951597.

[19] Z. Pan, Y. Wang, C. Wu, and L. Yin, “Oil price volatility and macroeconomic fundamentals: A regime switching GARCH-MIDAS model,” Journal of Empirical Finance, vol. 43, pp. 130–142, Sep. 2017, doi: 10.1016/j.jempfin.2017.06.005.

[20] O. R. Olayeni, A. K. Tiwari, and M. E. Wohar, “Global economic activity, crude oil price and production, stock market behaviour and the Nigeria-US exchange rate,” Energy Economics, vol. 92, p. 104938, Oct. 2020, doi: 10.1016/j.eneco.2020.104938.

[21] A. M. and J. Nayar, “Integration of oil with macroeconomic indicators and policy challenges in regard to Oman,” International Journal of Energy Sector Management, vol. 14, no. 1, pp. 172–192, Jan. 2020, doi: 10.1108/ijesm-08-2018-0006.

[22] P. K. Narayan, “Oil Price News and COVID-19—Is There Any Connection?,” Energy Research Letters, vol. 1, no. 1, Jun. 2020, doi:10.46557/001c.13176.

[23] K. Mokni, “A dynamic quantile regression model for the relationship between oil price and stock markets in oil-importing and oil-exporting countries,” Energy, vol. 213, p. 118639, Dec. 2020, doi: 10.1016/j.energy.2020.118639.

[24] D. G. McMillan, S. A. Ziadat, and P. Herbst, “The role of oil as a determinant of stock market interdependence: The case of the USA and GCC,” Energy Economics, vol. 95, p. 105102, Mar. 2021, doi: 10.1016/j.eneco.2021.105102.

[25] F. Malik and Z. Umar, “Dynamic connectedness of oil price shocks and exchange rates,” Energy Economics, vol. 84, p. 104501, Oct. 2019, doi: 10.1016/j.eneco.2019.104501.

[26] M. Mahmoudi and H. Ghaneei, “Detection of structural regimes and analyzing the impact of crude oil market on Canadian stock market: Markov regime-switching approach,” *Studies in Economics and Finance*, vol. 39, no. 4, pp. 722–734, Jun. 2022, doi: 10.1108/SEF-09-2021-0352.

[27] M. Mahmoudi and H. Ghaneei, “Detection of structural regimes and analyzing the impact of crude oil market on Canadian stock market: Markov regime-switching approach,” Studies in Economics and Finance, vol. 39, no. 4, pp. 722–734, Jun. 2022, doi: 10.1108/SEF-09-2021-0352.

[28] L. Liu, E.-Z. Wang, and C.-C. Lee, “Impact of the COVID-19 Pandemic on the Crude Oil and Stock Markets in the US: A Time-Varying Analysis,” Energy Research Letters, vol. 1, no. 1, May 2020, doi: 10.46557/001c.13154.

[29] F. Liu, M. Umair, and J. Gao, “Assessing oil price volatility co-movement with stock market volatility through quantile regression approach,” Resources Policy, vol. 81, p. 103375, Mar. 2023, doi:10.1016/j.resourpol.2023.103375.

[30] T.H. Le and Y. Chang, “Oil price shocks and trade imbalances,” Energy Economics, vol. 36, pp. 78–96, Mar. 2013, doi:10.1016/j.eneco.2012.12.002.

[31] N. Köse and E. Ünal, “The effects of the oil price and oil price volatility on inflation in Turkey,” Energy, vol. 226, p. 120392, Jul. 2021, doi: 10.1016/j.energy.2021.120392.

[32] L. Kilian, “Oil Price Shocks: Causes and Consequences,” Annual Review of Resource Economics, vol. 6, no. 1, pp. 133–154, Nov. 2014, doi: 10.1146/annurev-resource-083013-114701.

[33] M. Bohdalová and M. Greguš, “Impact Of Uncertainty on European Market Indices Quantile Regression Approach,” CBU International Conference Proceedings, vol. 5, pp. 57–61, Sep. 2017, doi:10.12955/cbup.v5.902.

[34] R. Ferrer, S. J. H. Shahzad, R. López, and F. Jareño, “Time and frequency dynamics of connectedness between renewable energy stocks and crude oil prices,” Energy Economics, vol. 76, pp. 1–20, Oct. 2018, doi: 10.1016/j.eneco.2018.09.022.

[35] Y. Guo, C. Yu, H. Zhang, and H. Cheng, “Asymmetric between oil prices and renewable energy consumption in the G7 countries,” Energy, vol. 226, p. 120319, Jul. 2021, doi:10.1016/j.energy.2021.120319.

[36] R. Aloui, S. Hammoudeh, and D. K. Nguyen, “A time-varying copula approach to oil and stock market dependence: The case of transition economies,” Energy Economics, vol. 39, pp. 208–221, Sep. 2013, doi:10.1016/j.eneco.2013.04.012.

[37] D. H. Husaini and H. H. Lean, “Asymmetric impact of oil price and exchange rate on disaggregation price inflation,” Resources Policy, vol. 73, p. 102175, Oct. 2021, doi: 10.1016/j.resourpol.2021.102175.

[38] X. Chen, “Are the shocks of EPU, VIX, and GPR indexes on the oil-stock nexus alike? A time-frequency analysis,” Applied Economics, vol. 55, no. 48, pp. 5637–5652, Oct. 2022, doi:10.1080/00036846.2022.2140115.

[39] Y. T. Chan and Y. Dong, “How does oil price volatility affect unemployment rates? A dynamic stochastic general equilibrium model,” Economic Modelling, vol. 114, p. 105935, Sep. 2022, doi:10.1016/j.econmod.2022.105935.

[40] A. M. Herrera, M. B. Karaki, and S. K. Rangaraju, “Oil price shocks and U.S. economic activity,” Energy Policy, vol. 129, pp. 89–99, Jun. 2019, doi: 10.1016/j.enpol.2019.02.011.

[41] H. Alqaralleh, “Stock return-inflation nexus; revisited evidence based on nonlinear ARDL,” Journal of Applied Economics, vol. 23, no. 1, pp. 66–74, Jan. 2020, doi: 10.1080/15140326.2019.1706828.

[42] J. W. Jurek and E. Stafford, “The Cost of Capital for Alternative Investments,” The Journal of Finance, vol. 70, no. 5, pp. 2185–2226, Sep. 2015, doi: 10.1111/jofi.12269.

[43] N. T. Hung, “Asymmetric connectedness among S&P 500, crude oil, gold and Bitcoin,” Managerial Finance, vol. 48, no. 4, pp. 587–610, Feb. 2022, doi: 10.1108/mf-08-2021-0355.

[44] Z. Dai and J. Kang, “Bond yield and crude oil prices predictability,” Energy Economics, vol. 97, p. 105205, May 2021, doi:10.1016/j.eneco.2021.105205.

[45] A. Dutta, D. Das, R. K. Jana, and X. V. Vo, “COVID-19 and oil market crash: Revisiting the safe haven property of gold and Bitcoin,” Resources Policy, vol. 69, p. 101816, Dec. 2020, doi:10.1016/j.resourpol.2020.101816.

[46] J. F. Hair, G. T. M. Hult, C. M. Ringle, M. Sarstedt, and K. O. Thiele, “Mirror, mirror on the wall: a comparative evaluation of composite-based structural equation modeling methods,” Journal of the Academy of Marketing Science, vol. 45, no. 5, pp. 616–632, Feb. 2017, doi:10.1007/s11747-017-0517-x.

[47] J. Beckmann and R. Czudaj, “Oil prices and effective dollar exchange rates,” International Review of Economics & Finance, vol. 27, pp. 621–636, Jun. 2013, doi: 10.1016/j.iref.2012.12.002.

[48] G. Bampinas and T. Panagiotidis, “Oil and stock markets before and after financial crises: A local Gaussian correlation approach,” Journal of Futures Markets, vol. 37, no. 12, pp. 1179–1204, Jul. 2017, doi:10.1002/fut.21860.

[49] D. Aloui, S. Goutte, K. Guesmi, and R. Hchaichi, “COVID 19’s impact on crude oil and natural gas S&P GS Indexes,” shs.hal.science, May 20, 2020. https://shs.hal.science/halshs-02613280.

[50] S. M. Hashmi, B. H. Chang, and N. A. Bhutto, “Asymmetric effect of oil prices on stock market prices: New evidence from oil-exporting and oil-importing countries,” Resources Policy, vol. 70, p. 101946, Mar. 2021, doi: 10.1016/j.resourpol.2020.101946.