Capability Study of Production Process Using SPC Tool: A Case Study

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Abstract

The three different capability measures give a different view of process capability. The normal curve application generally concentrates on the percentage of products that are out of specifications. The PCR is a very visual indicator of capability. When the ratio is less than 1 or the percentage is less than 100%, it indicates an image of distribution that are totally within specifications. When the percentage is more than 100%, an image of the distribution overlaps the specification limits. The combination of C_p *and* C_{pk} *is used extensively because it shows both the potential process capabilities. The process capability ratio (PCR) and process capability index (*C_{pk}*) for a drive shaft have been computed after stabilizing the manufacturing process. Data have been taken from the manufacturing process and the* $\overline{X}$ *& R control charts have been plotted to find whether the processes are with-in statistical control or out of statistical control. Process capability analysis has been done to find the effectiveness of the processes. It is found that the 7.35% of sprockets is beyond the control limits, which is really a higher rejection level and major concern for the industry.*

Key words: The $\overline{X}$ & R Control charts, Process capability index, capability Ratio, manufacturing industry.

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1. Introduction

Variation in the production process leads to quality defects and lack of product consistency. The Intel Corporation, the world's largest and most profitable manufacturer of microprocessors understands this. Therefore, Intel has implemented a program it calls "copy-exactly" at all its manufacturing facilities. The idea is that regardless of whether the chips are made in Arizona, New Mexico, Ireland, or any of its other plants, they are made in exactly the same way. This means using the same equipment, the same exact materials, and workers performing the same tasks in the exact same order. The level of detail to which the "copy-exactly" concept goes is meticulous. For example, when a chip making machine was found to be a few feet longer at one facility than another, Intel made them match. When water quality was found to be different at one facility, Intel instituted a purification system to eliminate any differences. Even when a worker was found polishing equipment in one direction, he was asked to do it in the approved circular pattern. Why such attention to exactness of detail? The reason is to minimize all variation. Now let's look at the different types of variation that exist.

No two products are exactly alike because of slight differences in materials, workers, machines, tools, and other factors. These are called common, or random, causes of variation. Common causes of variation are based on random causes that we cannot identify. These types of variation are unavoidable and are due to slight differences in processing. The second type of variation that can be observed involves variations where the causes can be precisely identified and eliminated. These are called assignable causes of variation. Examples of this type of variation are poor quality in raw materials, an employee who needs more training, or a machine in need of repair. In each of these examples the problem can be identified and corrected. Also, if the problem is allowed to persist, it will continue to create a problem in the quality of the product. Quality can mean different things to different people and can be interpreted in a variety of ways by an individual. From a manufacturing stand point quality is simply conformance to specification. Quality can also be linked to customer satisfaction. Some companies have used that definition for years, but there is now a broad move toward defining quality as total customer satisfaction.

SPC can be applied wherever work is being done. Initially it was applied to just production processes, but it has evolved to the point where it is applied to any work situation where data can be gathered. As companies work toward a total quality goal. SPC is used in more diverse situations. SPC involves the use of statistical signals to identify sources of variation, to improve performance, and to maintain control of processes at higher quality levels. The statistical concepts that are applied in SPC are very basic and can be learned by everyone in the organisation. All workers must know how SPC applies to their specific jobs and how it can be used to improve their output. Supervisors must be aware of the ways SPC can be used in their sections; they must create and maintain a management style that emphasizes communication and cooperation between levels and between departments. The work of various researchers on the applications of control charts and statistical process control in different applications is summarized in the next section.

2. Literature Review

Shewhart (1931) was the first, who suggested the $\overline{X}$ chart; using two control limits. Since then, various researchers did a lot of work on SPC techniques and control charts and it is discussed in this section.

Barnett (1984) explained the statistical essential involved in using C_p and C_{pk} indices to assess product quality. These capability indices particularly latter, have much appeal as they seemingly wrap up quality into the calculation of a single number. He discussed confidence interval, definitions and provided some quantitative results; He also urged to be cautious while using these indices in practice. Chaudhry and Higbie (1989) examined the implementation and use of statistical process control in a chemical and plastic firm. They studied the factors which are associated with the practical implementation of statistical process control, and discussed their effects on the output. They discussed the important components of SPC process in context of their achievements at a manufacturing facility. They also discussed the benefits achieved from the successful implementation of SPC. Wu (1996) presented an approach to determine the optimum control limits of the x-bar chart for skewed process distributions. The approach takes both the control limits of the x-bar chart and the specification limits of x-bar into consideration, and relates the out-of-control status directly with the nonconforming products. The proposed approach may be applied to industries to reduce the average number of scrap products, without increasing the type I error in statistical process control (SPC).

Goh (2000) outlined the functions of statistical tools and examined the steps in which they are adopted by non-statisticians in industry. A "seven S" approach is explained, highlighting a strategy for the effective deployment of statistical quality engineering. In a manufactured product attainment of superior quality and reliability depends upon the existence of a framework integrating an organisation's capabilities in management, technology and information utilization. With respect to information utilization, statistical tools are particularly essential for optimizing product and process performance. MacCarthy and Wasusri (2002) reviewed non-standard applications of SPC charts reported in the literature from the period 1989 to 2000, inclusive. Non-standard applications are analysed with respect to application domain, data sources used and control chart techniques employed. The principal application domain for statistical process control (SPC) charts has been for process control and improvement in manufacturing businesses. Chan et al. (2003) combined and studied the performance of individual X-charts and $\overline{X}$ charts. Traditionally, according to Juran and others, $\overline{X}$ charts are more sensitive than individual X-charts. They claimed that the finding seems to be useful for practitioners in quality control. Khoo (2005) proposed a semicircle control chart that can be used in detecting both increases and decreases in the mean and/ or variance. In his work, he proposed two modified semicircle charts for detecting a reduction in the process variance, a.k.a. process improvement. Each of these modified semicircle charts, namely, SC1 and SC2 has two limits, defined by the inner and outer semicircles. Prajapati and Mahapatra (2006) proposed the new design approach of $\overline{X}$ control chart for detecting the process shift by introducing two more limits known as warning limits. The concept of proposed $\overline{X}$ chart is based upon chi-square ($\chi 2$) distribution. They compared the performance of proposed chart with Shewhart $\overline{X}$ control chart, Dermon-Ross: two of two, Derman-Ross: two of three schemes. They found that ARLs of proposed $\overline{X}$ chart are lower than Shewhart $\overline{X}$ and Dermon-Ross and Klein's schemes at feasible range. Mattias et al. (2008) contributed to the understanding of how statistical process control (SPC) methodology can be implemented and used in organisational setting. An action model was used. Data were collected through formal meeting protocols, interviews and participant observation. Based on the results of an action research project, the paper emphasizes the need for: top management support with respect to roles such as infrastructural assistance, mentor, critic, financer; creating system validity through the involvement of people with experiential knowledge about the "world" in which SPC should be applied; keeping a small, highly knowledgeable development team with appropriate expertise together during the whole process from beginning to end; keeping the various end-users in focus but separate and prioritising between their different needs; and working with iterative design methodology. Shishebori and Hamadani (2009) investigated the statistical propertied of the estimated MC_p with respect to measurement capability considers the effect of gauge measurement capability on the lower confidence bound, hypothesis testing, critical value and power of testing for MC_p at the mentioned state. The aim of this paper is to consider the effect of gauge measurement capability on the multivariate process capability index (MC_p). The results show that gauge measurement capabilities will notably change the results of estimating and testing the process capability index. The research would help quality experts to determine whether their processes meet the required capability, and to make more reliable decisions.

Jose et al. (2010) demonstrated the relationship between the overall equipment effectiveness (OEE) and process capability (PC). These measures however are traditionally applied separately and with different purposes. They investigated the relationship between OEE and PC, how they interact and impact each other, and the possible effects that this relationship may have on decision making. They reviewed the OEE and PC background. Then a discrete-event simulation model of a bottling line is developed. Using the model, a set of experiments are run and the results interpreted using graphical trend and impact analyses. Abdolshah et al. (2011) presented a review of loss-based PCIs such as C_{pm}, C_{pmk}, PCI_θ, C_{pc}, L_e and L''_e. They also discussed the characteristics of loss-based PCIs such as reject based, asymmetric, bounded, loss based and target based. Finally they recommended development of a new loss-based process capability index with more excellent specifications. Das (2012) discussed the use of generalized lambda distribution to handle non-normal data. Traditional control chart has been established based on the assumption of normality. In many practical situation assumption of normality is violated. Under these situations, the use of traditional control chart gives erroneous conclusion. But for handling non-normal data one approach is use of non-parametric control charts which are not so efficient. Another approach is to use generalised distribution is very effective in non-normal data. Amiri et al. (2013) discussed the limitations of MEWMA control chart in spite of its ability to detect small shifts in the process with multiple quality characteristics due to its high cost of implementation. They applied and optimized two multi-objective approaches, an aggregative and a non-aggregative approach using a genetic algorithm. They evaluated the proposed approaches through a numerical example from the literature and the efficiency of the multi-objective approaches are verified in comparison with the previous methods. Prajapati and Singh (2014) studied the effect of the autocorrelation on the process dispersion. They suggested a modified R chart for sample size of four at different levels of correlation. The performance of the modified R chart in terms of Average Run Lengths (ARLs) is computed for sample sizes (n) of 4 for various shifts in the process standard deviation. They observed that for a particular sample size, when the level of correlation (Φ) increases, the performance of the modified R chart deteriorates.

3. Products and Industry

This firm is the collaboration of twelve industries and one of the largest firms of the fastener manufacturing. It is situated in the northern India. This company is established in 2011. This company supplies the products to: Hero Honda, Honda, Bajaj, Maruti-Suzuki, Yamaha, Bajaj, TVS and Mushashi etc.

There are many products which are manufactured by this firm, like axles, bearing, nut and bolts, cams, brakes, drive shaft, kick gear starter, sprocket cam drive, axles, bearings etc.

3.1 Sprocket cam drive manufacturing process

Sprocket is a fastener; which is used in motorcycles. It is supplied to T.V.S. and it is made of low carbon steel. The specifications of sprockets are: (i) Inner diameter: 20.032 mm -20.045 mm, (ii) Outer diameter – 22.3 mm (iii) Number of teeth =14

Sprockets for various applications are shown in Figure 1.

Figure 1: sprockets

The defects occur in the manufacturing of sprockets is oversize and undersize of its inner diameter that leads to rejection and rework and ultimately increase the production cost. The manufacturing process of drive shaft is shown in Figure 2.

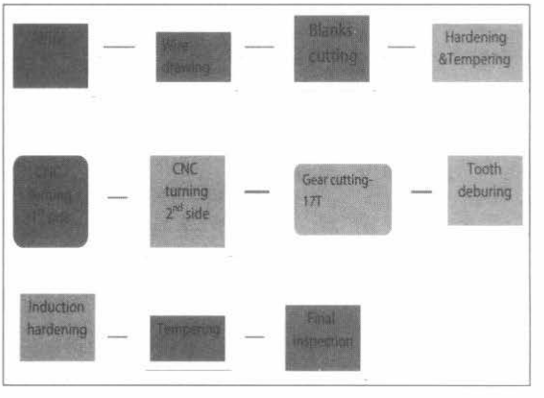

Figure 2 Flow diagram of sprocket cam drive manufacturing process

4. Methodology

The $\overline{X}$ and R charts has been used to know whether the process is with-in control or out of control. The measurements were taken from in-line process. The observations from the manufacturing line of 150 samples; using sample size of 4; are taken. These data is shown in the Table 1A in Appendix 'A'. The procedure of implementing of $\overline{X}$ and R charts is discussed in the following section.

4.1 Procedure of implementation of $\overline{X}$ & R charts

The following step-by-step procedure is followed to implementing of $\overline{X}$ and R charts.

Step 1 Determine the quality characteristics to be measured.

Step 2 Determine the sample size and lot size.

Step 3 Samples are taken at random basis.

Step 4 Calculate the average and range for each sample and record them in a tabular form.

Step 5 Calculate the process average ($\overline{X}$) and Upper control limit (UCL) and lower control limit (LCL) by using the following relation.

$$\overline{X} = \frac{\Sigma \bar{x}}{N}$$

Where,

$\overline{X}$ = Average of the averages of all samples

$\bar{x}$ = Average of each sample

N = Number of samples taken.

Next, find the average of range values by using the formulas given below

$$\overline{R} = \frac{\Sigma Ri}{N}$$

Ri = Range of individual samples = Max Value – Min Value

$\overline{R}$ = The average of ranges of all the samples range.

Control limits for $\overline{X}$ chart are:

$$UCL_{\overline{X}} = \overline{X} + A2\,\overline{R}$$

$$LCL_{\overline{X}} = \overline{X} - A2\,\overline{R}$$

Control limits for R chart are:

$$UCL_R = D4 \times \overline{R}$$

$$LCL_R = D3 \times \overline{R}$$

Where,

$UCL_{\overline{X}}$, $LCL_{\overline{X}}$ = Upper and lower control limits for $\overline{X}$ chart respectively.

UCLR, LCLR = Upper and lower control limits for R chart respectively.

D3, D4, A2 = Constants, depend upon sample size.

Step 6 Draw $\overline{X}$ and R charts by using above values.

Step 7 Observe and interpret the pattern of the points on $\overline{X}$ and R charts. If there are any points, falling beyond the control limits, delete/ignore them and find the new mean and control limits for $\overline{X}$ and R charts. This procedure may be continued till all the points are falling within control limits of $\overline{X}$ and R charts and the process may be called in statistical control.

Step 8 Calculate the process capability of the process that is in statistical control, as shown in section 4.3.

4.2 Plotting of data points on $\overline{X}$ and R charts

The 150 observations of drive shaft run-outs are taken and computations of various parameters are presented in this section as follows:

Mean ($\overline{X}$) Chart:

$$\overline{X} = \frac{\overline{X}1 + \overline{X}2 + \overline{X}3 + \overline{X}4}{4}$$

Similarly, Mean or Average of 150 samples can be calculated as,

$$\text{Samples mean } (\overline{X}) = \frac{\Sigma(\overline{XI}.........\overline{XN})}{N}$$

Where, N is the number of subgroups = 150

$$\overline{X} = \frac{3005.31}{15D} = 20.03854 \text{ mm and Average}$$

range can be calculated as:

$$\overline{R} = \frac{\Sigma(\overline{RI}..........\overline{RN})}{N} = \frac{2.0595}{15D} = 0.00653$$

Upper control limit (UCL_X) = $\overline{X} + A_2 \times \overline{R}$

$= 20.03854 + 0.729 \times 0.00653 = 20.04330$

Lower control limit =$LCL_X = \overline{X} - A_2 \times \overline{R}$

$LCL_X = 20.03854 - 0.729 \times 0.00653$
$= 20.03379$

Range (R) Chart

Range (R) = $X_{max} - X_{min}$

$$\overline{R} = \frac{\Sigma(\overline{RI}..........\overline{RN})}{N} = \frac{2.0595}{15D} = 0.00653,$$

Where, N is the number of subgroups = 150 (for this case)

Upper control limit on R chart,

$UCLR = D4 \times \overline{R} = 2.282 \times 0.00653 = 0.01489$

Lower control limit on R chart,

$LCLR = D_3 \times \overline{R} = 0 \times 0.0137 = 0$, where $A_2 = 0.738$, $D_4 = 2.28$, $D_3 = 0$ (values of these factor, corresponding to sample size, are available in all the books of Quality control). The plot for $\overline{X}$ and R charts for sprocket cam drive for initial data are shown in Figure 3.

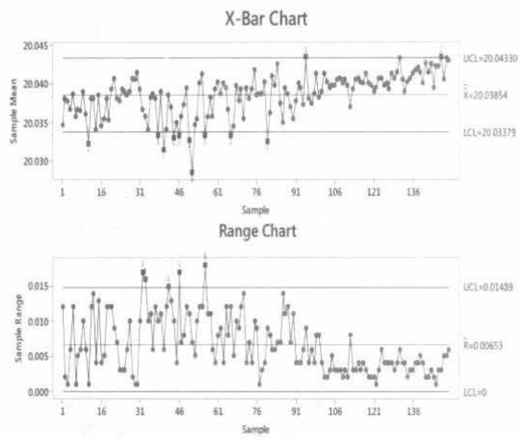

Figure 3: $\overline{X}$ and R charts for initial observed data

It is found from Figure 3 that observation numbers: 32, 33, 42, 46 and 56 are beyond upper control limit (UCL_R) on R chart. Similarly, 84-90, 101-111 and 113-150 points are below average line, making three Shifts (seven consecutive points above or below the R-bar line).

So the above observations need to be deleted and new values for range are calculated to check the statistical control of the process. The same procedure has been continued till all the points are within control limits on $\overline{X}$ and R charts. The final computed parameters of the statistical control of the process are as follows:

$\overline{X} = 20.03796$, $\overline{R} = 0.00738$

$\overline{X}$ - Chart

$UCL_{\overline{X}}$ = 20.04334 and $LCL_{\overline{X}}$ = 20.03259

R-chart

UCLR = 0.01683 and LCLR = 0

Figure 4 shows $\overline{X}$ and R charts of statistically controlled process of sprocket cam drive.

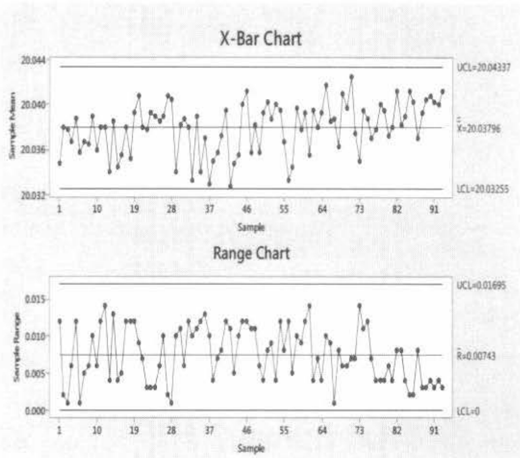

Figure 4: $\overline{X}$ and R charts of statistically controlled process

4.3 Process capability analysis

The capability analysis is the minimum spread of the process under controlled conditions. It is the measure that is frequently used in the normal distribution with the value of 3 times standard deviations from the mean. The R chart is a distribution of range (R) values and has a standard deviations SR. The upper control limit is 3 times standard deviations above the mean. The $\overline{X}$ chart is a distribution of $\overline{X}$ values and has a standard deviation $S_{\overline{X}}$. Normally, the control limits are set at 3 times the standard deviations from the mean.

$$\overline{X} = \frac{\Sigma(\overline{X1}.........\overline{XN})}{N}$$

LSL = Lower specification limit = 20.032

USL = Upper specification limit =20.045

From Figure 4,

$$\overline{X} = \frac{1863.53}{93} = 20.03796 \text{ mm}$$

For this case, N is the number of subgroups = 93 (for statistically controlled process for $\overline{X}$ chart)

$$\overline{R} = \frac{\Sigma(\overline{R1}........\overline{RN})}{N} = \frac{0.6882}{97} = 0.0074 \text{ mm}$$

For this case, N is the number of subgroups = 97 (for statistically controlled process for R chart)

Standard deviation is calculated as follow:

$$s = \frac{\overline{R}}{d2}$$

d2 =2.059 for sample size n =4.

$$s = \frac{0.0074}{2.059} = 0.0036$$

Upper control limit and lower control limit are calculated as follows:

Upper control limit = $UCL_x = \overline{X} + A2 \times \overline{R}$

= 20.03796 + 0.729 × 0.0074 = 20.04337

Lower control limit = $LCL_x = \overline{X} - A2 \times \overline{R}$

= 20.03796 - 0.729 × 0.0074 = 20.03255

$UCL_R = D4 \times \overline{R} = 2.282 \times 0.0074 = 0.01695$

$LCL_R = D3 \times \overline{R} = 0 \times 0.0074 = 0$

The area out of specification limits can be calculated as follow:

$$Z = \frac{X - \overline{\overline{X}}}{s}$$

$$Z = \frac{20.032 - 20.03796}{0.0036} = -1.66$$

The area corresponds to z = -1.66 to the lower side of the curve is 0.0485 (from normal distribution curve table). It means that 4.85 % is out of specification on the low side.

Now substitute the USL for the value of x.

$$Z = \frac{X - \overline{\overline{X}}}{s}$$

$$Z = \frac{20.045 - 20.03796}{0.0036} = 1.96$$

The area corresponds to z =1.96 to the upper side is 0.0250 (from normal distribution curve table). It means that 2.5% is out of specification limit. The total percentage of product out of specification is:

4.85% + 2.5%= 7.35%

4.3.1 Calculation of capability Ratio

Population standard deviation

$$s = \frac{\overline{R}}{d2}$$

Where d2 =2.059 (for sample size n =4)

$$s = \frac{0.0074}{2.059} = 0.0036$$

So, Process capability (Cp) = 6s
= 6×0.0036= 0.0216

To be process under control,

$(USL - LSL) \geq 6s$

$(20.045 - 20.032) \geq = 6 \times 0.0036 = 0.0216$

$0.013 \leq 0.0216$

So, process is out of control

The Capability ratio can also be calculated as:

$$PCR = \frac{6s}{USL - LSL}$$

$$PCR = \frac{6 \text{ x } 0.0036}{20.045 - 20.032}$$

$$PCR = \frac{6 \text{ x } 0.0036}{0.013}$$

PCR= 1.66

This indicates that 166% of tolerance is used by the distribution. This results an over lapping picture. Part of the product is out of control.

4.6.3 Calculation of process capability index (C_{pk})

There are two working version of capability index C_p and C_{pk}. The C_p is just the reciprocal of PCR.

$$C_p = \frac{Tolerance}{6s} = \frac{USL - LSL}{6s}$$

$$C_p = \frac{20.045 - 20.032}{6 \text{ x } 0.0036}$$

$$C_p = \frac{0.013}{6 \text{ x } 0.0036}$$

C_p = 0.60 or 60%

$$C_{pk} = \text{Minimum of}\left[\frac{USL - \overline{X}}{3s} \text{ or } \frac{\overline{\overline{X}} - LSL}{3s}\right]$$

The minimum occurs with the specification limit that is closest to $\overline{X}$. The $\overline{X}$ is closest to LSL.

$\overline{X}$ - LSL = 20.03796 – 20.032 = 0.00596

s = 0.0036

$$C_{pk} = \frac{\overline{X} - LSL}{3s}$$

$$C_{pk} = \frac{0.00596}{3 \times 0.0036}$$

$$C_{pk} = 0.552$$

$$\frac{1}{C_{pk}} = \frac{1}{0.552} = 1.81$$

It means that 181% of the tolerance is used and on the worst side, the distribution overlaps the specification limit.

According to the calculated values of PCR and C_{pk}, it may be designated as: "as a 'D' process as per the classification given in (Smith, 2003). Many industries are using a company standard of C_{pk} =1.33 and some have also set a goal of Cp = 2. So, the C_{pk} interpretation is that the company requirement is Cpk = 1.33 or more. If it is less than 1, then 100% inspection has to be instituted because there will always be some manufactured products which may be out-of-specifications.

5. Conclusions

Statistical process analysis helps the industry to improve the efficiency of the manufacturing processes to decrease the number of defective products and thus the industry may save a lot of re-work cost and valuable time in future, if it adopts the statistical process control. The $\overline{X}$ and R control charts have been used to find whether the processes are with-in statistical control or not. Process capability analysis has been done to find the effectiveness of the processes. It is found that the 7.35% of drive shaft run outs are beyond the control limits, which is really a higher rejection level and major concern for the industry. The process capability of this process is required to be improved by the management to reduce the major loss. Since, the calculations of process capabilities are based upon the normal distribution and one of the limitations of normal distribution is that it does consider the values of 'Z' beyond four.

References

1. Abdolshah, M., Yusuff, R.M., Hong, T.S. and Ismail, M.Y. (2011), "Loss-based process capability indices: a review", *International Journal of Productivity and Quality Management*, Vol. 7, No.1, pp. 1 – 21.

2. Amiri, A., Mogouie, H. and Doroudyan, M.H. (2013), "Multi-objective economic-statistical design of MEWMA control chart", *International journal of Productivity and Quality Management,* Vol. 11, No. 2, pp.131 – 149.

3. Barnett, N.S. (1984), "Process Control and Product Quality: The Cp and Cpk Revisited" *International Journal of Quality & Reliability Management,* Vol. 7 Issue: 5.

4. Chan C-I., Tse, A.C.B. and Yim, F.H.K. (2003), "Comparing and combining individual x-charts and x-bar charts", *International Journal of Quality & Reliability Management,* Volume 20, Number 7, pp. 827-835.

5. Chaudhary, S.S. and Higbie J.R. (1989), "Practical Implementation of Statistical Process Control in a Chemicals Industry", *International Journal of Quality & Reliability Management*, Vol. 6 Issue: 5.

6. Das, N. (2012), "Performance of control chart using generalised lambda distribution, *International journal of Productivity and Quality Management*, 2012 Vol.10, No.4, pp.411 – 427.

7. Goh, T.N. (2000), "Operating frameworks for statistical quality engineering", *International journal of quality and reliability management*, Vol. 17, issue 2, pp.180-188.

8. Jose, A., Steve, E., Barber, K.D. and Meier, H.S. (2010), "Overall equipment effectiveness (OEE) and process capability (PC) measures: A relationship analysis", *International Journal of Quality & Reliability Management*, Vol. 27 Issue 1, pp.48 – 62.

9. Khoo, M. B. C. (2005), "Two Modified Semicircle Control Charts for Detecting Process Improvement", *Quality Engineering*, Volume 17, Issue 3 , pages 453 – 458.

10. MacCarthy, B.L and Wasusri, T. (2002), "A review of non-standard applications of statistical process controls (SPC) charts", *International Journal of Quality & Reliability Management*, Vol. 19, Issue 3, pp. 295 – 320.

11. Mattias, E., Olsson, J. and Dahlgaard, J. J. (2008), "Implementing statistical process control: an organizational perspective", *International Journal of Quality & Reliability Management,* Vol. 25, Issue 6, pp.545 – 560.

12. Prajapati, D. R. (2006), "An Optimal Approach to Reduce the ARL of $\overline{X}$ Charts", *Journal for Practicing Managers (Udyog Pragati), NITIE, India,* Vol. 30, No. 3, pp. 44-51.

13. Prajapati, D.R. and Singh, S. (2014), "Optimal schemes of Standard Range (R) Chart for Auto-correlated Observations', *International Journal of Quality Engineering and Technology,* Vol. 4, No. 1, pp. 81-96,

14. Shishebori, D. and Hamadani, A. Z. (2009), "The effect of gauge measurement capability on MCp and its statistical properties", *International Journal of Quality & Reliability Management,* Vol. 26, Issue 6, pp.564 – 582.

15. Wu, Z. (1996), "Asymmetric control limits of the -bar chart for skewed process distributions", International Journal of Quality & Reliability Management, Vol. 13 Issue 9, pp.49 – 60.

Appendix A

Table 1A Observations of sprocket inner diameter (in mm)

Sample Nos.	X1	X2	X3	X4	$\overline{X}$	R
1	20.04	20.028	20.038	20.033	20.03475	0.012
2	20.037	20.039	20.038	20.038	20.038	0.002
3	20.038	20.037	20.038	20.038	20.03775	0.001
4	20.041	20.036	20.035	20.035	20.03675	0.006
5	20.045	20.033	20.033	20.044	20.03875	0.012
6	20.035	20.036	20.036	20.036	20.03575	0.001
7	20.035	20.035	20.04	20.037	20.03675	0.005
8	20.036	20.034	20.036	20.04	20.0365	0.006
9	20.045	20.041	20.035	20.035	20.039	0.01
10	20.034	20.04	20.034	20.036	20.036	0.006
11	20.033	20.032	20.032	20.032	20.03225	0.001
12	20.045	20.033	20.041	20.033	20.038	0.012
13	20.036	20.046	20.032	20.038	20.038	0.014
14	20.035	20.036	20.033	20.032	20.034	0.004
15	20.046	20.04	20.033	20.035	20.0385	0.013
16	20.035	20.035	20.032	20.036	20.0345	0.004
17	20.033	20.035	20.038	20.036	20.0355	0.005
18	20.033	20.035	20.045	20.039	20.038	0.012
19	20.028	20.04	20.039	20.034	20.03525	0.012
20	20.046	20.034	20.04	20.037	20.03925	0.012
21	20.036	20.041	20.045	20.041	20.04075	0.009
22	20.038	20.035	20.042	20.037	20.038	0.007
23	20.039	20.036	20.038	20.038	20.03775	0.003
24	20.038	20.041	20.039	20.039	20.03925	0.003
25	20.038	20.041	20.039	20.038	20.039	0.003
26	20.041	20.035	20.038	20.04	20.0385	0.006
27	20.035	20.045	20.04	20.036	20.039	0.01
28	20.042	20.04	20.041	20.04	20.04075	0.002
29	20.04	20.04	20.041	20.041	20.0405	0.001
30	20.041	20.042	20.041	20.042	20.0415	0.001
31	20.041	20.035	20.036	20.045	20.03925	0.01
32	20.028	20.035	20.039	20.045	20.03675	0.017
33	20.03	20.032	20.046	20.035	20.03575	0.016
34	20.028	20.035	20.038	20.035	20.034	0.01
35	20.038	20.045	20.034	20.036	20.03825	0.017
36	20.036	20.038	20.042	20.039	20.03875	0.006

Sample Nos.	X1	X2	X3	X4	$\overline{X}$	R
37	20.038	20.036	20.045	20.033	20.038	0.012
38	20.034	20.038	20.028	20.033	20.03325	0.01
39	20.037	20.045	20.04	20.034	20.039	0.011
40	20.028	20.034	20.033	20.031	20.0315	0.006
41	20.04	20.028	20.034	20.034	20.034	0.012
42	20.03	20.034	20.045	20.045	20.0385	0.015
43	20.046	20.033	20.033	20.036	20.037	0.013
44	20.027	20.037	20.035	20.033	20.033	0.01
45	20.033	20.035	20.037	20.035	20.035	0.004
46	20.045	20.032	20.028	20.028	20.03325	0.017
47	20.034	20.04	20.033	20.036	20.03575	0.007
48	20.036	20.042	20.034	20.037	20.03725	0.008
49	20.033	20.045	20.042	20.038	20.0395	0.012
50	20.031	20.028	20.033	20.039	20.03275	0.011
51	20.027	20.026	20.028	20.033	20.0285	0.007
52	20.033	20.034	20.038	20.034	20.03475	0.005
53	20.04	20.036	20.036	20.03	20.0355	0.01
54	20.04	20.046	20.034	20.04	20.04	0.012
55	20.045	20.034	20.04	20.046	20.04125	0.012
56	20.033	20.027	20.028	20.045	20.03325	0.018
57	20.029	20.039	20.035	20.04	20.03575	0.011
58	20.032	20.04	20.043	20.038	20.03825	0.011
59	20.034	20.033	20.039	20.037	20.03575	0.006
60	20.039	20.038	20.038	20.042	20.03925	0.004
61	20.045	20.037	20.04	20.039	20.04025	0.008
62	20.033	20.04	20.042	20.04	20.03875	0.009
63	20.038	20.041	20.039	20.042	20.04	0.004
64	20.039	20.039	20.034	20.046	20.0395	0.012
65	20.033	20.041	20.033	20.04	20.03675	0.008
66	20.028	20.037	20.04	20.028	20.03325	0.012
67	20.033	20.038	20.034	20.033	20.0345	0.005
68	20.037	20.045	20.042	20.035	20.03975	0.01
69	20.038	20.042	20.033	20.038	20.03775	0.009
70	20.045	20.033	20.04	20.039	20.03925	0.012
71	20.034	20.038	20.042	20.028	20.0355	0.014
72	20.042	20.039	20.038	20.039	20.0395	0.004
73	20.035	20.036	20.039	20.042	20.038	0.007
74	20.037	20.041	20.04	20.039	20.03925	0.004
75	20.042	20.048	20	20.039	20.04175	0.01

Sample Nos.	X1	X2	X3	X4	$\overline{X}$	R
76	20.042	20.039	20.04	20.033	20.0385	0.009
77	20.039	20.039	20.038	20.039	20.03875	0.001
78	20.038	20.04	20.037	20.04	20.03875	0.004
79	20.039	20.042	20.038	20.042	20.04025	0.004
80	20.037	20.033	20.032	20.028	20.0325	0.009
81	20.04	20.032	20.039	20.034	20.03625	0.008
82	20.045	20.04	20.04	20.039	20.041	0.009
83	20.039	20.042	20.042	20.036	20.03975	0.006
84	20.04	20.046	20.045	20.039	20.0425	0.007
85	20.039	20.033	20.038	20.04	20.0375	0.007
86	20.033	20.028	20.037	20.042	20.035	0.014
87	20.039	20.04	20.034	20.045	20.0395	0.011
88	20.04	20.045	20.037	20.033	20.03875	0.012
89	20.041	20.038	20.035	20.034	20.037	0.007
90	20.039	20.039	20.028	20.036	20.0355	0.011
91	20.036	20.04	20.036	20.039	20.03775	0.004
92	20.04	20.042	20.038	20.04	20.04	0.004
93	20.038	20.039	20.039	20.042	20.0395	0.004
94	20.037	20.04	20.034	20.038	20.03725	0.006
95	20.048	20.042	20.045	20.039	20.0435	0.009
96	20.039	20.035	20.039	20.039	20.038	0.004
97	20.038	20.04	20.038	20.034	20.0375	0.006
98	20.04	20.039	20.036	20.04	20.03875	0.004
99	20.046	20.04	20.038	20.041	20.04125	0.008
100	20.039	20.038	20.034	20.042	20.03825	0.008
101	20.039	20.037	20.039	20.041	20.039	0.004
102	20.041	20.042	20.04	20.042	20.04125	0.002
103	20.04	20.041	20.039	20.041	20.04025	0.002
104	20.038	20.04	20.039	20.041	20.0395	0.003
105	20.04	20.042	20.037	20.04	20.03975	0.005
106	20.038	20.04	20.04	20.041	20.03975	0.003
107	20.04	20.042	20.039	20.041	20.0405	0.003
108	20.039	20.042	20.04	20.042	20.04075	0.003
109	20.039	20.041	20.039	20.041	20.04	0.002
110	20.039	20.041	20.04	20.042	20.0405	0.003
111	20.039	20.041	20.039	20.04	20.03975	0.002
112	20.033	20.035	20.039	20.041	20.037	0.008
113	20.038	20.04	20.038	20.041	20.03925	0.003
114	20.039	20.041	20.04	20.042	20.0405	0.003

Sample Nos.	X1	X2	X3	X4	$\bar{X}$	R
115	20.04	20.043	20.039	20.041	20.04075	0.004
116	20.039	20.041	20.039	20.042	20.04025	0.003
117	20.04	20.042	20.038	20.04	20.04	0.004
118	20.04	20.042	20.04	20.043	20.04125	0.003
119	20.039	20.041	20.039	20.041	20.04	0.002
120	20.039	20.041	20.039	20.04	20.03975	0.002
121	20.038	20.04	20.038	20.04	20.039	0.002
122	20.039	20.04	20.039	20.04	20.0395	0.001
123	20.04	20.043	20.04	20.04	20.04075	0.003
124	20.04	20.044	20.038	20.041	20.04075	0.006
125	20.038	20.04	20.039	20.042	20.03975	0.004
126	20.038	20.042	20.038	20.042	20.04	0.004
127	20.037	20.04	20.039	20.041	20.03925	0.004
128	20.041	20.043	20.039	20.042	20.04125	0.004
129	20.039	20.042	20.04	20.042	20.04075	0.003
130	20.04	20.042	20.04	20.044	20.0415	0.004
131	20.044	20.046	20.04	20.043	20.04325	0.006
132	20.039	20.041	20.039	20.043	20.0405	0.004
133	20.037	20.041	20.038	20.04	20.039	0.004
134	20.04	20.041	20.039	20.041	20.0405	0.002
135	20.039	20.042	20.04	20.042	20.0405	0.003
136	20.04	20.042	20.04	20.043	20.0415	0.003
137	20.04	20.043	20.04	20.044	20.0415	0.004
138	20.04	20.043	20.041	20.044	20.042	0.004
139	20.039	20.042	20.041	20.044	20.0415	0.005
140	20.038	20.042	20.04	20.04	20.04	0.004
141	20.042	20.044	20.043	20.042	20.0425	0.002
142	20.043	20.041	20.041	20.041	20.0415	0.002
143	20.042	20.043	20.044	20.041	20.0425	0.003
144	20.039	20.041	20.039	20.039	20.0395	0.002
145	20.042	20.043	20.042	20.042	20.0425	0.001
146	20.041	20.044	20.041	20.043	20.0422	0.003
147	20.043	20.042	20.044	20.045	20.0435	0.003
148	20.042	20.043	20.038	20.039	20.0405	0.005
149	20.044	20.046	20.041	20.042	20.0435	0.005
150	20.046	20.04	20.042	20.044	20.043	0.006
					$\bar{\bar{X}}$ = 20.03854	R=0.00653