Refractive-index profiling of single-mode graded-index optical planar waveguides by the inverse Wentzel-Kramers-Brillouin method with improved accuracy

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Abstract. We demonstrate with examples a technique of improving the accuracy of the inverse Wentzel-Kramers-Brillouin (WKB) method for refractive-index profiling of single-mode graded-index optical planar waveguides.
[DOI: 10.1117/1.1907624]

Subject terms: graded-index waveguides; optical measurement; optical waveguide characterization; refractive-index profiling; thin-film characterization; Wentzel-Kramers-Brillouin method.

Paper 040523 received Aug. 05, 2004; revised manuscript received Oct. 31, 2004; accepted for publication Nov. 26, 2004; published online May 11, 2005.

1 Introduction

Dielectric thin films play an important role in modern science and technology. Numerous applications in the areas of optical coatings, protective coatings, integrated optics, etc. are concerned with thin films, and much chemistry and physics can be learned from studies of materials in thin-film form. While there are a large number of methods available for the measurement of the refractive index and the thickness of a thin film, only a few are capable of giving the refractive-index profile of a graded-index film.[1]

In integrated optics, several fabrication processes, such as ion exchange and ion diffusion, lead to graded-index waveguides. The Wentzel-Kramers-Brillouin (WKB) method (see, for example, Refs. 2 and 3) was once a popular method for the analysis of such waveguides. With the tremendous improvement in computational power over the years, the WKB method, which is an approximate method, has long been replaced by highly accurate numerical methods. The inverse WKB method for refractive-index profiling of graded-index waveguides,[4,5] however, remains popular, as the inverse problem cannot be solved conveniently by those numerical methods. In the inverse WKB method, the profile is calculated directly from the effective indices of the guided modes, which can be measured to a high accuracy by the prism-coupler method.[6] A version of the method[5] has in fact been implemented in a commercial prism-coupler system (Metricon Model 2010) for the characterization of thin films. However, the early methods[4,5] are applicable only to waveguides that support many guided modes. Recently, we have shown that, by combining the measurements for both the TE and TM modes and/or at different wavelengths,[7] the method can be applied to single-mode and two-mode waveguides. Unfortunately, the technique of combining the measurements for both mode types cannot be applied to waveguides that do not support both mode types or contain unknown material birefringence. The technique of combining the measurements at different wavelengths is more general, but it requires several laser sources and an accurate knowledge of the dispersion properties of the waveguide material. More recently, we have demonstrated a technique of profiling single-mode waveguides at a fixed wavelength by combining the effective indices measured with different external refractive indices.[8] However, because of the approximation in the WKB equation,[2,3,9] the accuracy of the method drops significantly for single-mode profiles with relatively large index differences. While a couple of techniques[10,11] have been proposed to improve the accuracy of the inverse WKB method, they have been developed only for multimode waveguides, in which case, the inverse WKB method is accurate enough and the improvement is not significant. In this paper, we demonstrate a technique of improving the accuracy of the inverse WKB method for the profiling of single-mode waveguides. A significant improvement in accuracy is achieved.

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2 Inverse WKB Method

We consider a graded-index profile $n(x)$, which is a monotonically decreasing function of x for $x \geqslant 0$ with a peak value n_0 at $x=0$. The substrate index at $x=+\infty$ and the external index for $x<0$ are denoted as n_s and n_e, respectively. The WKB equation is written as

$$k \int_0^{x_t(m)} [n^2(x)-N^2(m)]^{1/2}\,\mathrm{d}x=(m+0.25)\pi+\Phi(N,n_e), \tag{1}$$

with the phase change Φ given by

$$\Phi(N,n_e)=\arctan\{r_e[(N^2-n_e^2)/(n_0^2-N^2)]^{1/2}\}, \tag{2}$$

where $k=2\pi/\lambda$ is the free-space wave number with λ as the wavelength, m is the mode order, $N(m)$ is the effective index (propagation constant divided by k), and x_t is the turning point at which $n(x_t)=N(m)$. The constant r_e is equal to unity and $(n_0/n_e)^2$, respectively, for the TE_m and TM_m modes.

The inverse WKB method[5] is based on treating $N(m)$ as a continuous function of m (the effective-index function) so that it can be constructed by curve fitting of the measured effective indices $N(0)$, $N(1)$, $N(2)$,.... The profile $n(x)$ can then be recovered from the function $N(m)$ with an efficient algorithm.[5] The peak index at the surface n_0 is simply equal to $N(-0.75)$, i.e., by extrapolating[5] the curve $N(m)$ to $m=-0.75$. For a completely unknown profile shape, at least three effective indices must be available, although the accuracy increases with the number of effective indices available. To increase the number of points for curve fitting, we can combine the effective indices measured with different external refractive indices to construct the effective-index function[8] $N(m)$. To do that, we define an effective mode order $\bar{m}$ as

$$\bar{m}=m-\frac{\Phi(N,n_a)-\Phi(N,n_e)}{\pi}, \tag{3}$$

where n_a is a reference external index (air with $n_a=1$ is normally used as the reference). Using the new variable $\bar{m}$, the WKB equation, Eq. (1), can be written as

$$k\int_0^{x_t(\bar{m})}[n^2(x)-N^2(\bar{m})]^{1/2}\,dx=(\bar{m}+0.25)\pi+\Phi(N,n_a), \tag{4}$$

where $N(\bar{m})$ is the effective-index function that characterizes the refractive-index profile. When evaluated at the values of $\bar{m}$ given by Eq. (3), $N(\bar{m})$ gives the effective indices obtained with various external indices. With the introduction of the effective mode order, the effective indices obtained with different external indices become specific points of the same effective-index function. The effective-index function can therefore be constructed from a set of measurements with different external indices using the conventional algorithm, even if the waveguide supports only one guided mode.[8] In practice, to provide the external index needed, a suitable index-matching liquid can be applied thinly on the waveguide surface when the effective index is measured by the prism-coupler method.[8]

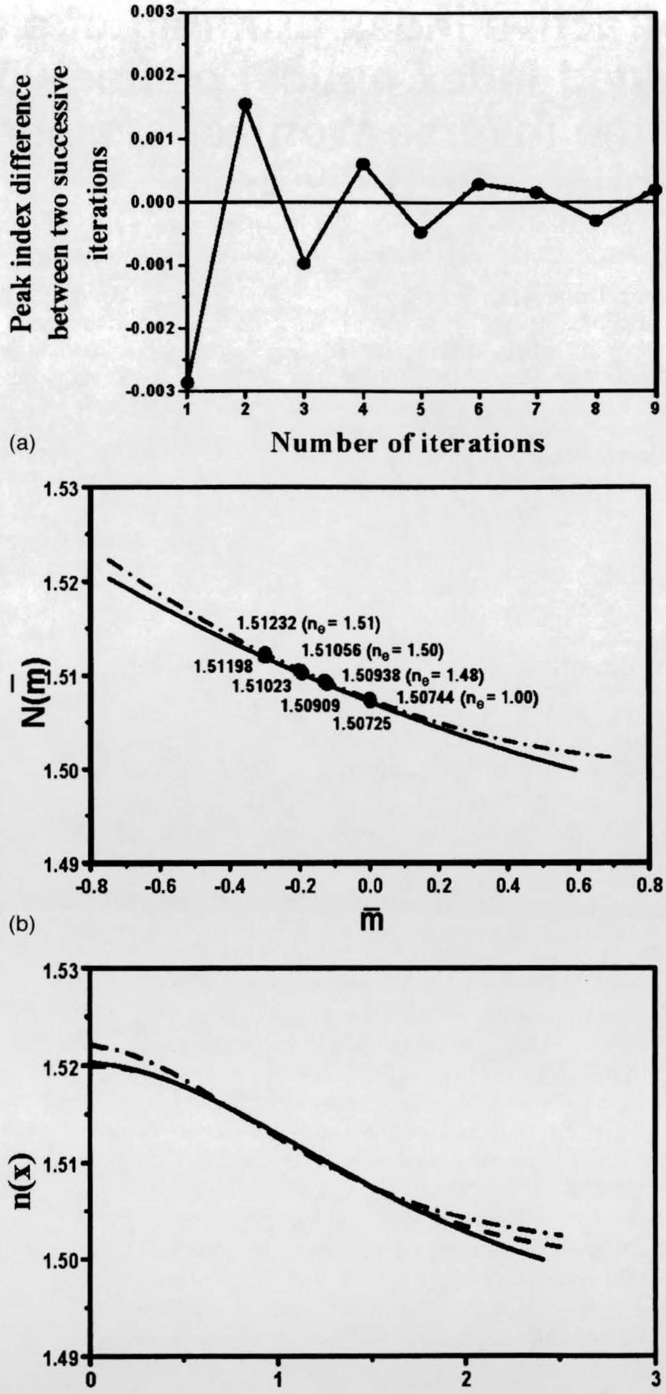

Fig. 1 Recovery of a single-mode Gaussian profile using effective indices obtained with different external refractive indices: (a) difference in the peak indices calculated from two successive iterations as a function of the number of iterations, (b) the effective indices (●) and the effective-index functions with error correction (solid line) and without error correction (dot-dashed line), and (c) the recovered profiles with error correction (solid) and without error correction (dot-dashed line) and the actual profile (dashed line).

3 Error Correction

In general, the effective index calculated by the WKB equation is smaller than the exact value and the discrepancy can be significant for single-mode waveguides.[9] Therefore, in applying the inverse WKB method, which relies on the one-to-one correspondence between the refractive-index profile and the effective-index function, if the effective-index function is constructed with exact effective indices, the recovered profile will contain errors due to the inaccuracy of the WKB equation. To eliminate such errors, the effective-index function should rather be constructed with the effective indices that are the solutions of the WKB equation (instead of the exact effective indices), so that the inverse of the WKB equation generates a profile that agrees more closely to the actual profile. In practice, we use the following procedure to correct such errors.

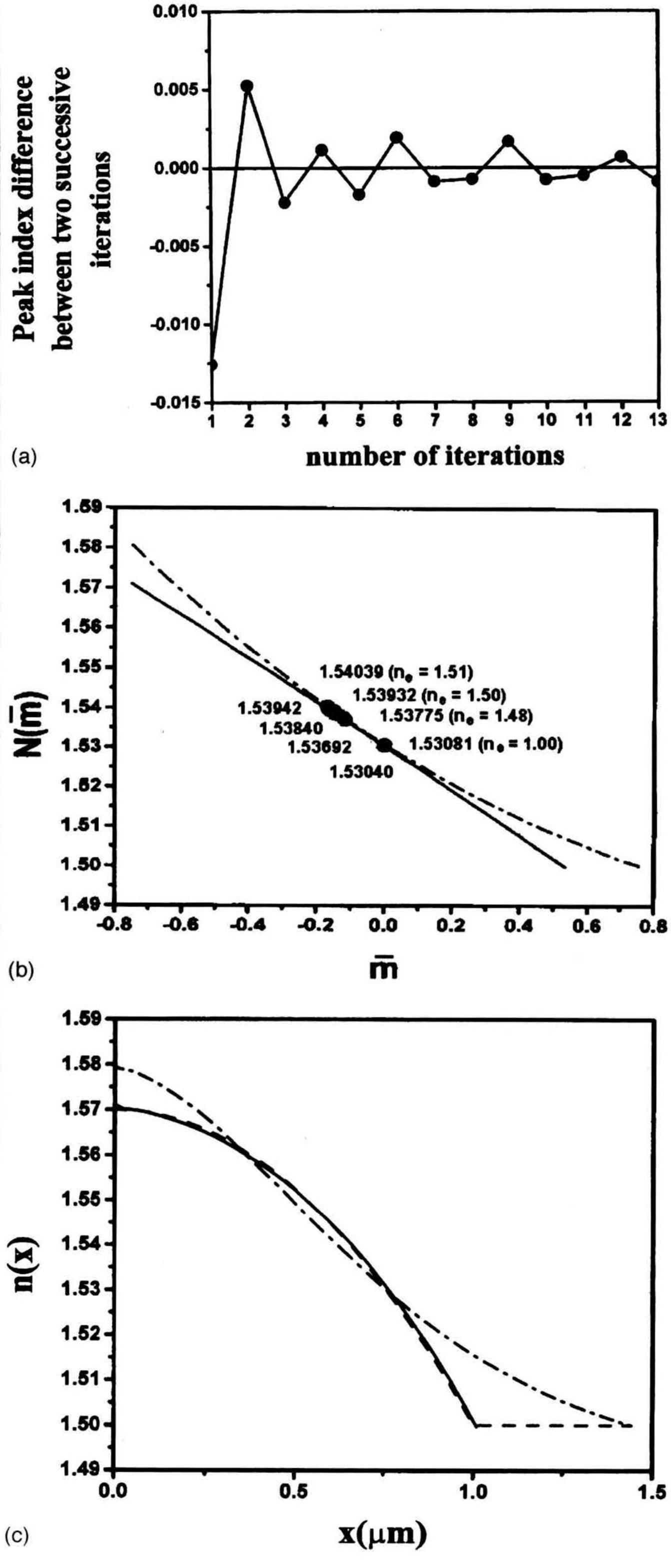

Fig. 2 Recovery of a single-mode truncated parabolic profile using effective indices obtained with different external refractive indices: (a) difference in the peak indices calculated from two successive iterations as a function of the number of iterations, (b) the effective indices (●) and the effective-index functions with error correction (solid line) and without error correction (dot-dashed line), and (c) the recovered profiles with error correction (solid line) and without error correction (dot-dashed line) and the actual profile (dashed line).

We start with the profile $n_1(x)$, which is constructed with the exact effective indices N_1 (in practice, the accurately measured effective indices) of the actual profile $n(x)$ using the inverse WKB method. The profile $n_1(x)$ thus contains errors due to the WKB approximation. To compensate for the errors, we calculate a set of corrected effective indices N_2 according to $N_2 = N_1 - (N_{E1} - N_1)$, where N_{E1} are the exact effective indices of the profile $n_1(x)$, which are obtained by solving the wave equation numerically. Here, $N_{E1} - N_1 (>0)$ are the errors in the WKB approximation for the profile $n_1(x)$ and N_2 should be closer to the solutions of the WKB equation for the actual profile than N_1. The profile constructed from the effective indices N_2, denoted as $n_2(x)$, is therefore expected to match the actual profile better. We can continue the process by constructing the profile $n_3(x)$ with the effective indices N_3, where $N_3 = N_2 - (N_{E2} - N_1)$ with N_{E2} being the exact effective indices of the profile $n_2(x)$. Since $n_2(x)$ is closer to the actual profile than $n_1(x)$, N_3 should be closer to the solutions of the WKB equation for the actual profile than N_2. Note that according to Eq. (3), the values of $\bar{m}$ are also modified slightly in each iteration. In general, we construct the profile $n_{j+1}(x)$ with the effective index N_{j+1}, where $N_{j+1} = N_j - (N_{Ej} - N_1)$ $(j = 1,2,\ldots)$. As N_{Ej+1} approach N_1, $n_{j+1}(x)$ approaches the actual profile $n(x)$. This condition could be used as a criterion to stop the iterative process. In practice, we use a simpler criterion, i.e., the convergence of the peak index calculated from the effective-index function as the criterion. A few iterations are usually sufficient to give good results. In the following, we present two numerical examples to demonstrate the performance of the method.

In the first example, we assume that the actual profile is a Gaussian profile given by $n(x) = 1.50 + 0.02 \exp[-(x/1.5)^2]$ for $x \geqslant 0$ in micrometers, which is single-moded in air at 632.8 nm. We first construct the profile with the effective indices for the TE_0 mode calculated exactly for four different external indices at 632.8 nm. We then use this profile as the initial profile to start the iterative procedure. In each iteration, the effective-index function is fitted with a second-order polynomial.[8] The difference in the peak indices calculated from two successive iterations as a function of the number of iterations is shown in Fig. 1(a). The calculated peak index converges rapidly, but the difference between two successive iterations does not decrease further when the number of iterations becomes large enough. This is due to the inherent errors arising from the determination of a continuous function with discrete points (i.e., interpolation and extrapolation), which are fundamental to the method and cannot be eliminated. The effective-index function and the effective indices calculated after six iterations are shown in Fig. 1(b) together with the initial ones without iteration. The corresponding recovered profiles are shown in Fig. 1(c), from which we can see that the recovered profile after six iterations matches the actual profile much better, compared with the initial recovered profile that has no error correction. Without error correction, the peak index of the recovered profile is $N(-0.75)=1.52224$, which is larger than the actual value 1.52 by 11% with respect to the index difference $n_0-n_s=0.02$. With error correction, the peak index obtained is $N(-0.75)=1.52036$, which corresponds to a discrepancy of only 1.8%.

In the next example, we consider a truncated parabolic profile given by $n(x)=1.50+0.07(1-x^2)$ for $0\leqslant x\leqslant 1$ and $n(x)=1.50$ for $1\leqslant x$, where x is in micrometers. The waveguide in air supports only the fundamental mode at 632.8 nm. The difference between the surface index and the substrate index is deliberately chosen to be large to amplify the errors in the WKB approximation and hence provides a more stringent test for the error correction method. Again, we first construct the profile with the effective indices for the TE_0 mode calculated exactly for four different external indices at 632.8 nm, and then use this profile as the initial profile to start the iterative procedure. In each iteration, the effective-index function is fitted with a second-order polynomial. The difference in the peak indices calculated from two successive iterations as a function of the number of iterations is shown in Fig. 2(a). Small oscillations after a few interactions similar to those shown in Fig. 1(a) can be seen. The effective-index function and the effective indices calculated after three iterations are shown in Fig. 2(b) together with the initial ones without iteration. The corresponding recovered profiles are shown in Fig. 2(c). The recovered profiles using more than three iterations do not differ much, and they all agree with the actual profile closely. On the other hand, the recovered profile that has no error correction deviates significantly from the actual profile. Without error correction, the peak index of the recovered profile is $N(-0.75)=1.58059$, which is larger than the actual value 1.57 by 15% with respect to the index difference $n_0-n_s=0.07$. With error correction, the peak index obtained is $N(-0.75)=1.57097$, which corresponds to a discrepancy of only 1.4%. The improvement in accuracy is similar to that in the previous example.

4 Conclusion

We outlined the inverse WKB method for index profiling of graded-index planar waveguides and demonstrated an iterative procedure to correct the errors in the recovered profile caused by the WKB approximation. This procedure can give significantly improved results for single-mode waveguides. With this development, the inverse WKB method becomes an even more powerful tool for the characterization of dielectric thin films.

References

1. P. W. Bohn and D. R. Miller, "Depth profiling in thin dielectric films," *Crit. Rev. Anal. Chem.* **22**(1–2), 455–470 (1991).
2. A. H. Hartog and M. J. Adams, "On the accuracy of the WKB approximation in optical dielectric waveguides," *Opt. Quantum Electron.* **9**, 223–232 (1977).
3. J. Janta and J. Ctyroký, "On the accuracy of WKB analysis of TE amd TM modes in planar graded-index waveguides," *Opt. Commun.* **25**(1), 49–53 (1978).
4. J. M. White and P. F. Heidrich, "Optical waveguide refractive index profiles determined from measurement of mode indices: A simple analysis," *Appl. Opt.* **15**(1), 151–155 (1976).
5. K. S. Chiang, "Construction of refractive-index profiles of planar dielectric waveguides from distribution of effective indices," *J. Lightwave Technol.* **LT-3**(2), 385–391 (1985).
6. P. K. Tien and R. Ulrich, "Theory of prism-film coupling and thin film light guides," *J. Opt. Soc. Am.* **60**, 1325–1337 (1970).
7. K. S. Chiang, C. L. Wong, H. P. Chan, and Y. T. Chow, "Refractive-index profiling of graded-index planar waveguides from effective indices measured for both mode types and at different wavelengths," *J. Lightwave Technol.* **14**(5), 824–832 (1996).
8. K. S. Chiang, C. L. Wong, S. Y. Cheng, and H. P. Chan, "Refractive-index profiling of graded-index planar waveguides from effective indices measured with different external refractive indices," *J. Lightwave Technol.* **18**(10), 1412–1417 (2000).
9. Q. Li and J. Wang, "A modified ray-optic method for arbitrary dielectric waveguides," *IEEE J. Quantum Electron.* **28**(12), 2721–2727 (1992).
10. R. Oven, S. Batchelor, D. G. Ashworth, D. Gelder, and J. M. Bradshaw, "Iterative refinement technique for reconstructing refractive index profiles from mode indices," *Electron. Lett.* **31**(3), 229–231 (1995).
11. Y. Ding, Z. Q. Cao, and Q. S. Shen, "Determination of optical waveguide refractive-index profiles with the inverse analytic transfer matrix method," *Opt. Quantum Electron.* **36**, 489–497 (2004).

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