

On Sensitivity of Structural Equation Modeling to Latent Relation Misspecifications

Tenko Raykov

*Department of Psychology
Fordham University*

By giving counterexamples where the covariance matrix provides crucial information about consequential model misspecifications, this article cautions researchers about overinterpreting Rogosa and Willett's (1985, p. 104) conclusion that "the covariance matrix is a severe summary of the longitudinal data, one that may discard crucial information about growth."

Structural equation modeling (SEM) methodology is an attractive means of studying behavioral, social, and educational phenomena. Research into its specifics has at the same time increased our awareness of its potential and limitations (e.g., Freedman, 1987; Rogosa, 1987; Rogosa & Willett, 1985). In a cautionary article 15 years ago, Rogosa and Willett (1985) used SEM in a longitudinal modeling context to fit a simplex structure to data arising from a constant rate of change (CRC) model. They concluded that "The covariance matrix is a severe summary of the longitudinal data, one that may discard crucial information about growth" (p. 104). The purpose of this article is to raise caution against overinterpreting this statement and particularly its generalization across longitudinal settings, by giving counterexamples where the covariance matrix provides crucial information about consequential model misspecifications. The intent is to contribute to the study of the potential of SEM to sense deviations from assumed latent relation patterns.

In the remainder of this article, a simulated linear change data set with two, rather than single, indicators at assessment points is considered. The example is

based on similar parameters as those presented by Rogosa and Willett (1985). Fitting a corresponding simplex structure, however, suggests serious model misspecifications. Subsequently, an example is presented with data following a quadratic true score relation in which model misspecifications cannot be sensed by the overall fit indexes or measures of variance accounted for both at the latent and observed levels. Rather, it is indicated by other parts of the output of the numerical minimization routine. Finally, empirical conditions are stated that can generally provide good chances for structural equation models to sense latent pattern misspecifications, and some issues pertaining to model fit evaluation in SEM are discussed.

FITTING A MULTIPLE-INDICATOR SIMPLEX STRUCTURE TO DATA FOLLOWING A CRC MODEL

Longitudinal modeling has recently attracted increased interest in the behavioral and social sciences, a trend that has followed a similar earlier tendency in biostatistics, applied statistics, and the life sciences (for a brief overview of leading lines of these research streams, see Collins & Horn, 1991; Gottman, 1996; Raykov, 1998).

CRC and Simplex Models

Within the repeated assessment context, Rogosa and Willett (1985) analyzed a covariance matrix arising from a CRC model that maximally violated Guttman's (1954, pp. 99–100, 105–106) condition for the simplex. The CRC model is based on the following latent relations:

$$\eta_{i,p} = \eta_{1,p} + \theta_p (t_i - t_1) \quad (1)$$

where $t_i = i$, $i = 1, 2, 3, 4, 5$ (using the same notation used in Rogosa and Willett's article). In Equation 1, η stands for latent variable, i is the repeated measurement index denoting five consecutive assessment occasions, and p is the person index ($p = 1, 2, \dots, 500$; $N = 500$ being the sample size). Thereby, $\eta_{1,p}$ is the true initial status of subject p ; that is, $\eta_{1,p}$ is his or her true score at i th assessment, $i = 2, 3, 4, 5$; and θ_p , $p = 1, 2, \dots, N$, is his or her rate of change over time, which remains constant across all repeated assessments. That is, at each of the five consecutive assessments there is possible variability among participants' rates θ_p of latent change, $p = 1, 2, \dots, N$, but within participants there is no variability over time in their rates of change between the measurement occasions.

Rogosa and Willett (1985) fit a simplex model (e.g., Jöreskog, 1970) defined with the latent relations

$$\eta_{i+1,p} = \beta_i \eta_{i,p} + \delta_{i+1,p}, i = 1, 2, 3, 4, p = 1, \dots, N \quad (2)$$

to the simulated covariance matrix presented in Table 1. In Equation 2, β_i is the occasion-specific true regression slope assumed constant across participants within assessment points, and δ_i is the structural residual at the i th measurement occasion, $i = 2, 3, 4, 5$. Rogosa and Willett (1985) postulated the observed variables as

$$Y_i = \eta_i + \varepsilon_i \quad (3)$$

where ε_i denotes measurement errors, $i = 1, 2, \dots, 5$; that is, the observed variables represented each a single indicator of the contemporaneous latent construct.

When fitted to the covariance matrix in Table 1, the simplex model defined by Equations 2 and 3 was found to exhibit close to perfect overall goodness-of-fit indexes (see Rogosa & Willett, 1985, p. 100). These were reported as follows: $\chi^2 = 2.13$, $df = 5$, $p = .831$, root mean square residuals less than .01, and adjusted goodness-of-fit index (AGFI) = .995. Rogosa and Willett (1985, p. 100) concluded that the "quasi-simplex covariance structure model provides an excellent fit to the covariance matrix in Table I."¹

Model Performance in a Multiple Indicator Context

A major characteristic of the simplex model fit by Rogosa and Willett (Equations 2 and 3), as well as the CRC model they used for data generation purposes (Equations 1 and 3), was that each construct was measured with only one indicator. Use of single indicators of latent variables has been frequently criticized in the psychometric literature because of the very limited information that a fallible measure usually contains about the latent construct of actual interest and hence about its temporal change (e.g., Bollen, 1989). It is therefore of interest to see if a similar general statement like Rogosa and Willett's (1985, p. 104, cited in the preceding section) about crucial limitations of the associated covariance matrix can be made also in cases

¹One covariance structure residual is in this author's view of importance. This is the residual of .009, which relative to the other residuals appears inordinately large. (The next largest residual is less than half its size; this comparison is sensible because the magnitudes of the observed variable variances are fairly comparable.) It is associated with the element that reflects the relation between two assessments furthest away from each other according to the fitted simplex model, the first and last measured variables. This observation suggested the example discussed later in this section.

TABLE 1
Observed Score Covariance Matrix From Rogosa & Willett (1985, Table I)

Variable	Y_1	Y_2	Y_3	Y_4	Y_5
Y_1	.619				
Y_2	.453	.595			
Y_3	.438	.438	.587		
Y_4	.422	.430	.438	.595	
Y_5	.406	.422	.438	.453	.619

Note. $Y_1, Y_2, Y_3, Y_4,$ and Y_5 represent single indicators of a latent construct following the constant rate of change model in Equation 1, which has been assessed five consecutive times. From "Satisfying a Simplex Structure Is Simpler Than It Should Be," by D. R. Rogosa and J. Willett, 1985, *Journal of Educational Statistics*, 10. Copyright 1985 by the American Educational Research Association and the American Statistical Association. Reprinted with permission.

where one uses at least two indicators per latent construct. Addressing this issue constitutes a main concern of this article.

To respond to it, a counterexample is considered where the associated covariance matrix is not necessarily a severe summary of the longitudinal data in the sense of being insensitive to deviations from the CRC model that are built in a fitted simplex model. This example is based on similar latent and observed level parameters as those cited by Rogosa and Willett (see Equations 1 and 3). The difference between the two examples is that here two, rather than only one, indicators are simulated at each assessment. Like Rogosa and Willett's example, this data set is based on the straight-line constant growth rate model in Equation 1, and uses the following measurement model:

$$\begin{aligned}
 Y_1 &= \eta_1 + \varepsilon_1 \\
 Y_2 &= \eta_1 + \varepsilon_2 \\
 Y_3 &= \eta_2 + \varepsilon_3 \\
 Y_4 &= \eta_2 + \varepsilon_4 \\
 Y_5 &= \eta_3 + \varepsilon_5 \\
 Y_6 &= \eta_3 + \varepsilon_6 \\
 Y_7 &= \eta_4 + \varepsilon_7 \\
 Y_8 &= \eta_4 + \varepsilon_8 \\
 Y_9 &= \eta_5 + \varepsilon_9 \\
 Y_{10} &= \eta_5 + \varepsilon_{10}
 \end{aligned}
 \tag{4}$$

Based on Equations 1 and 4, 10 normally distributed random variables were generated for $N = 500$ cases (Y_1 through Y_{10}). (Thereby, the reliabilities of the simulated indicators Y_1 to Y_{10} were at least .75; cf. Rogosa & Willett, 1985.) The resulting covariance matrix is displayed in Table 2.

TABLE 2
Observed Score Covariance Matrix Following the Constant Rate of Change Model With
Two Indicators per Assessment

Variable	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}
Y_1	1.20									
Y_2	.93	1.24								
Y_3	.96	.94	2.28							
Y_4	.99	.94	2.04	2.39						
Y_5	1.00	.87	3.00	3.12	5.45					
Y_6	1.05	.93	3.08	3.15	5.19	5.52				
Y_7	1.09	.88	4.12	4.20	7.34	7.39	10.91			
Y_8	1.06	.92	4.03	4.10	7.14	7.22	10.34	10.41		
Y_9	1.13	.87	5.07	5.19	9.32	9.34	13.55	13.23	17.77	
Y_{10}	1.14	.91	5.05	5.17	9.24	9.29	13.48	13.14	17.29	17.50

Note. Each consecutive pair of variables represents two indicators of a latent construct following the constant rate of change model in Equation (1), which has been assessed at five successive times.

Exploration of the raw data suggested that the assumption of normality could be considered plausible. Specifically, (a) Mardia's normalized multivariate kurtosis was nonsignificant (Bentler, 1995), (b) the simulated variables each appeared normally distributed (Jöreskog & Sörbom, 1993b), and (c) pairwise scatterplots of variables exhibited no marked deviations from linearity as well as cigar-shaped dispersion patterns (e.g., Tabachnick & Fidell, 1996). The application of the maximum likelihood method was therefore considered justified.

Fitting the simplex model given by Equations 2 and 4 yielded an unacceptably high chi-square value: $\chi^2 = 570.89$ for $df = 31$, with associated probability level ($p < .00001$) and very high root mean square error of approximation (RMSEA), $\pi = .19$. (LISREL indicated that it could not compute confidence interval limits due to the too small probability associated with the chi-square value; Jöreskog & Sörbom, 1993a.) In addition, even though the estimated factor loadings were close to the simulated ones, and the percentage explained variance for most latent variables was relatively high, the Q-Q plot of the standardized residuals was very negatively skewed (some 80% of the residuals being negative), suggesting lack of model fit. Specifically, the line (fitted) through their representative signs in the plot had a nearly horizontal slope. Furthermore, the modification indexes for the fixed parameters in the matrix reflecting regressions among the latent variables (the matrix "Beta" in LISREL's terminology) were very high (far above 10), particularly those for the parameters relating the first and last two latent variables. These results indicate that the fitted simplex model could not be accepted as a means of describing the covariance matrix in Table 2, and that (at least in the case of a pair of indicators per assessment occasion) the observed variable covariance

matrix contained crucial information about the pattern of temporal change. Interestingly, the model was also fit with three different sets of starting values, and identical results were obtained.

SENSITIVITY OF SEM TO NONLINEAR LATENT RELATION MISSPECIFICATIONS

The discussion thus far was concerned with the issue of sensitivity of SEM to such misspecifications of the underlying latent relation pattern that remained in the class of linear relations. This section's aim is to address the problem of sensitivity of SEM to nonlinear latent relation misspecifications. In particular, as in the preceding section, the interest here is in presenting a counterexample where the covariance matrix is not a severe summary of longitudinal data and does not discard (all) crucial information about model misspecifications in the sense of being insensitive to them.

To address this concern, in the remainder of this section the results of a simulation study based on a model with a quadratic relation at the true score level are presented. In this study, data on $N = 1000$ observations were simulated in accordance with the true relation

$$\eta_2 = 5\eta_1 - .5\eta_1^2 \quad (5)$$

with two indicators per true score, η_1 and η_2 . (The sample size was chosen to be twice as high as that in Rogosa & Willett, 1985, and that in the preceding section because the weighted least squares [WLS] fitting method will be used later due to the induced lack of multivariate normality in the simulated data; see later.) The extreme relation of 10 to 1 between the coefficients of the linear and quadratic terms in the right-hand side of Equation 5 was chosen to provide empirical evidence that, even under rather unfavorable conditions for detecting model misspecifications, the associated covariance matrix can contain crucial information suggesting lack of model fit. This extreme relation of 10 to 1 of the linear to quadratic coefficients in Equation 5 facilitates obtaining a high percentage of explained latent variability in a linear regression of the simulated η_2 upon η_1 to begin with. Thus it makes sensing deviations from latent linearity more difficult for SEM (which has generally been viewed as a linear modeling methodology). (In a preliminary regression analysis relating the simulated η_2 upon η_1 , the proportion of latent explained variance in the former was found to be 98%. Raykov & Penev, 1997, discussed a case where the relation of linear to quadratic coefficients is only 1 to 1 and the amount of explained latent variability was fairly low, viz. 30%; cf. later.)

When simulating the data, four error terms as well as a true score η_1 were obtained as realizations of zero-mean normal variates (the variance of η_1 was postulated to be unity). The measurement model used for data generation purposes was

$$\begin{aligned} Y_1 &= \eta_1 + \varepsilon_1 \\ Y_2 &= 2\eta_1 + \varepsilon_2 \\ Y_3 &= \eta_2 + \varepsilon_3 \\ Y_4 &= 2\eta_2 + \varepsilon_4 \end{aligned} \quad (6)$$

providing the covariance matrix presented in Table 3.

Initial data exploration suggested that the assumption of normality of the four indicators Y_1 , Y_2 , Y_3 , and Y_4 was not plausible. Specifically, the univariate distributions of the last two variables exhibited marked deviations from normality. This was not surprising because Y_3 and Y_4 by definition each represented the sum of two normal variables, η_1 ($2\eta_1$) and ε_3 (ε_4), respectively, and a (proportionate to) χ^2 variable with a single degree of freedom, η_1^2 . Given these findings, the WLS method was considered appropriate (e.g., Jöreskog & Sörbom, 1993a; the following results did not change substantially however when using the maximum likelihood method and suggested similar conclusions as those stated later).

The misspecified, simplex model in Equations 6 and 2 (for two assessments) was fit to the covariance matrix in Table 3. The resulting goodness-of-fit indexes were as follows: $\chi^2 = 1.174$, $df = 1$, $p = .279$, and RMSEA with its 90% confidence interval $\pi = .013$ (0; .086). These appear excellent (overall) fit indexes, and there does not seem to be indication of low power as judged by the length of the reported confidence interval (e.g., Jöreskog & Sörbom, 1993a). In addition, there are high proportions of explained latent (97%) and observed (in excess of 73%) variability. However, the residuals of the model were mostly positive (80% of them), and their distribution was very skewed. Moreover, the standardized residuals exhibited a very platykurtic distribution. Their Q-Q plot had a clear nonlinear pattern suggesting model misspecifications (e.g., Jöreskog & Sörbom, 1989). Further, although

TABLE 3
Observed Score Covariance Matrix Following a Nonlinear True Relationship Pattern

Variable	Y_1	Y_2	Y_3	Y_4
Y_1	1.286			
Y_2	1.934	5.226		
Y_3	4.599	9.616	32.110	
Y_4	9.218	18.802	46.502	123.520

Note. Y_1 , Y_2 and Y_3 , Y_4 = two pairs of simulated indicators of a latent construct that follows, over two assessments, the quadratic relationship in Equation 5; indicator reliabilities larger than .75 (cf. Rogosa and Willett, 1985).

with a single degree of freedom for the overall model test such residuals' behavior may not be very surprising, the standard error of the latent disturbance term variance was inordinately large relative to the remaining parameter standard errors pertaining to the latent structure of the model. (Note that this term is not present in Equation 5 but a linear structural model with such a term, viz. with the structural equation $\eta_2 = \beta\eta_1 + \zeta_2$, is fitted to the data here.) For example, the standard error of this residual variance was more than nine times larger than the standard error of the latent predictor variance (i.e., the variance of η_1). In addition, the standard errors of the error variances for the indicators Y_3 and Y_4 of the latent dependent variable η_2 were each more than 20 times higher than the standard errors of these variances for the corresponding indicators Y_1 and Y_2 , respectively, of the latent predictor η_1 . These three inordinately large standard errors suggested considerable lack in precision of estimation of their corresponding parameters, which in this empirical case reflects deviations from linearity. This finding of large parameter estimate imprecision, too, was taken as an indication of nonlinearity misspecifications at the ability level of the fitted linear model. (This model was also fit with three different sets of initial starting values, and identical results were obtained.)

Thus the conditions underlying this example were rather unfavorable for SEM to detect the built-in model misspecifications, relative to the fitted model, in the data simulation process. Specifically, the ratio of latent linear to quadratic coefficients was very high, namely 10 to 1. As such, it ensures substantial domination of the linear over quadratic term in Equation 5 defining the data simulation model, in addition to there being only two assessment points.² Nonetheless, the associated covariance matrix in Table 3 was found to contain important information about model misspecifications. Consequently, this matrix is not insensitive to the serious build-in model misspecifications, and hence cannot be considered a severe summary of the analyzed two-wave data.

DISCUSSION AND CONCLUSION

This article contributes to the discussion of sensitivity of SEM methodology to model misspecifications. In particular, it provides counterexamples in which the covariance matrix is not a severe summary of longitudinal data to the extent of rendering it incapable of sensing latent pattern misspecifications. As indicated previously, the article does not make general statements but only aims at cautioning against such, particularly against generalizations of possible interpretations of

²It is stressed that the concern of this structural equation modeling (SEM) application is not with fitting a latent growth curve based on only two consecutive assessment points, but rather to explore the potential of SEM to sense deviations in the analyzed data from latent linearity.

some of Rogosa and Willett's (1985) statements to suggestions that the covariance matrix is in its very nature insensitive to the important aspects of latent change.

At the same time, the developments in this article do not invalidate the message by Rogosa and Willett (1985) that fundamentally distinct patterns of latent change over time can generate observed covariance matrices that are (nearly) identical. In fact, this message is plausible in light of a broader interpretation of the problem of equivalent models in SEM, which has received wide recognition in the behavioral and social sciences (e.g., Lee & Hershberger, 1990; MacCallum, Wegener, Uchino, & Fabrigar, 1993; Raykov & Penev, 1999; Stelzl, 1986; see also Raykov, 1998). Accordingly, more than a single structural equation model can fit a given data set (statistically) acceptably well (e.g., Breckler, 1990). Instead of aiming at general statements, which are very hard to make in this complex area of model choice, this article provided counterexamples against a potential overinterpretation of Rogosa and Willett's findings as proposing that covariance structure analysis is inherently insensitive to latent change patterns.

In this context, this article also raises the issue of enhancing the ability of structural equation models to sense underlying misspecifications, particularly at the latent level. This is a major theoretical and empirical issue in applications of SEM, and one that awaits more comprehensive theoretical developments. The approach followed here was intuitively appealing, namely increasing the amount of empirical (sample) information against which a model is tested. Specifically, more than a single indicator was used per latent construct in this discussion. This represents a direct—though not always easily followed in practice—method of such empirical information increase. The article also illustrates in a longitudinal setting the widely accepted view that more than one indicator per construct are highly desirable in empirical practice.

This major issue of increasing likelihood of sensing model misspecifications has been addressed by insightful developments in modeling longitudinal data, which have taken place in the more recent methodological literature. In particular, the development and popularization of the so-called latent curve analysis (LCA; Meredith & Tisak, 1990; see also Raykov, 1998) has provided applied researchers in the behavioral and social sciences with a comprehensive instrument for modeling developmental processes. A specific feature of LCA is the inclusion of the indicator means in the modeling approach, which was not a concern of Rogosa and Willett's (1985) discussion and hence not followed in this article dealing with their contribution. LCA achieves the aforementioned accumulation of empirical (sample) information about the studied phenomenon by incorporating the longitudinal means, in addition to variances and covariances of the analyzed variables. It is emphasized, however, that LCA accomplishes this in a specific way that follows a general modeling framework based on the assumption of existence of a number of basis curves representing main features of temporal change in a studied group (Meredith & Tisak, 1990). This yields such a parsimonious way of inclusion of

variable means into the analysis, which allows their parameterization only in terms of latent construct means. As a result, the manifest variable means' structure of an LCA model is not saturated, but rather over identified. In this way, saturation of this structure when simply including the observed means into the analysis is avoided, which otherwise leads to a mean structure model that is just as informative as a model with the same covariance structure specification and not including any variable means.

Being concerned with the findings in Rogosa and Willett (1985), this article focused on the longitudinal modeling context used by those authors. In this regard, it is interesting to know if the lack of discernable implied covariance matrices by structural equation models—as exemplified by those authors—is unique to the longitudinal design context. To respond to this query in a more qualified manner requires a more comprehensive investigation. However, it can be said here that at least for some parameter values of appropriately constructed models this lack of discernable covariance matrices can hold with other than longitudinal data. This is perhaps most easily seen by realizing that the covariance matrix of any longitudinal data set does not know where it is coming from and hence is unaware of its resulting in a repeated assessment setting rather than say from a cross-sectional design. Thus, because repeatedly observed constructs can be formally viewed—after their assessment—also as latent variables in a fictitious cross-sectional design, it is possible that in a cross-sectional study and for certain parameter values two structural equation models yield nearly identical implied covariance matrices.³ Furthermore, at a general level, equivalent models for a given data set (studied phenomenon) can be considered representing yet another case of (currently) statistically indistinguishable means of description and explanation. Thus, for a nonlongitudinal data set resulting from a fixed member of a class of equivalent models, any other member of the class can be considered a “misspecified” model that fits the data equally well (e.g., MacCallum et al., 1993).

In addition to a notable body of pertinent literature (e.g., Bollen, 1989), this article exemplified a limitation of overall Goodness-of-Fit Indexes, such as the chi-square value and a host of related descriptive fit indexes (e.g., GFI, AGFI, and many other derivatives of the chi-square; Jöreskog & Sörbom, 1993a). Specifically, their value cannot be generally interpreted as providing evidence for fit in all portions of a given model. As in Rogosa and Willett's (1985) finding, a good

³The discussion in this paragraph of the main text is distinct from, though related to, the problem of equivalent models in structural equation modeling (e.g., MacCallum, Wegener, Uchino, & Fabrigar, 1993). Although equivalent models yield identical reproduced covariance matrices for any set of admissible values of their parameters, the near identity of implied covariance matrices (mentioned in that paragraph) can hold only for some sets of values of the parameters of two (or more) models (see Raykov, 1998, for this relation between the particular constant rate of change and simplex models in Rogosa & Willett, 1985). Furthermore, unlike the definition of equivalent models (e.g., Stelzl, 1986), the discussion in the main text does not require (complete) identity of these implied matrices.

overall fit index does not imply lack of serious misspecifications across all parts of the model. As this article illustrates, a careful study of model residuals and parameter standard errors (as well as modification indexes) for signs of model misspecification(s) is always recommended when evaluating the fit of a structural equation model under consideration. Thereby, this recommendation is valid regardless of how favorable (acceptable) its overall goodness-of-fit indexes are.

In conclusion, the developments in this article suggest that the following actions by applied researchers can substantially enhance the likelihood of an SEM application to sense consequential model misspecifications (cf. Bollen, 1989; Jöreskog & Sörbom, 1989; Raykov & Penev, 1997). These are (a) an increase in the number of valid indicators of (repeatedly assessed) latent constructs under investigation; (b) a thorough examination of all parts of the output of the numerical minimization procedure underlying routine utilizations of SEM, not only those pertaining to the overall measures of fit; (c) increased measurement precision; (d) using SEM for purposes of individual latent change modeling (e.g., Meredith & Tisak, 1990; Raykov, 1998; Rogosa, 1987; Rogosa, Brandt, & Zimowski, 1982; Rogosa & Willett, 1985; Willett & Sayer, 1994); and (e) extensive preliminary data exploration to determine appropriate data modeling approaches and modeling classes. When dealing with a substantively sound structural equation model that capitalizes on all relevant previous research and accumulated theoretical knowledge in a domain of application, these points can ensure a good chance of detecting model misspecifications in empirical behavioral and social research using the popular SEM methodology.

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REFERENCES

- Bentler, P. M. (1995). *EQS structural equations program manual*. Encino, CA: Multivariate Software.
- Bollen, K. A. (1989). *Structural equations with latent variables*. New York: Wiley.
- Breckler, S. (1990). Applications of covariance structure modeling in psychology: Cause for concern? *Psychological Bulletin*, *107*, 260-273.
- Collins, L., & Horn, J. L. (Eds.). (1991). *Best methods for the analysis of change*. Washington, DC: American Psychological Association.

- Freedman, D. A. (1987). As others see us: A case study in path analysis. *Journal of Educational Statistics*, 12, 101-128.
- Gottman, J. M. (Ed.). (1996). *The analysis of change*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- Guttman, L. A. (1954). A new approach to factor analysis: The radex. In P. F. Lazarsfeld (Ed.), *Mathematical thinking in the social sciences* (pp. 1-54). New York: Columbia University Press.
- Jöreskog, K. G., & Sörbom, D. (1989). *LISREL 7: A guide to the program and its applications*. Chicago: SPSS.
- Jöreskog, K. G., & Sörbom, D. (1993a). *LISREL 8: User's guide*. Chicago: SPSS.
- Jöreskog, K. G., & Sörbom, D. (1993b). *PRELIS 2: User's guide*. Chicago: SPSS.
- Lee, S., & Hershberger, S. (1990). A simple rule for generating equivalent models in covariance structure modeling. *Multivariate Behavioral Research*, 25, 313-334.
- MacCallum, R. C., Wegener, D. T., Uchino, B. N., & Fabrigar, L. R. (1993). The problem of equivalent models in applications of covariance structure analysis. *Psychological Bulletin*, 114, 185-199.
- Meredith, W., & Tisak, J. (1990). Latent curve analysis. *Psychometrika*, 55, 107-122.
- Raykov, T. (1998). Satisfying a simplex structure is simpler than it should be: A latent curve analysis revisit. *Multivariate Behavioral Research*, 33, 343-363.
- Raykov, T., & Penev, S. (1997). Structural equation modeling and the latent linearity hypothesis in social and behavioral research. *Quality & Quantity*, 31, 57-78.
- Raykov, T., & Penev, S. (1999). On structural equation model equivalence. *Multivariate Behavioral Research*, 34, 199-244.
- Rogosa, D. R. (1987). Myths about longitudinal research. In K. W. Schaie, R. T. Campbell, W. M. Meredith, & S. C. Rawlings (Eds.), *Methodological issues in aging research* (pp. 171-209). New York: Springer.
- Rogosa, D., Brandt, D., & Zimowski, M. (1982). A growth curve approach to the measurement of change. *Psychological Bulletin*, 92, 726-748.
- Rogosa, D. R., & Willett, J. (1985). Satisfying a simplex structure is simpler than it should be. *Journal of Educational Statistics*, 10, 99-107.
- Stelzl, I. (1986). Changing a causal hypothesis without changing the fit: Some rules for generating equivalent path models. *Multivariate Behavioral Research*, 21, 309-331.
- Tabachnick, B. G., & Fidell, L. S. (1996). *Using multivariate statistics*. New York: HarperCollins.
- Willett, J. B., & Sayer, A. G. (1994). Using covariance structure analysis to detect correlates and predictors of individual change over time. *Psychological Bulletin*, 116, 363-381.