Advanced Lyapunov control of a novel laser beam tracking system

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Abstract. Laser communication systems developed for mobile platforms, such as satellites, aircraft, and terrain vehicles, require fast wide-range beam-steering devices to establish and maintain a communication link. Conventionally, the low-bandwidth, high-steering-range part of the beam-positioning task is performed by gimbals that inherently constitutes the system bottleneck in terms of reliability, accuracy and dynamic performance. Omni-Wrist™, a novel robotic sensor mount capable of carrying a payload of 5 lb and providing a full 180-deg hemisphere of azimuth/declination motion is known to be free of most of the deficiencies of gimbals. Provided with appropriate controls, it has the potential to become a new generation of gimbals systems. The approach we demonstrate describes an adaptive controller enabling Omni-Wrist™ to be utilized as a part of a laser beam positioning system. It is based on a Lyapunov function that ensures global asymptotic stability of the entire system while achieving high tracking accuracy. The proposed scheme is highly robust, does not require knowledge of complex system dynamics, and facilitates independent control of each channel by full decoupling of the Omni-Wrist™ dynamics. We summarize the basic algorithm and demonstrate the results obtained in the simulation environment. [DOI: 10.1117/1.1917409]

Subject terms: optical communications; beam steering; tracking; adaptive control.

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1 Introduction

Laser communications, including intersatellite cross-links and aircraft-to-aircraft and air-to-surface communication, is a technology that has the potential to provide the material base for the information revolution. Lasers provide the most logical connectivity channels and have unsurpassed advantages over the traditional rf technology in terms of speed and secure operation. Indeed, lasers enable a concentration of energy within a very small spatial angle and, as a result, lead to very low power consumption, low weight, and small size. However, these advantages do not come without a price: due to the low divergence, the laser beam must be very accurately positioned on the target or the receiver station. In many aerospace applications, when the optical platform is placed on board an aircraft, the ability to track the target is affected by the resident vibration of the airframe and the complex maneuvers performed by the air vehicle, often at supersonic speed. A laser-positioning task must comply with the bandwidth, accuracy, and range-of-operation requirements prompted by a particular application.

Very often, the full functionality of the entire system cannot be achieved without gimbals. Commonly used systems, such as Schaeffer gimbals, are two-degree-of-freedom mechanical steering devices, designed to achieve maximum decoupling between azimuth and declination channels. Driven by rotary dc brush motors, they exhibit fairly wide steering range, but are flawed by singularity inherent from their mechanical structure.[1,2] In addition, their design is responsible for the narrow bandwidth, limited accuracy, complex, nonlinear friction phenomena that interferes with operation, especially at low-range steering. Moving electrical wires and complex coupling of the motors to the moving parts adversely affect the reliability and energy efficiency and contribute to friction and nonlinear behavior. Omni-Wrist III, a novel robotic manipulator, was developed by Ross-Hime Designs[3] to address these issues. It is a two-degree-of-freedom system capable of a full 180-deg hemisphere (plus 5 deg below horizon) of singularity-free yaw/pitch motion with up to 5 lb of payload. In comparison to traditional gimbals positioning devices, Omni-Wrist III enjoys increased bandwidth due to a greater power/mass ratio and reduced inertia and friction. However, its mechanical design does not eliminate nonlinearities and cross-coupling, thus complicating the controls task.

This paper demonstrates a solution to decoupling and improved dynamic performance of Omni-Wrist III. The approach, which was originally suggested by Seraji[4] and Slotine and Li[5,6] for robotic manipulators and developed further for laser beam tracking systems,[7] is based on the method of Lyapunov functions. The proposed control scheme does not require knowledge of complex system dynamics, and facilitates accurate beam positioning while

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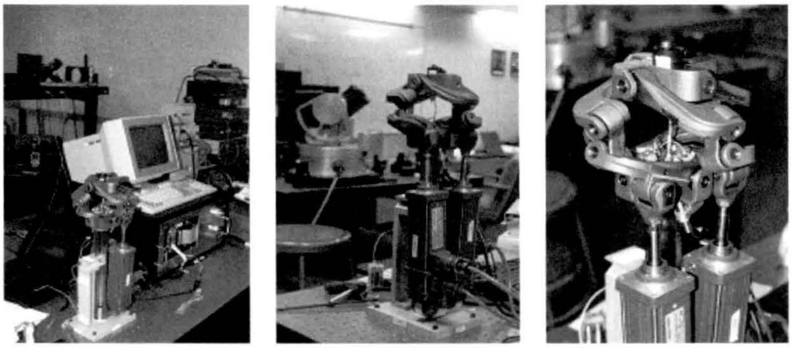

Fig. 1 Omni-Wrist III sensor mount.

providing independent (decentralized) control of the azimuth and declination channels. Note that the Lyapunov approach could be beneficial for a variety of mechanical devices with complex dynamics, including other gimbals, galvanometer mirrors, piezoelectric mirrors, etc.

2 Omni-Wrist III Model

The Omni-Wrist™ device was developed with the aim to address the inherent problems of commonly used gimbal systems. It features an innovative solution that emulates the kinematics of a human wrist, the ingenious manipulator design honed to perfection by million-year evolution.[8] In contrast to traditional robotic manipulators, the actuators driving the Omni-Wrist are not located in the joints but rather are attached to the links, just like muscles are attached to bones in biological structures. The moving sensor mount is connected to the stationary platform through 12 links and 16 joints (Fig. 1) constituting a singularity-free two-degree-of-freedom pointing system. Incremental optical encoders embedded within the linear brushless actuators provide the position feedback. Linear displacement of each of the two actuators results in angular displacement of both the azimuth and the declination coordinates of the Omni-Wrist platform. The relationship between the positions of the linear actuators and azimuth and declination platform coordinates is complex, nonlinear and clearly exhibits cross-coupling.

The development of an accurate mathematical model[8] is a crucial starting point for the design of an efficient control system. Figure 2 represents the proposed configuration of the mathematical model of Omni-Wrist that comprises two modules. The first, DYNAMICS, represents the dynamics of two independently operating linear actuators coupled to the sensor mount through a series of links and joints. It includes two transfer functions, $G_1(s)$ and $G_2(s)$, describing the typical linear relationships between the control efforts, voltages U_1 and U_2, and the resultant linear displacements x and y. Our laboratory experiments demonstrate that the actuators exhibit the velocity response of a first-order system.[8] While the magnitude of the response differs in the positive and negative direction, it is possible to model the dynamics of the system using only two transfer functions, one for each actuator, with voltage input and position output as

$$G(s)=\frac{b}{s(s+a)}, \tag{1}$$

where s is the Laplace transform variable, and a and b are denominator and numerator coefficients, respectively.

The KINEMATICS module describes the nonlinear static relationship between the linear actuator displacements x and y and the angular displacements of the platform Θ_1 and Θ_2. Elements $\Phi_{ij}(x,y)$, $i, j=1, 2$, reflect the complex kinematics of the Omni-Wrist structure. Analysis of the system has resulted in a system of trigonometric equations; however, these equations are too complex for any direct use, and it seems to be more practical to represent both the kinematics and inverse kinematics of the device by a sequence of three transformations.[8] Development of this module implies the solution of the direct pose kinematics problem utilizing the Denavit-Hartenberg approach and finding a transformation of the linear encoder readings into joint coordinates; then into the yaw, pitch, and roll angles; and finally, into the azimuth and declination coordinates.

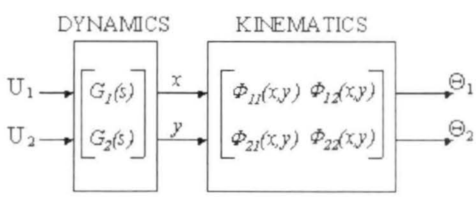

Fig. 2 Configuration of the Omni-Wrist model.

3 Control Synthesis

When used as a steering device for pointing, acquisition, and tracking (PAT), Omni-Wrist III could be viewed as a two-input, two-output system that positions the laser beam over a wide range of azimuth and declination angles. Its model, with the structure an shown in Fig. 2, could be presented with a single nonlinear transformation that accounts for both kinematic and dynamic properties as follows:

$$\begin{bmatrix}\Theta_1\\ \Theta_2\end{bmatrix}=\begin{bmatrix}\Theta_{11}(x,y) & \Theta_{12}(x,y)\\ \Theta_{21}(x,y) & \Theta_{22}(x,y)\end{bmatrix}\begin{bmatrix}G_1(s) & 0\\ 0 & G_2(s)\end{bmatrix}\begin{bmatrix}U_1\\ U_2\end{bmatrix}$$
$$=\begin{bmatrix}g_{11}(s,x,y) & g_{12}(s,x,y)\\ g_{21}(s,x,y) & g_{22}(s,x,y)\end{bmatrix}\begin{bmatrix}U_1\\ U_2\end{bmatrix}. \tag{2}$$

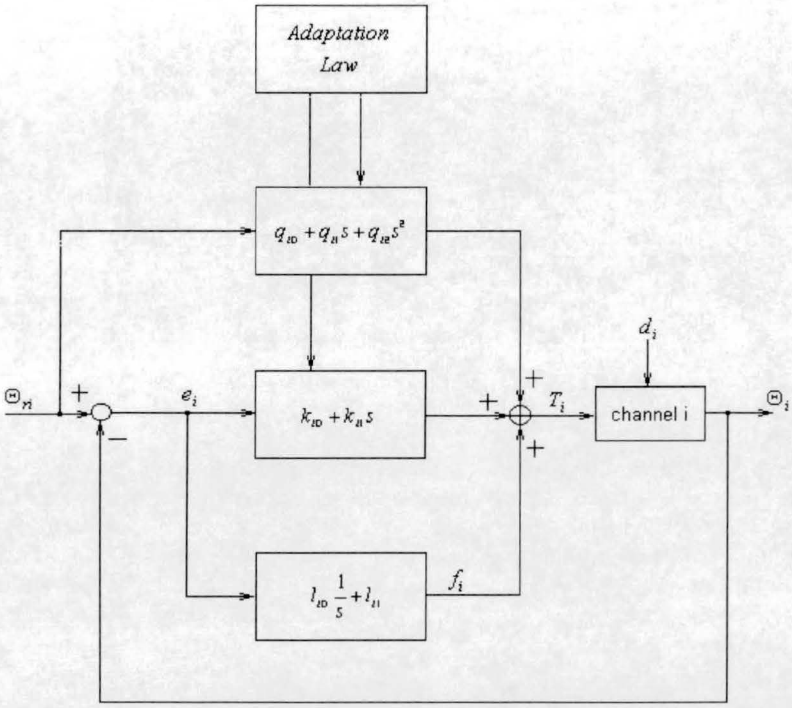

Fig. 3 Decentralized adaptive control system.

Hence, the output of either dynamic channel can be found as

$$\Theta_i = g_{ii}U_i + g_{ij}U_j . \tag{3}$$

Let $\bar{d}_i$ (disturbance signal) represent cross-coupling effects, nonlinearities, and other unmodeled dynamics,[4] such that Eq. (3) becomes

$$\Theta_i = G_i U_i + \bar{d}_i \equiv \frac{N_i}{D_i} U_i + \bar{d}_i = \frac{b}{s^2 + as} U_i + \bar{d}_i , \tag{4}$$

where N_i and D_i are the numerator and denominator polynomials of G_i, respectively.

Then, the suggested control signal is formed as follows

$$T_i = N_i U_i = D_i \Theta_i - D_i \bar{d}_i . \tag{5}$$

For simplicity the subscript *i* identifying the dynamic channel can be omitted, resulting in the following differential equation when Eq. (5) is presented in the time domain:

$$T = \Theta'' + a\Theta' + d, \tag{6}$$

where a new disturbance term is $d = D_i \bar{d}_i$.

The proposed control system for each dynamic channel has a unity gain feedback and three modules: a conventional controller, adaptive feedback, and feedforward controllers in the following form

$$T = \left(l_0 \int e\,\mathrm{d}t + l_1 e \right) + (k_0 e + k_1 e') + (q_0 \Theta_r + q_1 \Theta_r' + q_2 \Theta_r''), \tag{7}$$

where l_i, k_i, and q_i are controller gains. The system configuration for one channel is presented in Fig. 3.

Substituting the first term of Eq. (7) with *f*, as shown in Fig. 3, and combining Eqs. (6) and (7) results in

$$e'' + (a + k_1)e' + k_0 e = d - f - q_0 \Theta_r - (q_1 - a_1)\Theta_r' - (q_2 - 1)\Theta_r'' . \tag{8}$$

This second-order differential equation has a matrix-vector equivalent that could be obtained by introducing $\mathbf{X} = [ee']^T$, as follows:

$$\dot{\mathbf{X}} = \begin{bmatrix} 0 & 1 \\ -k_0 & -a-k_1 \end{bmatrix} \mathbf{X} + \begin{bmatrix} 0 \\ d-f \end{bmatrix} + \begin{bmatrix} 0 \\ -q_0 \end{bmatrix} \Theta_r + \begin{bmatrix} 0 \\ a_1 - q_1 \end{bmatrix} \Theta_r' + \begin{bmatrix} 0 \\ 1-q_2 \end{bmatrix} \Theta_r'' . \tag{9}$$

If vector $\mathbf{X}_m = [e_m e_m']^T$ represents the desired error signals, then its dynamics of convergence toward zero can be described by the following state equation written in the canonical-controllable form.

$$\dot{\mathbf{X}}_m = \mathbf{D}\mathbf{X}_m = \begin{bmatrix} 0 & 1 \\ -a_0^m & -a_1^m \end{bmatrix} \mathbf{X}_m . \tag{10}$$

Since Eq. (10) is a stable system, there exists a solution to the Lyapunov equation[4]

$$\mathbf{PD}+\mathbf{D}^T\mathbf{P}=-\mathbf{Q},$$

where $\mathbf{P}$ and $\mathbf{Q}$ are positive definite matrices.

Introduction of vector $\mathbf{E}=\mathbf{X}_m-\mathbf{X}$ leads to a formal mathematical definition of the process of error convergence

$$\dot{\mathbf{E}}=\begin{bmatrix}0 & 1\\ -a_0^m & -a_1^m\end{bmatrix}\mathbf{E}+\begin{bmatrix}0 & 1\\ k_0-a_0^m & a+k_1-a_1^m\end{bmatrix}\mathbf{X}+\begin{bmatrix}0\\ f-d\end{bmatrix}$$
$$+\begin{bmatrix}0\\ q_0\end{bmatrix}\Theta_r+\begin{bmatrix}0\\ q_1-a\end{bmatrix}\Theta_r'+\begin{bmatrix}0\\ q_2-1\end{bmatrix}\Theta_r''. \tag{11}$$

A control law based on a Lyapunov function obtained from Eq. (11) would ensure that, given any initial condition, $\mathbf{E}$ converges asymptotically, and; therefore, the actual error trajectory $\mathbf{X}$ will track the desired error trajectory $\mathbf{X}_m$ that converges to zero. Consider the following positive-definite function as a Lyapunov candidate.

$$V=\mathbf{E}^T\mathbf{PE}+Q_0(f-d)^2+Q_1(k_0-a_0^m)^2+Q_2(a+k_1-a_1^m)^2$$
$$+Q_3q_0^2+Q_4(q_1-a)^2+Q_5(q_2-1), \tag{12}$$

where Q_i are positive scalars. Its derivative must be negative-definite to claim that Eq. (12) is a Lyapunov function. To obtain an analytical expression the assumption that the controlled plant is "slowly time varying" compared to the control effort is suggested;[4] therefore, $\dot{d}=0$, and differentiating Eq. (12) along the trajectory defined by Eq. (11) results in

$$\dot{\mathbf{V}}=-\mathbf{E}^T\mathbf{QE}+2Q_0(f-d)\dot{f}-(f-d)r+2Q_1(k_0-a_0^m)\dot{k}_0$$
$$-(k_0-a_0^m)re+2Q_2(a+k_1-a_1^m)\dot{k}-(a+k_1-a_1^m)re'$$
$$+2Q_3q_0\dot{q}_0-q_0r\Theta_r+2Q_4(q_1-a)\dot{q}_1-(q_1-a)r\Theta_r'$$
$$+2Q_5(q_2-1)\dot{q}_2-(q_2-1)r\Theta_r'', \tag{13}$$

where

$$r=w_0e+w_1e', \tag{14}$$

and w_0 and w_1 are positive weighting coefficients. Grouping terms of the Eq. (13) results in

$$\dot{\mathbf{V}}=-\mathbf{E}^T\mathbf{QE}+(f-d)(2Q_0\dot{f}-r)+(k_0-a_0^m)(2Q_1\dot{k}_0-re)$$
$$+(a+k_1-a_1^m)(2Q_2\dot{k}-re')+\cdots+q_0(2Q_3\dot{q}_0-r\Theta_r)$$
$$+(q_1-a)(2Q_4\dot{q}_1-r\Theta_r')+(q_2-1)(2Q_5\dot{q}_2-r\Theta_r''). \tag{15}$$

While there could be multiple solutions to the control synthesis problem that result in Eq. (15) being negative-definite, the most natural way to select the adaptation law is as follows:

$$2Q_0\dot{f}-r=0,$$
$$2Q_1\dot{k}_0-re=0,$$
$$2Q_2\dot{k}_1-re'=0,$$
$$2Q_3\dot{q}_0-r\Theta_r=0, \tag{16}$$
$$2Q_4\dot{q}_1-r\Theta_r'=0,$$
$$2Q_5\dot{q}_2-r\Theta_r''=0.$$

Hence, the time derivative of function $\mathbf{V}$ becomes

$$\dot{\mathbf{V}}=-\mathbf{E}^T\mathbf{QE}. \tag{17}$$

Solving Eq. (16) for unknown variables provides expressions for the conventional controller

$$f=\delta\int r\,\mathrm{d}t=\delta w_0\int e\,\mathrm{d}t+\delta w_1e=l_0\int e\,\mathrm{d}t+l_1e, \tag{18}$$

and equations for the adjustable gains of adaptive controllers

$$k_0=\alpha_1\int re\,\mathrm{d}t+k_0(0),$$
$$k_1=\alpha_2\int re^{(1)}\,\mathrm{d}t+k_1(0),$$
$$q_0=\gamma_1\int r\Theta_r\,\mathrm{d}t+q_0(0), \tag{19}$$
$$q_1=\gamma_2\int r\Theta_r'\,\mathrm{d}t+q_1(0),$$
$$q_2=\gamma_3\int r\Theta_r''\,\mathrm{d}t+q_2(0),$$

where δ_i, α_i, and γ_i are positive adaptation gains selected by the system designer.

The results of the preceding mathematical analysis make it evident that this approach does not require knowledge of the Omni-Wrist dynamics. In addition, there is no explicit definition of a reference model to specify the desired behavior of the system. However, signal Θ_r applied to the input represents the desired dynamics of the system response; hence, this signal could be generated by a reference model G_M selected to satisfy the design specifications.

4 System Implementation

Application of the method of Lyapunov functions results in a highly robust controller design. However, before proceeding with a prototype implementation a couple of important issues, pertaining to system stability and performance, should be discussed. Note that the control law defined by Eq. (7) generates signal T_i, while the physical input to the dynamic channel is U_i. A correspondence between the two signals is established by Eq. (5), hence

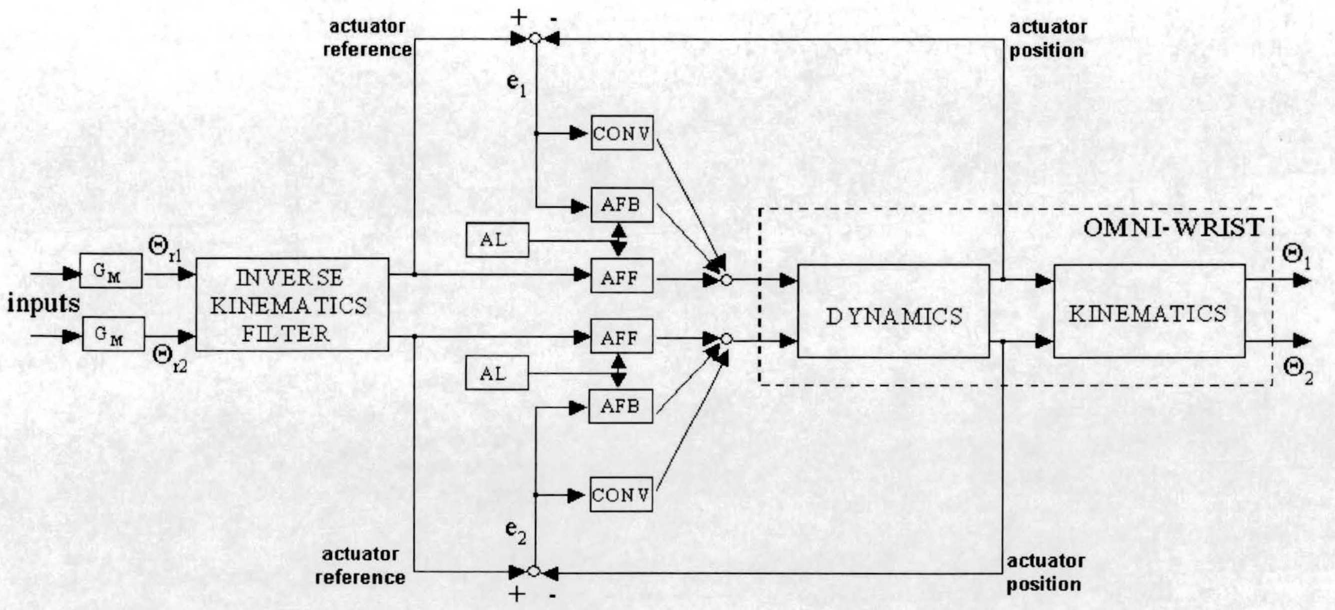

Fig. 4 Decentralized control system: CONV, conventional controller; AFB, adaptive feedback; AFF, adaptive feedforward; AL, adaptation law.

$$U_i = N_i^{-1}(s) T_i. \tag{20}$$

It appears that by applying T_t rather than U_i the residual signal $[N_{ii}^{-1}(s)-1]T_i$ is ignored.[9] Even if this signal is regarded as a part of component d_i; it cannot be considered "slowly time varying," since it includes signal T_i that has high-frequency content. On the other hand, N_i is a constant coefficient, and it could be demonstrated that even if Eq. (6) is scaled by a factor of N_i and the proposed controller is described by Eq. (7), we could still obtain the same expressions for the conventional controller of Eq. (18) and adjustable gains of Eq. (19).

Another problem could be encountered because the cross-coupling effects from the actuator inputs to the azimuth/declination outputs, included in component d, are very strong. Indeed, changing only one output coordinate requires the motion of both linear actuators, and therefore, the application of voltage signals to both motors. The adaptive algorithm presented in this paper is adequately suited only for loosely coupled systems;[5,6] therefore, additional steps must be taken to reduce the coupling effects. A decoupling filter must be introduced in the input of the system. This filter is based on the solution of the inverse pose kinematics problem[8] and transforms desired azimuth/declination angles into corresponding linear actuator coordinates. Configuration of the entire decentralized system is presented in Fig. 4.

The linear actuators, represented in Fig. 4 as DYNAMICS, take voltage as inputs and provide actuator position as outputs, generated by optical incremental encoders, which is the reason why the controller is built around the actuators. The KINEMATICS block represents the kinematical structure of the device coupling the outputs, while the INVERSE KINEMATICS FILTER transforms the reference azimuth and declination coordinates into reference actuator positions. Each channel is controlled individually, thus facilitating decentralized operation.

5 Simulation Results

The model of the Omni-Wrist™ gimbal[8] was used to test the performance of the Lyapunov-based decentralized adaptive system described by Eqs. (7), (18), and (19). The design goal is to achieve the settling time $T_{set} = 50$ ms with no overshoot, which places the poles of the closed-loop system at $-80 \pm j$, and the reference model is

$$G_m(s) = \frac{6401}{s^2 + 180s + 6401}. \tag{21}$$

Obtaining an open-loop response of the Omni-Wrist™ manipulator is meaningless, because applying voltage to a linear motor without any feedback signal will drive its shaft to the end position. To provide a comparative example, a conventional feedback system was designed and tested in the simulation environment. For these purposes a constant-coefficient proportional-integral (PI) controller was implemented and tuned to achieve the best possible performance. The results are presented in Figs. 5 and 6.

Figures 7 and 8 present simulation results for the Lyapunov-based decentralized adaptive control system. As the results in Figs. 5 and 6 demonstrate, conventional control scheme ensures steady state decoupling; however, it does not meet the desired performance requirements. The adaptive control system facilitates full decoupling and enhances tracking capabilities to a considerable extent (see Figs. 7 and 8).

6 Conclusion

A decentralized adaptive control approach was presented in this paper. The designed laser beam tracking system is based on a Lyapunov function to ensure global asymptotic stability. Application of this technique provides channel decoupling and significant enhancement of the dynamic performance. The approach does not require knowledge of the

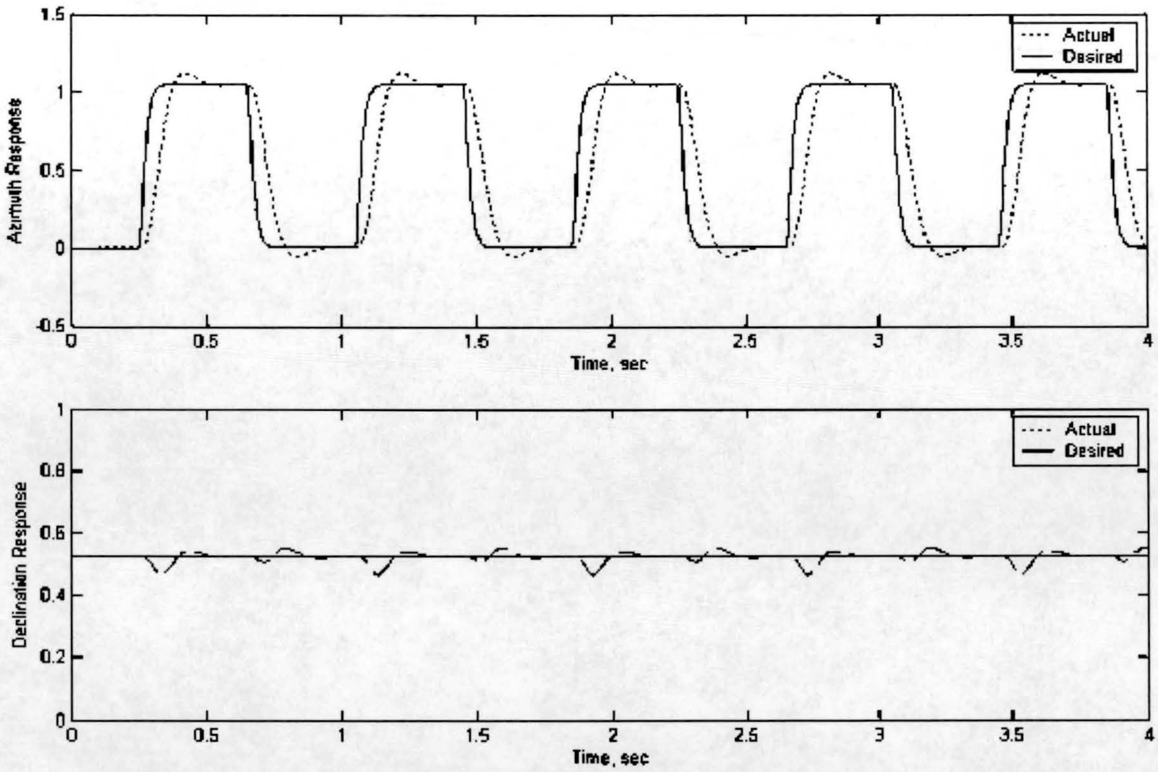

Fig. 5 Response of the conventional control system to a square wave signal applied to the azimuth channel.

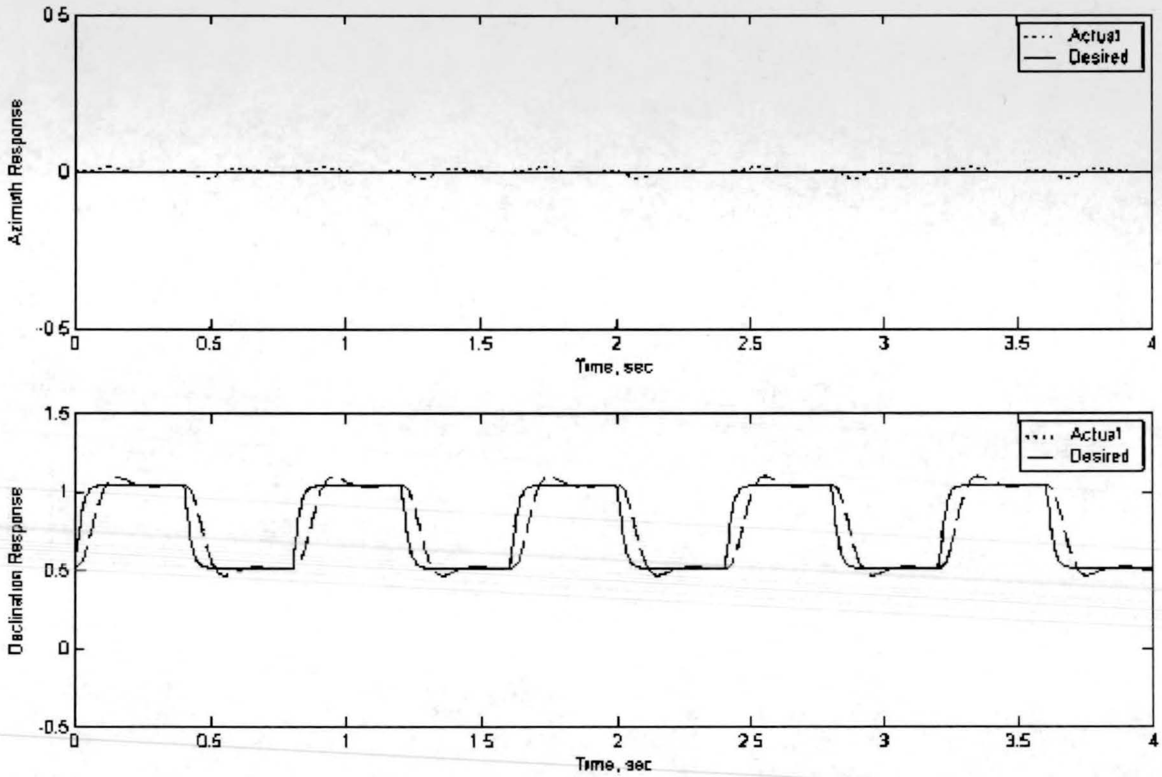

Fig. 6 Response of the conventional control system to a square wave signal applied to the declination channel.

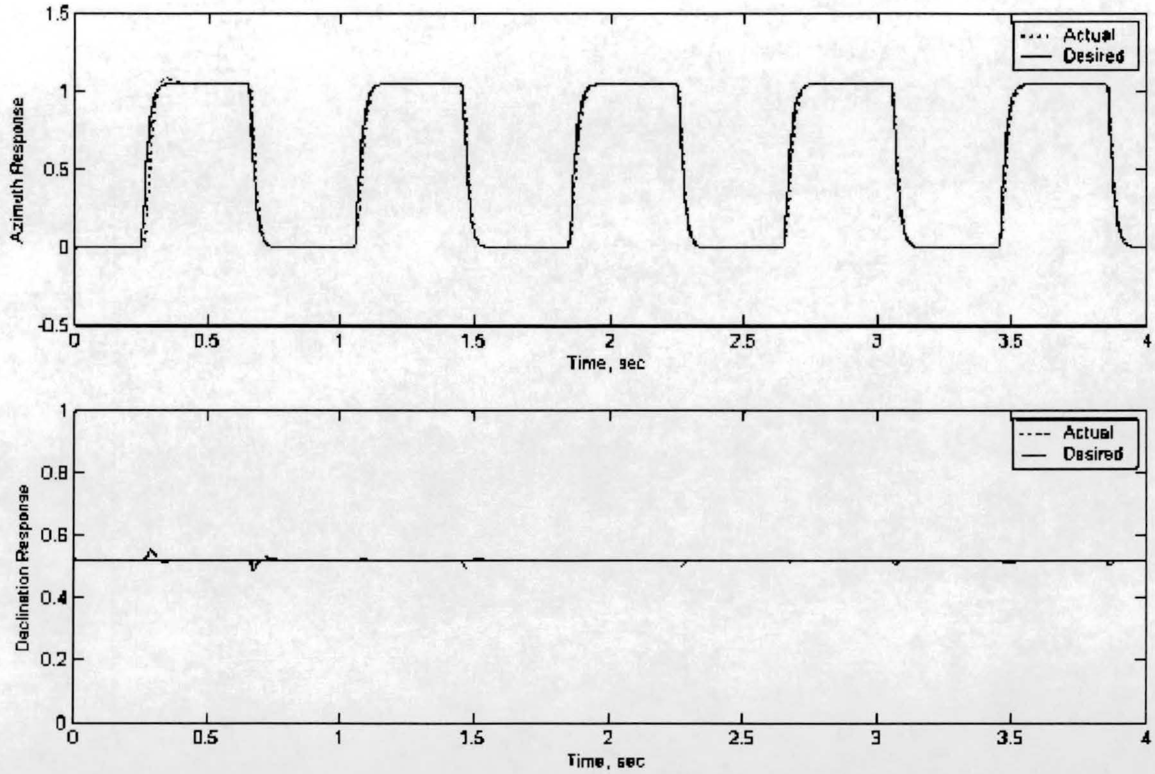

Fig. 7 Response of the adaptive control system to a square wave signal applied to the azimuth channel.

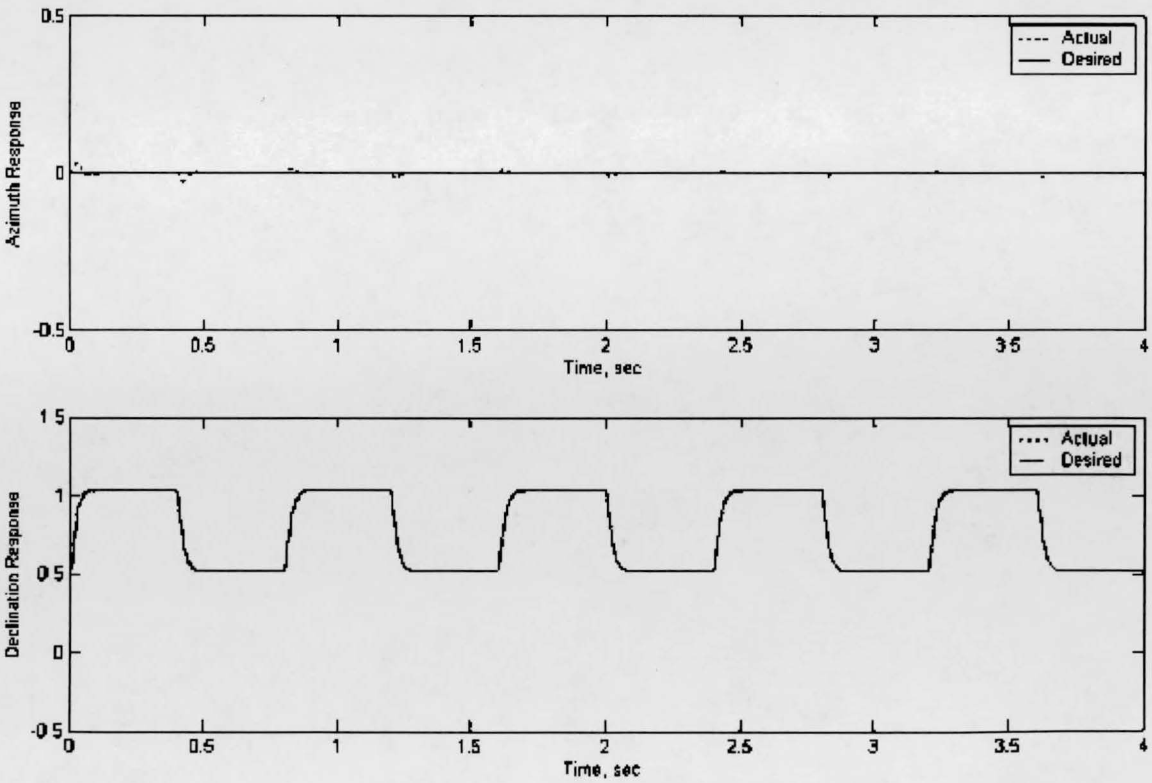

Fig. 8 Response of the adaptive control system to a square wave signal applied to the declination channel.

gimbal dynamics; neither does it rely on the system identification. As demonstrated by the simulation results, the decentralized system enjoys good robustness, while achieving high tracking accuracy over an extended range of pointing angles. In addition, as the laser beam tracking system operates, the adaptation mechanism keeps updating controller parameters, thus, facilitating online tuning of the system to alleviate the effects of nonlinearities and unmodeled dynamics parameter drift and maintaining a reliable laser link.

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