Lightweight mirror design method using topology optimization

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Abstract. Lightweight mirrors experience optical image degradation due to mechanical loadings such as self-weight, polishing pressure, and vibration. Optical surface deformation of a lightweight primary mirror is an important factor that affects optical performance. We use topology optimization to design a lightweight primary mirror under self-weight and polishing pressure. For the optimization, we used a 3-D model of the mirror and based our calculations on the rms surface error of the mirror as an objective function constrained by the maximum weight of the mirror. In the first example of topology optimization, we consider the mirror's self-weight loading. In the second example, we include the polishing pressure. We present the results of the optimized design topology for the mirror with various mass constraints. To examine the optimal design results, we manufacture a prototype of the mirror. [DOI: 10.1117/1.1901685]

Subject terms: mirrors; optomechanical design; topology optimization.

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1 Introduction

In space optics, the performance of a design should be tested after the design has experienced the loadings of a launch, such as vibration and shock, in a space environment such as microgravity, vacuum, and radiation. In keeping with the launch capability and cost, space optics should also be compact and light. Space optics, particularly large mirrors, should therefore be stiff and stable, and the weight should be kept to a minimum.

Many studies have analyzed and optimized lightweight mirrors. Valente and Vukobratovich made a comparative study of optical performance according to the shape of a primary mirror's substrate.[1] Cho, Richard, and Vukobratovich proposed design criteria to minimize optical aberrations and to obtain the optimum support points of a lightweight primary mirror under a self-weight loading.[2] Genberg and Cormany used a nonlinear programming method for the optimum design of a lightweight primary mirror.[3] In all these works, however, because parametric optimization was used, the given geometry of the mirror limited the final solution. In contrast, we optimized a lightweight mirror using a topology optimization method that overcomes the initial geometry limitation. The topology optimization of a continuum structure was first attempted by Bendsoe and Kikuchi.[4] Until now, however, such optimization has been conducted only on nonoptical structures under conditions of static loading.[5]

We used our topology optimization with various weight constraints to optimize several lightweight mirrors. In the manufacturing process, we based our calculations on rms surface deformation errors and the Strehl ratio as an objective function under self-weight loading and polishing pressure. That is, by using the suggested method of mirror design, we optimized the shape of a lightweight mirror that simultaneously satisfied the required optical performance and manufacturing constraints. The optimization method can therefore overcome the limitation of parametric studies on lightweight mirror design, and achieve the optimum shape of a lightweight mirror within the design specifications and operating conditions.

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2 Topology Optimization Procedure

2.1 *Fundamental Concepts*

We can represent the shape of a structure with parameters of the structure's boundary or with an indicator function. With parameters, we cannot change the topology of the structure during the optimization process, thereby making it difficult to obtain a truly optimized structure through the optimization process. However, if we use an indicator function, we can obtain a truly optimized structure. Thus, for the topology optimization, we introduced the following indicator function:

$$\chi(x)=\begin{Bmatrix} 1 & \text{if } x\in\Omega^m \\ 0 & \text{if } x\notin\Omega^m \end{Bmatrix}, \tag{1}$$

where Ω^m is the region occupied by the material.

By using this indicator function, we can represent the elasticity tensor at point x as follows:

$$E_{ijkl}(x)=\chi(x)E_{ijkl}^m, \tag{2}$$

where E_{ijkl}^m is the material's elasticity tensor.

To find the optimized shape of a structure with the help of the indicator function, we must find the value of the indicator function at all points in the structure. In general, however, the solution of the problem is not guaranteed,

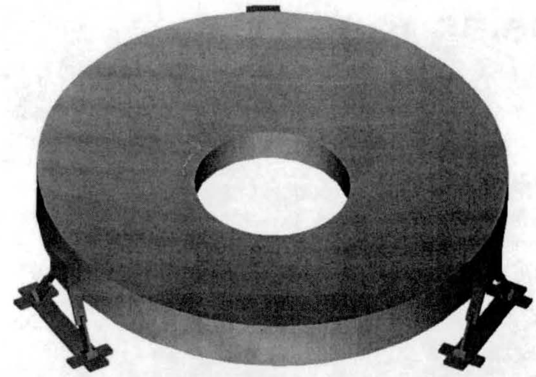


Fig. 1 Full primary mirror model.

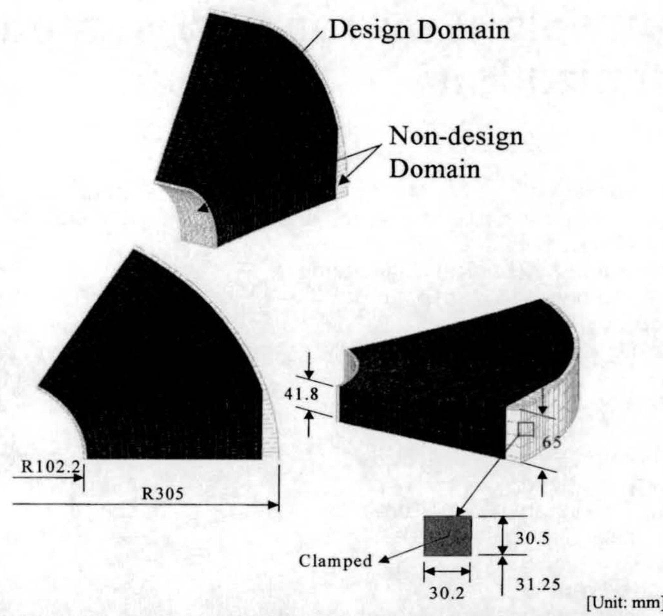


Fig. 2 The 1/6 primary mirror model used in the topology optimization.

because the indicator function is not smooth. We therefore need to use homogenization. By using material density instead of the indicator function, we can state the general problem of shape optimization in terms of finding the material density at all points in the structure.

We divided the structure into a finite number of elements and sought the material density of each element through the optimization process. On the assumption that the material density in each element was uniform, we deduced that, if the material density ρ had a value between 0 and 1, we would obtain the functional relationship between the elastic moduli and the material density.

There are several methods of obtaining the functional relationship between the moduli and the material density. We can apply the homogenization method after introducing a microstructure, or we can use an artificial material model that arbitrarily relates the elastic moduli to the material density without introducing any specific microstructure. The homogenization method has been used since it was introduced by Bendsoe and Kikuchi.[4] However, the patterns of the optimal density distributions depend strongly on the microstructure used. This approach also requires an additional finite element analysis to obtain the homogenized material properties. Although the artificial material model is simple, the quality of its solution is better than the homogenization method if an adequate functional relationship is used.[6] We therefore used the following artificial material model, which is based on the Hashin-Shtrikman lower bound:[6]

$$\frac{E_{ijkl}}{E^m_{ijkl}}=\frac{\rho_{idv}}{1+\alpha(1-\rho_{idv})}, \tag{3}$$

where ρ_{idv} is the density in the i'th element and α is a constant whose intermediate density is penalized differently. In the computation, we used $\alpha=20$.

2.2 *Definition and Formulation of the Optimization Problem*

After introducing the concept of material density, we can state the problem of topology optimization as a typical optimization problem. The material densities in the structure's element groups are treated as the following design variables:

$$\begin{aligned} &\text{minimize } f(X) \\ &\text{subject to } h_i(X)\leq 0, \quad 1\leq i\leq M, \\ &g_j(X)=0, \quad 1\leq j\leq N, \end{aligned} \tag{4}$$

where $X=[\chi_1,\chi_2,\cdots,\chi_{ndv}]^T$ is the design variable vector and ndv is the number of element groups in the design domain. The objective function and constraint conditions in Eq. (4) can be defined differently depending on the purpose of the optimization. Our purpose was to minimize the rms surface deformation errors and the Strehl ratio under the constraint of the total mass of the design domain. Thus, we can define the objective function and the constraint condition as follows:

$$f(X)=\left(\frac{\int_{\Omega_s}U_n^2 d\Omega}{\int_{\Omega_s}d\Omega}\right)^{1/2}=\left(\frac{\Sigma_{e=1}^{n_{se}}\int_{\Omega_e}U_n^2 d\Omega}{\Sigma_{e=1}^{n_{se}}\int_{\Omega_e}d\Omega}\right)^{1/2}, \tag{5}$$

$$h(X)=\int_{\Omega_d}\rho d\Omega_d-M_0=\sum_{idv=1}^{ndv}\rho_{0idv}\chi_{idv}\Omega_{idv}-M_0\leq 0, \tag{6}$$

where Ω_s is the optical surface, U_n is the normal component of the mirror surface displacement, n_{se} is the number of optical surface elements, Ω_e is the optical surface of the optical surface element, Ω_d is the design domain, ρ is the density, ρ_{0idv} is the density of the material in the i'th element group, χ_{idv} is the material density $(0\leq\chi_{idv}\leq 1)$, Ω_{idv} is the volume of the i'th element group, and M_0 is the specified limit of mass.

To obtain a 2-D density distribution from a 3-D model, we used an element grouping procedure. That is, we gave each group of elements the same design variable in the optimization procedure. We then used a direct differentia-

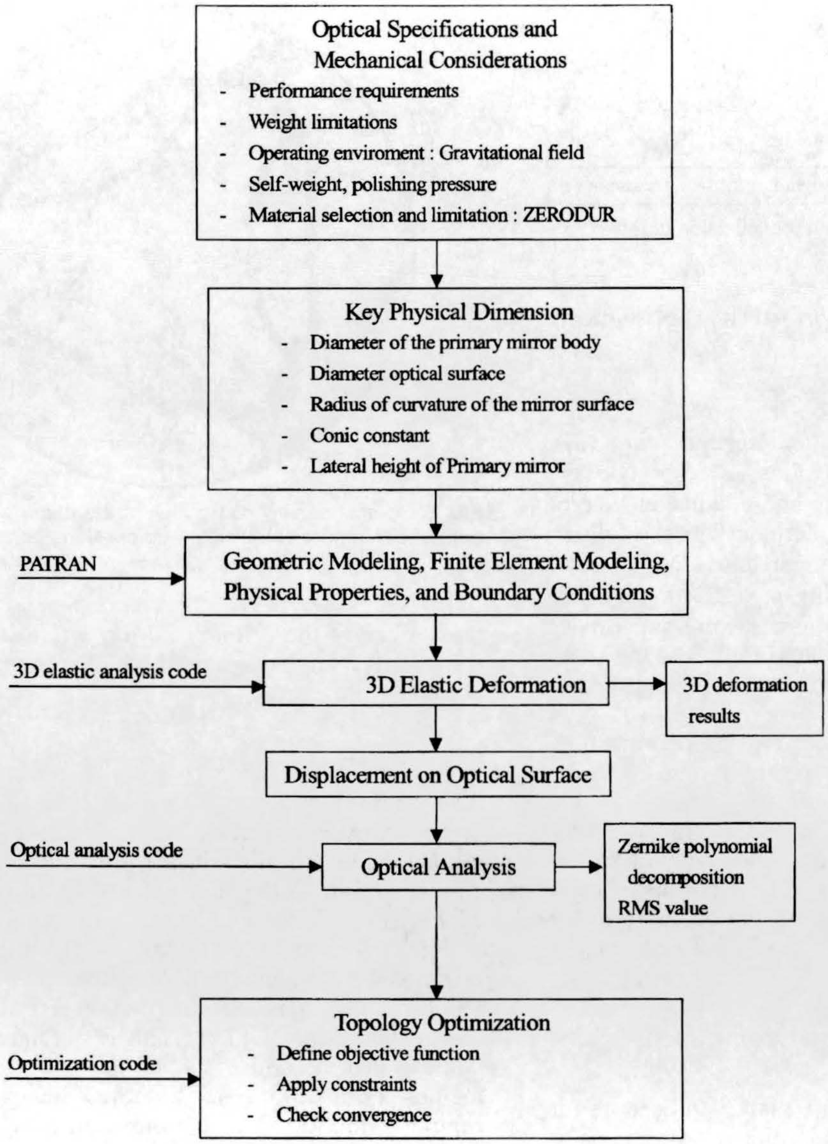


Fig. 3 Flowchart of the analysis procedure.

tion method to calculate the sensitivities of the objective function and constraint. The sensitivities of the objective function were calculated as follows:

$$\frac{\partial f(X)}{\partial \chi_{idv}}=\frac{\partial}{\partial \chi_{idv}}\left(\frac{\Sigma_{e=1}^{n_{se}}\int_{\Omega_e}U_n^2 d\Omega}{\Sigma_{e=1}^{n_{se}}\int_{\Omega_e}d\Omega}\right)^{1/2}$$

$$=\frac{1}{(\Sigma_{e=1}^{n_{se}}\int_{\Omega_e}d\Omega)^{1/2}}\frac{1}{(\Sigma_{e=1}^{n_{se}}\int_{\Omega_e}U_n^2 d\Omega)^{1/2}}\times\left(\int_{\Omega_e}U_n\frac{\partial U_n}{\partial \chi_{idv}}d\Omega\right). \tag{7}$$

2.3 *Optimization Algorithm*

In the present optimization problem, the number of design variables is large because we needed fine finite element meshes to help resolve the shape of the structure. On the other hand, the constraints are simple. As a result, the optimality criteria method is an adequate updating rule for the optimization problem in this category. The updating rule is based on the optimality suggested by Bendsoe and Kikuchi.[4] Although the updating rule is simple and efficient for typical static problems, it is inadequate in cases of self-weight loading. For typical static problems, the objective function has negative sensitivities to the design variables. In other words, we can decrease the objective function by increasing the material density of the elements in the design domain of the structure. However, for the self-weight loading problem, the objective function's sensitivities to design variables can be negative or positive, depending on the elements. After considering this feature of the self-weight loading problem, Ma, Kikuchi, and Hagiwara proposed a new algorithm, which they derived by using a new convex generalized-liberalization approach via a shift parameter.[7] In this work, we used the same algorithm.

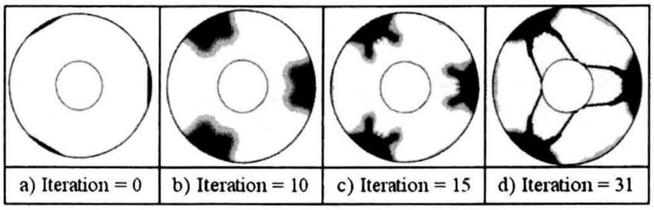


Fig. 4 Topology of the mirror as the optimization iterates.

3 Application to the Design of a Lightweight Primary Mirror

3.1 *Primary Mirror Model*

For this study, we considered a primary mirror from a Cassegrain space telescope. The mirror is an f/1 $\cdot x$ concave mirror with a diameter of 610 mm and a diameter hole of 204.4 mm. The mirror is made of Zerodur with the following mechanical properties: Young's modulus, $E_0 = 91$ GPa; Poisson's ratio, $\upsilon = 0.24$; and density, $\rho_0 = 2530\,\text{kg/m}^3$. The mirror is supported at three A-shaped flexure supports at the edge of the surface, as indicated in Fig. 1. The thickness is 41.8 mm for the inner edge and 65 mm for the outer edges.

To reduce the computing cost of topology optimization, we used a 1/6 model. Figure 2 shows the symmetry of the model. We then excluded the edge of the surface from the design domain for ease of support, manufacturing, and handling. We also distinguished the design domain with different grades of shading. Finally, we used eight-node linear solid elements to calculate the deformation of the primary mirror, and we used four-node linear surface elements on the optical surface.

3.2 *Application of the Topology Optimization Method*

Figure 3 presents a flowchart of the analysis procedure for topology optimization of the primary mirror. After considering the element grouping in the height direction, we obtained 2-D patterns of the density distribution. Furthermore, because gravitational force and polishing pressure displace

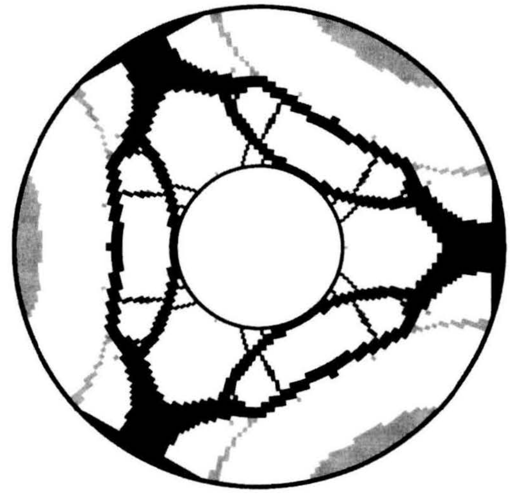


Fig. 6 Optimal density distribution of the primary mirror for a mass ratio of 22% under self-weight and polishing pressure loading.

the surface of the primary mirror, we used 3-D finite element analysis to extract the normal component of this displacement.

3.3 *Application Results*

As a preliminary example, we optimized a primary mirror under self-weight loading. For our space application, we needed the mass ratio, which is the ratio of the mass of the current design to the mass of a solid-filled design, to be 22%.

Figure 4 shows the topology of the mirror as the optimization iterates. Figure 4(d) shows that, for the optimal topology, the material distribution is high around the clamped boundary regions. This result agrees with the hypothesis that the mirror should be rigid around the support regions. Furthermore, six branches are extended into the central region to reduce the maximum displacement of the optical surface. In Fig. 5, the objective function (the rms surface error) monotonically converges from 90 nm to a minimum value of 39.4 nm. From these results, we can

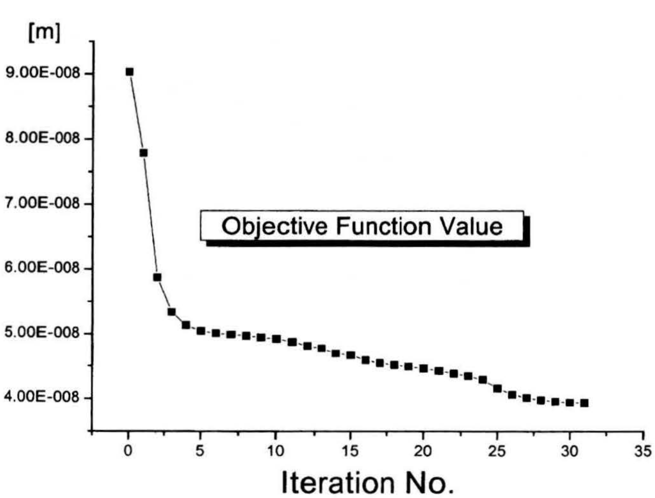


Fig. 5 Objective function values for a mass ratio of 22% under self-weight loading.

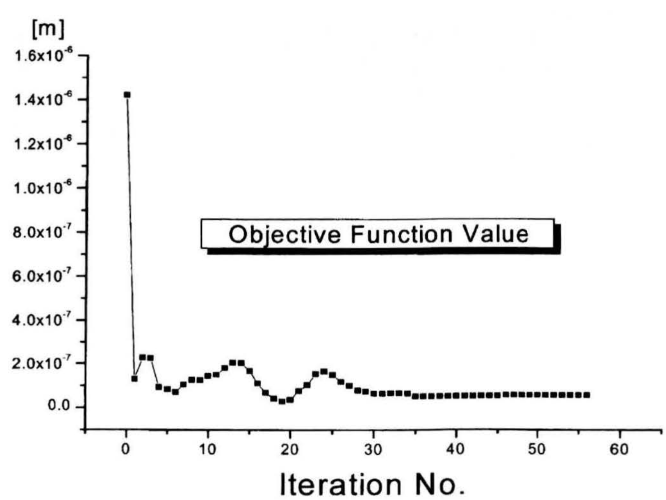
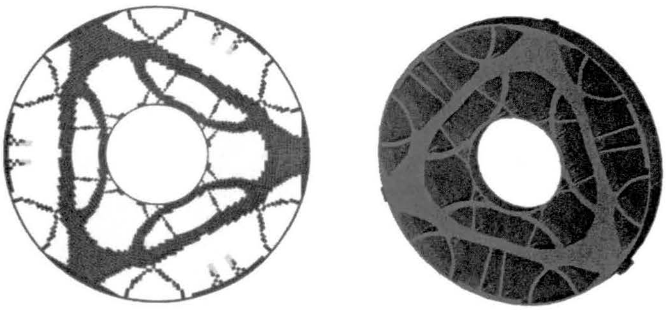
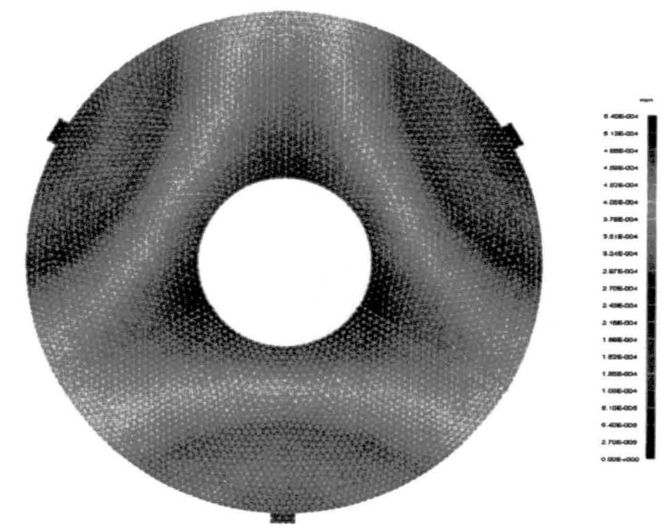


Fig. 7 Objective function values for a mass ratio of 22% under self-weight and polishing pressure loading.

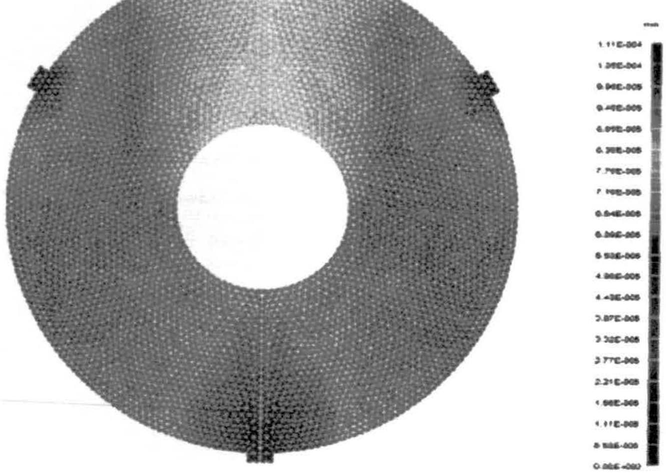


a) Optimized topology   b) Verification model

Fig. 8 Optimized topology and its verification model for a mass ratio of 35% under self-weight and polishing pressure loading.

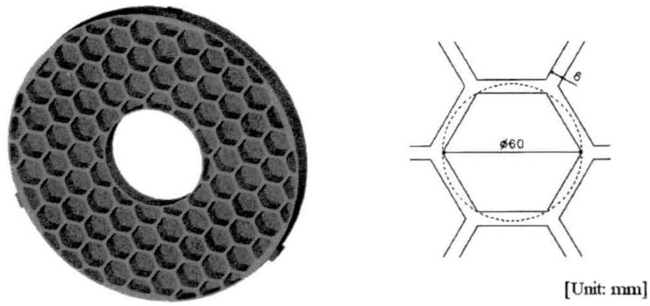


a) Under parallel gravity to the optical axis and polishing pressure



b) Under vertical gravity to the optical axis

Fig. 9 Surface deformations of the verification model.



a) Lightweight hexagonal cell pattern   b) Dimension of the unit hexagonal cell

Fig. 10 Classical lightweight mirror model with a hexagonal pattern.

confirm that, with respect to the primary mirror with a self-weight loading, the optimal topology has a good optical performance.

However, depending on the manufacturing method, quilting of the optical surface poses a serious damage risk to the quality of the optical image. In the optimization process, we must therefore give practical consideration to how the polishing pressure deforms the optical surface. In this work, we assumed that the polishing pressure was 0.2068 kPa (0.3 psi).

Figure 6 shows the optimal density distribution of the primary mirror for a mass ratio of 22% under a self-weight and polishing pressure loading. Figure 7 shows that although the value of the objective function for these loadings does not decrease monotonically, the value of the objective function converges into a certain value.

3.4 *Comparison with a Classical Mirror and Development of a Prototype*

As a verification process, we used the results of the optimization to construct a model of a geometrical mirror. The optimized results featured a mass ratio of 35%, which was due purely to the limitation of the manufacturing capability.

Figure 8 shows the optimized topology and its verification model. We modified the verification model in line with manufacturing constraints such as the minimum thickness of the lightweight patterns.

Figure 9 shows the deformations in the optical surface of the verification model, first, under parallel gravity to the optical axis and polishing pressure, and second, under vertical gravity to the optical axis.

To compare the rms value of the optimized mirror, we constructed a lightweight hexagonal cell mirror with the same design parameters—namely, the thickness of the optical surface, the inner diameter, the outer diameter, the inner-hole diameter, the radius of curvature, and the 35% mass ratio. Figure 10 shows the geometry and dimensions of the hexagonal cell mirror. We maintained the cell thickness at 6 mm to correspond to the minimum thickness of the verification model of the optimized mirror.

We then compared the rms values of the optimized mirror and the hexagonal cell mirror under two different loadings. Table 1 shows the rms values of the mirrors. For the verification model of the optimized mirror, the rms value is 134.4 nm under the parallel gravity and polishing pressure loading. For the vertical gravity loading, the rms value is 10.5 nm. However, the rms value of the hexagonal cell

Table 1 Analysis results: the rms value of the optimized mirror and the hexagonal cell mirror.

	Optimized mirror	Hexagonal cell mirror
rms value for parallel gravity to the optical axis and polishing pressure loading (nm)	134.4	156.6
rms value for vertical gravity to the optical axis (nm)	10.5	8.93

mirror is 156.5 nm under the parallel gravity and polishing pressure loading, and 8.93 nm under the vertical gravity loading. From the results, we conclude that the optimized mirror has 22.1-nm improvements in the rms value for the parallel gravity and polishing pressure loading, and 1.57-nm immaterial differences for the vertical gravity loading.

Based on analysis of the optimized mirror model, we manufactured a prototype mirror. Figure 11 shows photographs of the prototype. Because we are still preparing an optical setup for testing the manufactured mirror, we have omitted the optical test results in this work.

4 Conclusion

Using a topology optimization method, we design a lightweight primary mirror under self-weight and polishing pressure loading. For the self-weight loading, the objective function shows monotonic convergence to a minimum value during the iterations. For the optimization, we give practical consideration to how the polishing pressure deforms the optical surface. Our optimization produces patterns of a lightweight mirror under self-weight loading and polishing pressure. To verify the topology optimization design method, we compare the optical performance to that of a classical lightweight mirror with a hexagonal cell pattern. The optical performance of the optimized mirror is better. We therefore manufacture a prototype of the optimized mirror on the basis of these results.

In the future, we expect to research the topology optimization of a lightweight primary mirror under other mechanical conditions such as thermal and dynamic loading. Furthermore, for the optimization process, we will consider other manufacturing constraints such as the minimum thickness of the lightweight pattern.

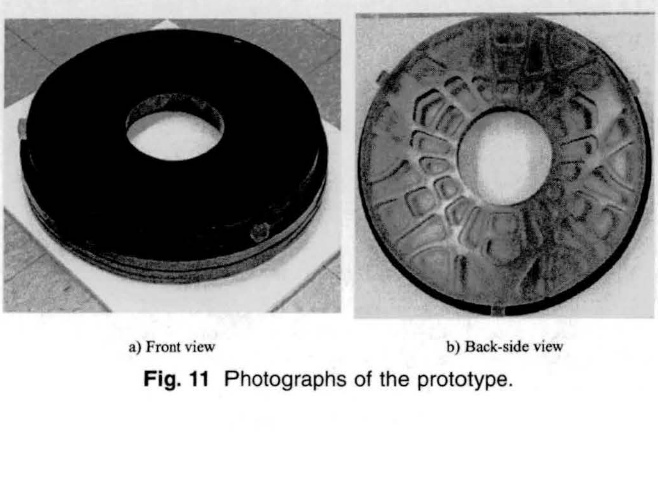


Fig. 11 Photographs of the prototype.

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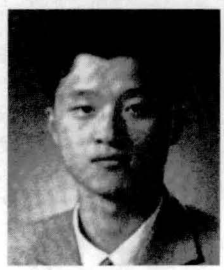

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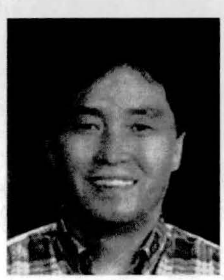

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