Is Investment –Uncertainty Relationship Monotonic ?

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INTRODUCTION

A considerable number of empirical studies exist which incorporate different measures of uncertainty in analyzing its impact on investment. Whereas Caballero (1991), Hartman (1972), Abel (1983), and others point to the positive impact of uncertainty on capital intensity of production under irreversibility of investment, McDonald and Siegel (1986), Dixit and Pindyck (1994) find that uncertainty is damaging to investment. Their argument is that in the long-run, the user-cost effect would dominate and that increased uncertainty in the long run would increase the expected capital stock under irreversibility and can increase it even more under reversibility. Actually, however, whether the increase in the expected long-run capital stock is larger under reversibility or under irreversibility depends on the choice of parameter values[1]. Almost the entire investment literature assumes a linear relationship between uncertainty and investment but the uncertainty- investment may be non-linear. Sarkar (2000) and later Abel and Eberly (1999) provide an excellent interpretation of the non-linear relationship between investment and uncertainty. Sarkar (2000)[2] points out that an increase in uncertainty increases the probability that the investment threshold will be crossed so that the probability of investment taking place within a specified time period increases. Sarkar's approach to investment (which is an option pricing approach) suggests that the investment- uncertainty relationship can be described by an inverted U- shaped curve. It is strange that the entire empirical literature has ignored this.

The purpose is to examine the empirical relevance of the uncertainty-investment link which is non- linear. The scheme of the paper is as follows: Section 1 provides a succinct version of the Sarkar's model. Section 2 provides two types of estimates on the impact of uncertainty measured by the volatility of stock market returns in India, Pakistan and Bangladesh. Section 3 analyses the results. In section 3, we provide estimate of an investment model that includes a linear and a quadratic term. Section 4 explains an investment model in which uncertainty becomes a threshold variable. Both types of estimates provide evidence on the empirical relevance of a non-linear effect of uncertainty on investment. Section 5 concludes the paper.

SARKAR'S MODEL

The present value of the expected future cash flows is assumed to follow a geometric Brownian motion with drift:

$$\partial V_t = \mu_t \partial_t = \sigma V_t \partial z_t$$

Where μ is the expected growth rate of the cash flow ∂V, σ the standard deviation of the growth rate (the uncertainty effect) and ∂z is an increment of a Weiner process. The latter assumption implies that the present value of the project is known when the firm invests immediately. However, when the firm invests, the future value becomes uncertain with a variance that grows linearly with the time horizon. There is a growth effect represented by a positive μ and an uncertain effect given by σ.

The problem is to find a critical value of V *(say* V^**)* such that firms will invest once $V > V^*$. This is solved by determining the value of the option to invest. More formally, this works as follows. The pay-off from investing at time t is $V_t - I$ where I are investments costs (assumed constant). The problem then is to maximize.

$$X(V) = \max E\,[V_t - I]e^{rt}$$

subject to the equation for the present value of the expected future cash flows. $x(v)$ denotes the value of the option to invest, E denotes the expectation operator, r is a constant discount rate, and t is the unknown future date. It is assumed that $\mu<r$, since otherwise waiting would always be better than investing and V would increase indefinitely as a function of t.

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The optimization problem can be solved by using dynamic programming. The solution can be described by the following valuation equation[3]:

$$X(V) = AV^n, \text{ where}$$
$$\alpha = 1/2 - \mu / \sigma^2 = ([\mu/\sigma^2 - 0.5]^2 + 2r/\sigma^2]^{1/2} \text{ and}$$
$$A = [(\alpha - 1)^{\alpha-1}]/[\alpha^{\alpha}/I^{\alpha-1}]^2$$

By differentiating α with respect to σ it can be easily shown to be equal to

$$V^* = (\alpha/\alpha - 1)I$$

Since $\partial\alpha / \partial\sigma < 0$, an increase in σ leads to an increase in $\alpha / (\alpha - 1)$. Therefore, an increase in uncertainty σ will lead to an increase in V^* and thus increases the trigger value of investment.

Sarkar derives the following expression for the probability of investing within some time period, T.

$$\text{Prob}(I) = \phi(In(V_0/V^*) + (\mu - 1/2\sigma^2)T)\ \sigma(T)0.5)$$
$$+ (V^*/V_0)\beta\ \sigma(In(V_0/V^*) - (\mu - 1/2\sigma^2)T)/\sigma(T)^{0.5})$$

where $\beta = 2\sigma/\sigma^2 - 1$ and V_0 is the starting value of V and ϕ is the area under the standard normal distribution. By substituting V^* in Prob (I), the overall effect of an increase in uncertainty on investment is derived. Since there is a positive and negative effect on investment, the overall effect becomes ambiguous.[4]

It may be relevant to spell out the differences between this model and the CAPM model. The CAPM model measures risk by the covariance of a firm's return with the market as whole. According to the CAPM model, uncertainty affects investment only if affects covariances in the returns between investment projects. In the irreversible investment models and in this option model of ours, uncertainty affects investment[5].

RESULTS

We empirically analyze the impact of the uncertainty on the investment to GDP ratio, INVGDP, for a panel of three countries in Asia, India, Pakistan and Bangladesh covering a time period from 1980 through 2004. To protect the dynamic properties of the model we employ annual data.

We estimate the following equations:

$$INVGDP = \beta_{i,1} + \beta_2\, LGDPPCI + \mu + \beta_3\, UNCER + \mu \quad (1)$$
$$INVDDP = \beta_{i,2} + \beta_4\, LGDPPCI + \beta_5\, UNCER + \beta_6\, UNCER^2 + \mu \quad (2)$$
$$INVGDP = \beta_{i,3} + \beta_7\, LGDPPCI + \beta_8\, UNCER + \beta_9\, UNCER^2 + \beta_{10}\, GROI + \mu \quad (3)$$
$$INVGDP = \beta_{i,4} + \beta_{11}\, LGDPPCI + \beta_{12}\, LGDPPCI^2 + \beta_{13}\, UNCER + \beta_{14}\, UNCER^2 + \beta_{15}\, GROI + \beta_{16}\, GOV + \mu \quad (4)$$
$$INVGDP = \beta_{i,5} + \beta_{17}\, LGDPPCI + \beta_{19}\, UNCER + \beta_{20}\, UNCER^2 + \beta_{21}\, GROI + \beta_{22}\, GOV + \beta_{23}\, AVGRET + \mu \quad (5)$$

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[1] For this result to occur, the marginal revenue product of a firm should be a decreasing function of the capital stock. If the marginal revenue product does depend on the capital stock, then the current and future marginal revenue products are unaffected by today's investment, so the link from today's investment to future returns is broken. The firm is then no more reluctant to invest under irreversibility than with reversible investment.

[2] Sarkar's model is a modified version of McDonald and Siegel (1986). McDonald and Siegel show that investment irreversibility and uncertainty drive a wedge between the value of the project and the investment costs. By deriving an exact expression of the wedge, they are able to show to what extent the standard net present value of investment has to be adjusted. The model assumes that the firm controls the timing of a totally irreversible investment problem and in each period, therefore, chooses between investment (stopping) or waiting (continuation) for one more period.

[3] The entire derivation is not shown here. For derivation, See Lensink *et al* (2001) or Dixit and Pindyck (1994).

[4] Sarkar gives an example by which he compares the probability of investing for different values of α. This example shows that the additional positive 'hitting' effect dominates the negative threshold effect for low levels of uncertainty. More in particular, the numerical example suggests that the investment-uncertainty relationship can be described by an inverted U-shaped curve. In this case, the probability of investing is an increasing function of volatility for $\sigma < 0.39$. For $\sigma < 0.39$, it becomes a decreasing function of volatility.

[5] See Leahy and Whited (1996) and compare the effects on investment of an uncertainty measure proxyed by the volatility of the stock market return and the CAPM model.

Where *LGDPPCI* is the logarithm of per capita real *GDP, GROI* is the one year lagged growth rate of real *GDP, AVGRET* is the average stock return, *GOV* is the share of government expenditure in *GDP* and is an error term. The uncertainty measure of the countries is obtained through standard deviation of monthly stock returns for the countries under study. Before submitting the regression results, we submit the correlation matrix in Table 1.

Table 1: Correlation Matrix

	GROI	LGDPPC	INVGDP	GOV	UNCER	AVGRE
GROI	1					
LGDPPC	-0.30	1				
INVGDP	0.44	-0.35	1			
GOV	-0.44	0.26	0.49	1		
UNCER	0.21	-0.38	-0.22	-0.66	1	
AVGRET	-0.55	0.22	-0.23	0.11	0.43	1

Fixed effects estimators allow the intercepts to vary by country. Because heteroskedasticity could be important across countries, the standard errors for the coefficients are based on White's heteroskedasticity - consistent covariance matrix. Table 2 presents 'fixed effect' estimates of the different models. Equation 1 contains only the starting value of the real GDP per capita and the uncertainty proxy. LGDPPCI captures the effect of conditional convergence[6]. In Table 2, the increase in UNCER has a negative effect on INVGDP but the effect is not found to be statistically significant at the 5% level of significance.

The detrimental effect of uncertainty on investment is consistent with the overwhelming findings of the recent studies.[7] [8] The next step is to re- estimate the equation (1) by using a different specification and using a quadratic term for UNCER. This allows for testing non-linear effects of uncertainty. The result is presented in equation (2). In Table 2, it appears that the linear term is found to be statistically significant, whereas the quadratic term is significantly negative. This argues for the existence of non-linear relationship between uncertainty and investment. Low levels of uncertainty exhibit positive effects but above a certain level of uncertainty, uncertainty begins to create a negative effect.

The regression results presented in Table 2, through the right signs of the linear and quadratic terms, do indeed establish a case for an inverted U-shaped curve[9] and thus endorse Sarkar's findings. We estimate the basic equation (eq. 3) of Table 2 by allowing for country-specific coefficients for the linear and quadratic uncertain terms by applying SUR (Seemingly Unrelated Regression) method. We assumed common coefficients for LGDPPCI and GROI but country-specific coefficients for UNCERT and $UNCER^2$ .We also assumed country-specific constants. Table 3 provides the results.

The estimates confirm the investment-uncertainty relationship and the U-shaped curve for India and Pakistan.

[6] Barro and Sala- i- Martin (1995) uses a logarithmic expression.

[7] There are not many studies where the effects of uncertainty on investment is tested. Besides, these studies are difficult to compare with each other and also with the present study, since they differ in terms of empirical approach and specification. For instance, Aizenman and Marion (1993) as well as Serven (1997) have focused on private investment, whereas others, including the present author concentrate on the aggregate investment rate. Ghuru and Grennes (1999) use exchange rate volatility as a measure of uncertainty. Serven as well as Ghuru and Grennes consider African countries; Pindyck and Solimana (1993) consider Latin American countries. Despite all these differences, there is a surprising similarity in outcomes; all studies, except a few, provide ample empirical evidence of a negative relationship between uncertainty and investment.

[8] A smaller strand of the literature is concerned with the effect of political uncertainty on investment. The study by Barro (1991) for instance finds that measures of government instability (the number of revolutions) and political violence (the number of assassinations) are significantly related to cross-country differences in investment.

[9] The robustness or sensitivity of the tests is not affected by a change in the estimation techniques. We re-estimated eq. (3) which we used as the base equation (see Lensink (2002)) for other sensitivity tests, assuming random instead of fixed effects (eq 2). To test for hetroskedasticity, we used Generalized Least Squares (GLS) with cross section weights (results are not reported). Qualitatively, the results are not any way different – the linear term is significantly positive and the quadratic term is significantly negative.

Table 2 : Uncertainty and Investment[10]

	1	2	3	4	5
LGDPPCI	-7.785 (-7.78)	-11.332 (-8-44)	-7.768 (-6-65)	-6.903 (-5.43)	76.222 (7.88)
$LGDPPCI^2$					-9.888 (-7.90)
UNCER	-.5.652 (-0.33)	15.431 (2.56)	33.211 (4.545)	21.431 (4.531)	45.321 (6.721)
$UNCER^2$		-231.311 (-5.45)	-101,111 (-5.64)	-134.231 (-7.56)	-233.45 (-9.85)
GROI			0.123 (2.45)	0.654 (3.67)	5.41 (6.44)
GOV				-0.555 (-9.56)	- 0.653 (-7.59)
AVGRET					32.333 (5.68)
Adj. R^2	0.72	0.86	0.78	0.83	0.80
F	344.431	389.52	411,0	367.44	368.31
Top % above	15	10	10	7	7
N	70	70	70	70	70

Table 3: Country-Specific Effects For The UNCER Measure

	UNCER	$UNCER^2$
India	42.344 (6.99)	-98.650 (-5.77)
Pakistan	19.222 (3.67)	- 45. 344 (-6.45)
Bangladesh	7.321 (0.67)	11.642 (-1.32)

THRESHOLD REGRESSION EFFECTS

To differentiate between the high and low levels of investment, we estimated a threshold model of investment[11]. We use *UNCER* as a threshold variable to classify countries into groups and to estimate the threshold value. The following model was estimated:

$$INVGDP = \beta_{i,6} + \beta_{24}\, LGDPPCI + \beta_{25}\, GROI + \beta_{26}\, UNCER\ 1\ (UNCER < THR)$$
$$+ \beta_{27}\, UNCER\ 1\ (UCER > THR) + \mu \qquad (6)$$

where *THR* is the threshold level of *UNCER* and *I* is the indicator function that has a value one if the argument is true and zero otherwise. The equation was estimated with individual specific fixed effects. Least square estimation of the threshold and regression slopes are proposed using fixed-effects transformation. The computation of the least squares estimate of the threshold involves a minimization problem. The optimum value is the value that minimizes the sum of squared residuals.[12]

[10] Some of the variables which are important at the firm level, for example, investment costs, proxies capturing the accelerator effect. Nevertheless, GROI variable is included here to pick up some of accelerator effects. The sign is expected to be positive. Table 2 (eq 3) does show the importance of this effect. The inclusion of the variable GOV is in line with Barro (2000) .Although it is difficult to account for Tobin's Q, Table 2 (eq 4) , we made an attempt to incorporate this effect by AVGRET which is found to be positive, unlike in Lensink (2002).

[11] The observed data are from a balanced panel $(y_{it}, q_{it}, x_{it}, 1 < i < n, 1 < t < T)$. The subscript *I* indexes the individual and the subscript t indexes time. The dependent variable y_{it} is scalar; the threshold variable q_{it} is scalar and the regressor x_{it} is a *k* vector. The structural equation of interest in a compact representation is to set

$$xx_{it}(\gamma) = \begin{bmatrix} x_{it}\, I\,(q_{it} \le \gamma) \\ x_{it}\, I\,(q_{it} \ge \gamma) \end{bmatrix}$$

Where $\beta = (\beta'_1, \beta'_2)'$ such that

$\gamma_{it} = \mu_t + \beta_{xit}(\gamma) + \varsigma_{it}$

The observations are divided into two 'regimes' depending on whether the threshold vaiable q_{it} is smaller or larger that the threshold γ. The regimes are distinguished by differing slopes, β_1 and β_2. We assume that the threshold variable q_{it} is not time invariant. The error ς_{it} is assumed to be independent and identically distributed with mean zero and finite variance σ^2. The analysis is asymptotic with fixed T as $n \rightarrow \infty$
The least square estimator of γ is

$\gamma = min\ S1\ (\gamma)$

[12] The search for the thresholds is restricted to specific quantiles. The advantage of this is that the number of regressions is educed but still yielding proper estimates. See Hansen (1999) and Lensink (2002).

The estimation is obtained as follows:

$$INVGDP = -\ 9.521\ LGDPPC + 0.4005\ GROI\ +\ 23.412\ UNCER\ I\ (\ UNCER\ <\ 0.19) -$$
$$(-3.67) \qquad (8.43) \qquad (1.45)$$
$$24.121\ UNCERT\ I\ (\text{UNCER}\ > 0.19)$$
$$(-3.03)$$

It is important to determine whether the threshold effect is statistically significant. The hypothesis of no threshold effect can be represented

$$\mathbf{H_0 : \beta_i = \beta_j}$$

Under H_0 the threshold is not identified, so classical tests will have non-standard distributions. This is typically called the 'Davies' problem (Davies (1987), and Andrews and Ploberger (1994). The value of the threshold obtained here is 0.19 and is found to be statistically significant by a likelihood ratio test. The sum of the squared residuals is found to be 678.45 which is lower than that of the linear model with no threshold. The model we have estimated has a single threshold, although in some applications, there may be multiple thresholds where thresholds could be ordered[13].

CONCLUSION

The paper demonstrates the empirical relevance of non-linear effects of uncertainty, measured by the volatility of stock market returns, on aggregate investment for a set of countries in Asia. The results strongly suggest that there are differences between the impact of low and high levels of uncertainty on the aggregate investment levels. It appears that high-levels of uncertainty have less favorable effects on investment than low levels of uncertainty. Two types of estimation confirm the non-linear impact on investment. First, we estimate the investment equations by including a linear and a quadratic term for uncertainty. The linear term in most of the cases is found to be significantly positive, whereas the quadratic term is significantly negative. Second, we estimated a threshold regression in which proxy for uncertainty is the threshold variable. Our threshold regression models specify that individual observations can be divided into classes or groups based on the value of an observed variable. Despite their intuitive appeal, econometric techniques have not been well developed for a threshold regression approach. We provided appropriate econometric technique for threshold regression with panel data. It is important for future research to extend the impact of uncertainty to a variety of countries to improve the quality of results. Perhaps, it is also important to determine if macroeconomic instability is more important than political instability or corruption.

APPENDIX

List of variables and sources

AVGRET = Average yearly stock market returns derived from monthly stock market returns (based on monthly stock market indices. IFS

GOV = The ratio of government expenditure to GDP. World Bank

GROI = The one year lagged growth rate of GDP , World Bank

INFL = The inflation rate, IFS

INVGDP = The investment to GDP ratio World Bank

LGDPPC = The logarithm of the real GDP per capita in constant dollar (international prices base year 1978. World Bank.

UNCER = Uncertainty proxy represented by standard deviation of monthly stock market returns IFS

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[13] If we compare the outcome of the of the threshold model with the egression results presented in Table 2, it appears that the coefficients for LGDPPC and GROI are similar. This provides further credence to the reliability of the estimate.