Prediction Of Stock Option Prices Using Volatility (Garch (1, 1)) Adjusted Black Scholes Option Pricing Model

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Abstract

This paper attempts to predict the option prices for the future date using the adjusted volatility to the traditional Black-Scholes option pricing model using GARCH (1, 1) in pricing the stock option contracts for the selected eight companies. The study uses the Black-Scholes model along with its basic parameters and the best known time series model GARCH (1, 1) for predicting volatility in order to estimate the future stock option contract prices. This helps in knowing how the prices of stocks would be in the near future. The study finally attempts to identify the pricing errors between the market price of the option contracts and the calculated option prices. This is done with the help of mean absolute percentage error and mean absolute deviation tools. The results of the study indicate that there was only a small difference between the calculated prices and the market price of the option contracts.

Keywords: **stock option pricing, GARCH (1, 1), Black-Scholes model, MAPE, MAD**
JEL Classification: C1, C13, G12

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There have been significant developments in the securities market in India during recent years, particularly with the introduction of derivative products since June 2000. The introduction of derivatives was well received by stock market players since derivatives serve as a risk reducing tool for the high volatile financial markets. An option contract is a type of derivatives contract, which has gained much attention among the investors. An option contract is a right to buy/sell a specified quantity of the underlying asset for a certain agreed price at a specified future date. There are different models for option pricing and valuation that are proposed by different researchers and academicians. Among them, the Black-Scholes (1973) option pricing model serves as an important and widely used option pricing model.

Research on Option Valuation

Constant elasticity of variance diffusion processes of Cox call option valuation model was tested against the Black-Scholes option valuation model. It was found that the Cox valuation model fits market prices of call options significantly better than the Black-Scholes Model (Macbeth & Merville, 1979). The Black-Scholes model overprices the options, and the degree of overpricing increases with the time to maturity. The Black-Scholes model implies volatility of an at-the-money option approximately equaling the expected future volatility over the life of the option (Hull & White, 1987). Severe mispricing was found in the volatility pricing of the index options with the help of the Black Scholes option pricing model and the GARCH (1, 1) model in Indian index options. Significant differences in volatility smiles for call and put options were also established (Varma, 2002).

Hedging positions of the options were examined by comparing GARCH deltas with the help of Black – Scholes's model with the GARCH (1, 1) Model with respect to moneyness of the option contract (Hsing, 2003). The forecasting models were examined - whether they are capable of outperforming implied volatility in forecasting, using high end-frequency data and incorporating both long and short-term memory effects (Kinlay, 2005). A study to find out the determinants of the implied volatility resulted in higher volatility for deeply in-the-money and deeply out-of-the-money options than the at-the-money options (Misra & Kannan, 2006). GARCH models were used to forecast underlying stock volatility, and the researchers used the forecasted volatility in the Black-Scholes model in order to determine whether the corresponding options are fairly priced or not (Dash, Dagha, Sharma, & Singhal, 2009). The

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implied volatility function for selected individual equity call options from the Indian stock market were examined and analyzed to ascertain the extent of mispricing volatility (Mundhra & Agarwal, 2009). The predictive accuracy of the Black-Scholes model in pricing the Nifty index option contracts were tested, and it was examined whether the skewness and kurtosis adjusted Black-Scholes model gives better results than the original Black-Scholes model (Tripathi & Gupta, 2010). The reasons for the differences between the theoretical option pricing formula and the prevailing market prices were examined and the conformance of implied volatilities to volatility smile/skew were also studied (Agarwal, Mukhtar, Nataraj, Agarwal, & Arora, 2010). Savitha and Deepika (2013) examined the value of the put-call ratio as an indicator of the future stock market trend, and they also analyzed the stock and option strategies in different market conditions and their pay-offs. Optimal portfolio of options for speculators was constructed, and its performance was compared with that of an optimal stocks portfolio to find whether the options portfolio gives better returns on average than the stocks portfolio (Dash, Babu, & Kodagi, 2007).

Methodology

For the prediction of prices of stock options using the adjusted Black-Schloes model, selected stocks were chosen from the National Stock Exchange using judgmental sampling technique. For conducting the study, eight companies' stock prices were collected for three years from NSE's Website (http://www.nseindia.com) (Table 1).

Table 1 : List of the Selected Companies

S.no	Name of the Company
1	Axis Bank Ltd.
2	Bajaj Auto Ltd.
3	ICICI Bank Ltd.
4	Infrastructure Development Finance Co. Ltd.
5	Jai Prakash Associates Ltd.
6	Punjab National Bank
7	Sesa Goa Ltd.
8	Tata Consultancy Services Ltd.

Source : Compiled by the Authors

For the eight companies as mentioned in the Table 1, stock prices were collected for three years starting from January 1, 2009 to December 31, 2011. From the collected data, the stock prices' volatility was calculated. The calculated volatility of all the selected companies were used to calculate stock option prices using the volatility adjusted Black-Scholes option pricing model. Using the volatility adjusted Black-Scholes option pricing model, the European call option and European put options prices for four different strike prices for all the contracts for all the eight companies were calculated. The study considered the stock prices for the study period from January 1, 2009 till December 31, 2011, and the predicted prices were compared with that of the January 2012 prices of the concerned contracts respectively. Finally, the calculated option prices were compared with the market prices using the mean absolute deviation and mean absolute percentage error for finding out the efficiency of the volatility adjusted Black-Scholes option pricing model.

Black-Scholes Option Pricing Model : Fisher Black and Myron Scholes made a major contribution in the subject matter of derivatives when they developed the theoretical model for the pricing of European options on non-dividend paying stocks. The model influenced the academicians and practitioners in a great way to price and hedge European options. The Black-Scholes model for pricing of European options assumes constant volatility and Gaussian log-returns (Black & Scholes, 1973).

This study alters the normal volatility (σ) in the traditional Black-Scholes option pricing model (Hulland & Basu, 2007) to the GARCH (1, 1) volatility denoted for the study as (σ_n), which is estimated using the GARCH (1, 1) model. Thus, the new formula can be stated as follows :

For Call option, the formula is,

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

For put option, the formula is,

$$p = Ke^{-rT} N(d_2) - S_0 N(d_1)$$

Where,

$$d_1 = \frac{ln\ (S_0/K) + (r + \sigma_n^2/2)\ T}{\sigma_n \sqrt{T}}$$

$$d_2 = \frac{ln\ (S_0/K) + (r + \sigma_n^2/2)\ T}{\sigma_n \sqrt{T}}$$

or

$$d_1 = d_2 - \sigma_n\ T$$

Where, the variables are,
S_0 is the stock price of the underlying stock at time zero,
K is the strike price,
e has the value 2.7128,
r is the risk free rate,
T is the time to expiry,
σ_n is the stock price volatility calculated using GARCH (1, 1) model,
ln is the Natural logarithm,
$N(x)$ is the cumulative probability distribution function.

Estimation of Volatility using GARCH (1, 1) Model : The GARCH (1, 1) model was proposed by Bollerslev in 1986 (Bollerslev, 1986). In GARCH (1, 1), σ^2_n is calculated from along the run average variance rate V_L, as well as from σ_{n-1} T and u_{n-1}. The equation for GARCH (1, 1) is :

$$\sigma^2_n = \gamma V_L + \alpha u^2_{n-1} + \beta \sigma^2_{n-1}$$

Where, γ is the weight assigned to V_L, α is the weight assigned to u^2_{n-1}, and ß is the weight assigned to σ^2_{n-1}. Since the weights must sum to unity, it follows that:

$$\gamma + \alpha + \beta = 1$$

The "(1, 1)" in GARCH (1, 1) indicates that σ^2_n is based on the most recent observation of u^2 and the most recent estimate of the variance rate. The most general GARCH (p, q) model calculates σ^2_n from the most recent p observations on u^2 and the most recent q estimates of the variance rate. GARCH (1, 1) is by far the most popular of the GARCH models.
Setting, $\omega = \gamma V_L$, the GARCH (1, 1) model can also be written as:

$$\sigma^2_n = \omega + \alpha u^2_{n-1} + \beta \sigma^2_{n-1}$$

This is the form of the model that is usually used for the purposes of estimating the parameters. Once ω, α, and ß have been estimated, the γ can be calculated as 1-α-ß. The long-term variance V_L can then be calculated as ω/γ. For stable GARCH (1, 1) process, the condition must be $\alpha + \beta < 1$. Otherwise, the weight applied to the long-term variance is negative.

Mean Absolute Deviation and Mean Absolute Percentage Error : The mean absolute deviation and mean absolute percentage error were used to test the check the efficiency of the predicted values of share prices and option prices with the market prices.

Results

Estimation of Volatility using GARCH (1, 1) : GARCH (1, 1) model was used to predict the volatility of the selected stocks. GARCH (1, 1) model helps in predicting the volatility pertaining to the future time period of the stocks. Hence,

Table 2 : Estimation of Volatility Using GARCH (1, 1)						
S. No	Company	K	Constant	ARCH Coefficient	GARCH Coefficient	GARCH (1, 1)
1	Axis Bank Ltd.	0.0001	1.0068	0.1091	0.8909	0.0008
2	Bajaj Auto Ltd.	0.0017	1.0010	0.0000	0.1944	0.0019
3	HDFC Bank Ltd.	0.0025	0.9977	0.5140	0.0000	0.0026
4	ICICI Bank Ltd.	0.0001	1.0023	0.0686	0.9314	0.0008
5	IDFC Ltd.	0.0008	1.0082	1.0000	0.0000	0.0009
6	JP Associates Ltd	0.0001	1.0074	0.1391	0.8609	0.0014
7	Punjab National Bank	0.0002	1.0049	0.1624	0.8376	0.0006
8	Sesa Goa Ltd.	0.0023	1.0000	0.0000	0.0000	0.0023
9	TCS Ltd.	0.0022	1.0006	0.4133	0.0000	0.0022

Source : Compiled by the Authors

the decisions of investment in the stocks can be taken by knowing the volatility that is going to be persistent towards the stock. The results of the GARCH (1, 1) are presented in the Table 2. Table 2 shows the estimation of the volatility with the help of GARCH (1, 1) model. The GARCH (1, 1) results show that the value was highest for HDFC Bank Ltd. (0.0026), Sesa Goa Ltd. (.0023), and Tata Consultancy Services (0.0022), and the remaining companies' variance were below 0.0020.

↳ **Estimation of Option Prices Using the Black-Scholes Model :** The option prices for both call option and put options were calculated using the GARCH (1, 1) adjusted Black-Scholes option pricing formula. The stock option prices were estimated for 4 different strike prices, that is, two strike prices above the market price of the share and two strike prices lesser than the market share price of the stock. The Table 3 shows the estimated call option prices and the put option prices along with its strike price. The call and put option prices were predicted using the volatility estimated using the GARCH (1, 1) model instead of the historical volatility in the traditional Black-Scholes Model.

↳ **Comparison of the Market Prices and the Calculated Option Prices :** The calculated option prices were compared with the market prices of the options under the different strike prices chosen for the study. This helps in knowing the accuracy of the estimated option prices. The comparison was done between the market prices and the option prices that estimated historical volatility. The comparison was done for the call and put options under different strike prices. The comparison was done by estimating the deviation and error of estimation using the tools - mean absolute deviation and the mean absolute percentage error. The Table 4 shows that the mean absolute deviation was between 198.4 and 0.02. The mean absolute deviation was below 0 (Zero)% for 12 options of different strike prices and companies. The highest mean absolute percentage error was noted for HDFC Bank Ltd. as 1491%, 734%, 351%, and 174%.

↳ **Comparison of the Calculated Put Options Prices with the Market Prices of the Put Options :** The estimated GARCH (1, 1) volatility is used to predict the option prices by altering the historical volatility in Black-Scholes model to GARCH (1, 1,) volatility. The newly estimated prices are compared with the market prices of the put options to know the efficiency of the prediction of option prices. The Table 5 shows that the mean absolute deviation was between 19.74 and 0.00. The mean absolute deviation was below 0 (Zero) % for 12 options of different strike prices and companies. The highest mean absolute percentage error was noted for HDFC Bank Ltd., Bajaj Auto Ltd., and Tata Consultancy Services Ltd.

Findings

The study reveals a major finding that the stock option prices can be predicted for a contract at certain limitations. GARCH (1, 1) volatility adjusted Black-Scholes option pricing model resulted in a finding that the prices of the contracts can only be predicted for the lower strike prices than that of the higher strike prices. From Tables 4 and 5, it is inferred that the mean absolute deviation and mean absolute percentage error denotes higher deviation for the higher strike prices, while for the lower strike prices, the predicted prices were nearer to the market prices of the option contracts. Hence, this model of GARCH (1, 1), the adjusted Black Scholes model helps in attempting to predict the option prices of lower strike contracts.

Table 3 : Estimation of Option Prices				
S.No	**Name of the Company**	**Strike Price**	**Calculated Call Option Price**	**Calculated Put Option Price**
1	AXIS BANK	760	63.20	22.45
		780	51.36	30.46
		800	41.09	40.03
		820	32.35	51.13
2	BAJAJ AUTO	1350	172.72	36.65
		1400	139.90	53.44
		1450	111.34	74.49
		1500	87.06	99.82
3	HDFC BANK	420	53.89	43.76
		440	45.10	54.81
		460	37.51	67.06
		480	31.03	80.43
4	ICICI BANK	660	60.67	18.96
		680	48.55	26.68
		700	38.13	36.11
		720	29.40	47.21
5	IDFC	85	9.58	1.96
		90	6.43	3.77
		95	4.06	6.36
		100	2.41	9.67
6	JP ASSOCIATES	45	8.90	0.90
		50	5.47	2.42
		55	3.05	4.97
		60	1.55	8.43
7	PNB	750	43.03	20.91
		760	37.40	25.20
		780	27.65	35.29
		800	19.86	47.34
8	SESA GOA	140	20.66	2.46
		150	13.73	5.45
		160	8.48	10.13
		170	4.88	16.45
9	TCS	1000	198.44	12.01
		1050	158.64	21.82
		1100	123.47	36.26
		1150	93.49	55.89

Source : Compiled by the Authors

Table 4 : Comparison of Market Prices and the Calculated Call Option Prices

S.No	Name of the Company	Strike Price	Market Price	Calculated Call Option Price	MAD	MAPE
1	AXIS BANK	760	60.9	63.20	1.15	4%
		780	42.45	51.36	4.46	21%
		800	32.8	41.09	4.14	25%
		820	24.9	32.35	3.72	30%
2	BAJAJ AUTO	1350	150.4	172.72	11.16	15%
		1400	113.7	139.90	13.10	23%
		1450	73.4	111.34	18.97	52%
		1500	50.35	87.06	18.36	73%
3	HDFC BANK	420	19.65	53.89	17.12	174%
		440	10	45.10	17.55	351%
		460	4.5	37.51	16.51	734%
		480	1.95	31.03	14.54	1491%
4	ICICI BANK	660	57.6	60.67	1.54	5%
		680	45	48.55	1.77	8%
		700	33.35	38.13	2.39	14%
		720	23.9	29.40	2.75	23%
5	IDFC	85	9.45	9.58	0.06	1%
		90	6.15	6.43	0.14	5%
		95	3.55	4.06	0.25	14%
		100	1.95	2.41	0.23	23%
6	JP ASSOCIATES	45	8.95	8.90	0.02	1%
		50	5.1	5.47	0.18	7%
		55	2.5	3.05	0.27	22%
		60	1	1.55	0.28	55%
7	PNB	750	43.8	43.03	0.38	2%
		760	30.7	37.40	3.35	22%
		780	28.7	27.65	0.53	4%
		800	13.05	19.86	3.40	52%
8	SESA GOA	140	20.25	20.66	0.20	2%
		150	11.5	13.73	1.11	19%
		160	6.5	8.48	0.99	31%
		170	2.85	4.88	1.02	71%
9	TCS	1000	185	198.44	6.72	7%
		1050	138	158.64	10.32	15%
		1100	94	123.47	14.73	31%
		1150	62.75	93.49	15.37	49%

Source : Compiled by the Authors

Table 5 : Comparison of Market Prices and the Calculated Put Option Prices						
S.No	Name of the Company	Strike Price	Market Price	Calculated Put Option Price	MAD	MAPE
1	AXIS BANK	760	28	22.45	2.77	20%
		780	35.7	30.46	2.62	15%
		800	46.1	40.03	3.04	13%
		820	56.60	51.13	2.73	10%
2	BAJAJ AUTO	1350	16.7	36.65	9.98	119%
		1400	29.65	53.44	11.90	80%
		1450	35	74.49	19.74	113%
		1500	72.05	99.82	13.89	39%
3	HDFC BANK	420	11.85	43.76	15.95	269%
		440	22.05	54.81	16.38	149%
		460	38	67.06	14.53	76%
		480	52.75	80.43	13.84	52%
4	ICICIBANK	660	17.3	18.96	0.83	10%
		680	24	26.68	1.34	11%
		700	32	36.11	2.05	13%
		720	41.40	47.21	2.91	14%
5	IDFC	85	2.1	1.96	0.07	6%
		90	3.6	3.77	0.09	5%
		95	6	6.36	0.18	6%
		100	8.80	9.67	0.44	10%
6	JP ASSOCIATES	45	0.9	0.90	0.00	0%
		50	2.15	2.42	0.14	13%
		55	4.5	4.97	0.23	10%
		60	8.05	8.43	0.19	5%
7	PNB	750	32.55	20.91	5.82	36%
		760	27.4	25.20	1.10	8%
		780	48.7	35.29	6.70	28%
		800	49.80	47.34	1.23	5%
8	SESA GOA	140	2.95	2.46	0.24	17%
		150	5.9	5.45	0.22	8%
		160	9.75	10.13	0.19	4%
		170	16.65	16.45	0.10	1%
9	TCS	1000	6.15	12.01	2.93	95%
		1050	11.15	21.82	5.33	96%
		1100	20.45	36.26	7.90	77%
		1150	35.40	55.89	10.25	58%

Source : Compiled by the Authors

Conclusion

The present study tests the predictive accuracy of the volatility adjusted Black Scholes option pricing model in pricing the stock options for the selected companies and examines whether the estimated stock option prices are the same with that of the original market prices. The results conclude that the volatility adjusted model helped in finding and predicting the option prices in a nearer term. The predicted prices were almost the same as that of the market price when the strike prices are lower, while the difference becomes more as the strike price becomes larger than that of the underlying asset price. This is evidenced by the mean absolute deviation results and the mean absolute percentage error results. Thus, it can be concluded that the market prices of the stock option contracts can be predicted in the near term, and when the strike prices are lower than the underlying price of the contracts, it is a good indicator that the exchange allows a buyer friendly environment.

Research Implications

The study will help the investors to act as rational investors who use derivatives for the purpose of hedging. Since the findings of the study implied that the model can predict option prices with lower strike contracts at a greater accuracy, it can caution the users of derivatives when they want to act as gamblers. Hence, this can prove itself as a risk reducing tool. But the study was conducted only with the stocks of eight companies, and to generalize the findings, more number of companies should be subjected to this analysis, and their behaviour should be predicted.

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