



On gg-Separation Axioms in Topological Spaces

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Abstract

Study and investigate a class of separation axiom namely $gg-T_k$ space ($k=0, 1, 2$), gg -regular and gg -normal space. Meanwhile, some of their properties are obtained.

Keywords: $gg-T_k$ spaces ($K=0, 1, 2$), gg -regular space and gg -normal space.

1. Introduction

From the literature survey on separation axiom, we observed that there has been significant work on T_0 space, T_1 space, T_2 space, regular space and normal space. In 1980, R. C. Jain [5] introduced a new class of separation axioms namely δT_i space ($i=0, 1, 2$). Since then different types of separation axioms are investigated. Using g -closed sets of Levine [6], T. Noiri and Papa [7] and Sheik John [8] contributed their work in the field of separation axiom. Recently, Basavaraj M. Ittanagi [1], studied basic properties of gg -closed sets, gg -continuous functions and gg -closed maps in topological spaces. In present paper, $gg-T_0$, $gg-T_1$ and $gg-T_2$ spaces are studied. The relation with existing separation axioms is discussed.

2. Prerequisites

In study, (X, T_1) , (Y, T_2) (or X, Y) denote topological spaces. Let $g: X \rightarrow Y$ (or simply g) always denote map. For $B \subseteq X$, $\text{int}(B)$, $\text{cl}(B)$, $X-B$ or B^c represent interior, closure and complement of B respectively. The basic definitions required for this research work are listed below.

Definition 2.1 Let $B \subseteq X$. Then B is said to be

- g -closed [5] if $\text{cl}(B) \subseteq W$ whenever $B \subseteq W$ and W is open.
- gg -closed [1] if $\text{ggcl}(B) \subseteq W$ whenever $B \subseteq W$ and W is regular semiopen. Also, $\text{ggC}(X)$ (resp. $\text{ggO}(X)$) always denote family of gg -closed (resp. gg -open) sets in X .
- δT_0 space [4] if for every explicit points p, q of X , $q \notin M$, $p \in M$, or $p \notin M$, $q \in M$, where M is a δ -open set of X .
- δT_1 -space [4] if for every explicit points p, q of X , $q \notin M$, $p \in M$ and $p \notin N$, $q \in N$ where M and N are two δ -open sets of X .
- δT_2 -space [4] if for every explicit points p, q of X , $q \notin H_1$, $p \in H_1$ and $p \notin H_2$, $q \in H_2$ where H_1, H_2 are two δ -open sets with $H_1 \neq H_2$.

Definition 2.2 A map g is known as gg -continuous [2] (gg -irresolute[2]) if $f^{-1}(F) \in \text{ggC}(X)$, \forall closed (gg -closed) set F in Y .

3. $gg-T_0$, $gg-T_1$ and $gg-T_2$ Spaces

Definition 3.1 Let X be

- $gg-T_0$ if for every explicit points p, q of X , $q \notin M$, $p \in M$ or $p \notin M$, $q \in M$ where M is an open set of X .
- $gg-T_1$ space if for every explicit points p, q of X , $q \notin M$, $p \in M$ and $p \notin N$, $q \in N$ where $N, M \in \text{ggO}(X)$.
- $gg-T_2$ space (gg -Housdorff space) if for every explicit points p, q of X , $q \notin H_1$, $p \in H_1$ and $p \notin H_2$, $q \in H_2$ where $H_2, H_1 \in \text{ggO}(X)$ $\exists H_2 \neq H_1$.

Theorem 3.2

- If X is T_0 , then X is $gg-T_0$
- If X is T_1 , then X is $gg-T_0$ and $gg-T_1$
- If X is T_2 , then X is $gg-T_2$
- If X is $gg-T_1$, then X is $gg-T_0$
- If X is $gg-T_2$, then X is $gg-T_1$

Proof. i) Given, X is T_0 space. For distinct points of every pair p, q of X \exists open set U where $q \notin U$, $p \in U$, or $p \notin U$, $q \in U$. Then $U \in \text{ggO}(X)$ with $q \notin U$, $p \in U$ or $p \notin U$, $q \in U$. Therefore X is $gg-T_0$ space.

Similarly we can prove ii), iii), iv) and v).

Example 3.3 Let $X = \{n, m, l, k\}$ and $T_1 = \{\emptyset, \{l, k\}, \{n, m\}, X\}$ be a topology on X . Then $\text{ggC}(X) = \text{P}(X) = \text{ggO}(X)$.

Here (X, T_1) is a

- $gg-T_0$ space but not a T_0 space. For explicit points k, l of X , no open set M exist $\exists k \in M$, $l \notin M$ or $l \in M$, $k \notin M$.
- $gg-T_0$ space but not a T_1 space. For explicit points k, l of X , no two open sets M and N exist $\exists k \in M$, $l \notin M$ and $l \in N$, $k \notin N$.
- $gg-T_1$ space but not a T_1 space. For explicit points k, l of X , no two open sets M and N exist $\exists k \in M$, $l \notin M$ and $l \in N$, $k \notin N$.

(iv) $gg-T_2$ space but not a T_2 space. For explicit points k, l of X , no two distinct open sets H_1 and H_2 exist $\exists k \in H_1, l \notin H_1$ and $l \in H_2, k \notin H_2$.

Theorem 3.4

- 1) If X is δT_0 , then it is $gg-T_0$
- 2) If X is δT_1 , then it is $gg-T_1$
- 3) If X is δT_2 , then it is $gg-T_2$.

Proof. Based on fact, every δ - open set is in $ggO(X)$.

Remark 3.5 Converse of this theorem is untrue.

Example 3.6 Let $T_1 = \{\varphi, X, \{1\}, \{m, n\}\}$ be a topology on $X = \{n, m, 1\}$. Then

$ggO(X) = P(X)$

$\delta-O(X) = \{\varphi, X, \{1\}, \{m, n\}\}$

Here (X, T_1) is a

- (1) $gg-T_0$ but not a δT_0 space. For explicit points m, n of X , no δ open set exist with $n \notin M, m \in M$ or $m \notin M, n \in M$
- (2) $gg-T_1$ space but not a δT_1 space. For explicit points m, n of X , no two δ open sets M and N exist with $n \notin M, m \in M$ and $m \notin N, n \in N$.
- (3) $gg-T_2$ space but not a δT_2 space. For explicit points m, n of X , no two distinct δ -open sets H_1 and H_2 exist with $m \in H_1, n \notin H_1$ and $n \in H_2, m \notin H_2$.

Theorem 3.7 Let X is $gg-T_0$ iff $ggcl\{x_1\} \neq ggcl\{x_2\}, \forall x_1 \neq x_2$ of X .

Proof. Given that X is $gg-T_0$. Let $x_2, x_1 \in X$ and $x_2 \neq x_1$, then $\exists V \in ggO(X), x_2 \notin V$ and $x_1 \in V$ implies $x_1 \notin V^c, x_2 \in V^c$. But $ggcl\{x_2\}$ is the smallest gg -closed containing x_2 . Therefore $ggcl(\{y\}) \subseteq V^c$. Hence $x \notin ggcl\{x_2\}$. Thus $ggcl\{x_1\} \neq ggcl\{x_2\}$.
 Conversely, suppose $ggcl\{x_1\} \neq ggcl\{x_2\}, \forall x_1 \neq x_2$ of X . Let $x_3 \in X$ with $x_3 \in ggcl(\{x_1\})$ but $x_3 \notin ggcl(\{x_2\})$. If $x \in ggcl(\{y\})$ then $ggcl(\{x_1\}) \subseteq ggcl(\{x_2\})$. Hence $x_3 \in ggcl(\{x_2\})$. This contradicts our assumption. Thus $x_1 \notin ggcl(\{x_2\})$ implies $x_1 \in (ggcl(\{x_2\}))^c$ implies $(ggcl(\{x_2\}))^c$ is gg -open containing x_1 and not x_2 . Therefore X is $gg-T_0$.

Theorem 3.8 If g is bijective, strongly gg -open and X is $gg-T_0$, then Y is $gg-T_0$ space.

Proof Take y_2 and y_1 of Y with $y_2 \neq y_1$. By hypothesis $y_1 = g(x_1)$ and $y_2 = g(x_2)$ where x_2 and x_1 are explicit points of X . By hypothesis, $M \in ggO(X)$ with $x_1 \in M$ and $x_2 \notin M$. Therefore $g(x_1) \in g(M)$ and $g(x_2) \notin g(M)$. As X is strongly gg -open, $g(M) \in ggO(Y)$. Therefore $g(M)$ is gg -open set in Y with $y_1 \in g(M)$ and $y_2 \notin g(M)$. Thus Y is a $gg-T_0$ space.

Theorem 3.9 If g is gg -irresolute, injective, Y is $gg-T_0$ then X is $gg-T_0$.

Proof. Take explicit points x_2 and x_1 of X . As g is injective implies $g(x_1) \neq g(x_2)$. As Y is $gg-T_0$, exists $U \in ggO(Y)$ such that $g(x_1) \in U, g(x_2) \notin U$ or exists a $V \in ggO(Y)$ such that $g(x_1) \notin V, g(x_2) \in V$ with $g(x_2) \neq g(x_1)$. Since g is gg -irresolute then $g^{-1}(U) \in ggO(X) \ni x_2 \notin g^{-1}(U), x_1 \in g^{-1}(U)$ or $g^{-1}(V) \in ggO(X) \ni x_1 \notin g^{-1}(V), x_2 \in g^{-1}(V)$. Hence X is $gg-T_0$.

Theorem 3.10 Let X is $gg-T_1$ iff $x_2 \in X$ singleton $\{x_2\} \in ggC(X)$.

Proof Let X is $gg-T_1, x_1 \in X$. Then $x_2 \in X - \{x_1\} \Rightarrow x_1 \neq x_2 \in X$. But X is $gg-T_1$ space implies their exist $G_1, G_2 \in ggO(X) \ni x_1 \notin G_1, x_2 \in G_2 \subseteq (X - \{x_1\})$. Also $x_2 \in G_2 \subseteq (X - \{x_1\})$ implies $(X - \{x_1\}) \in ggO(X)$. Thus $\{x_1\}$ is gg -closed.

Conversely, Let $x_1 \neq x_2 \in X$ then $\{x_1\}$ and $\{x_2\}$ are gg -closed sets and $\{x_1\}^c$ is gg -open. Clearly $x_1 \notin \{x_1\}^c$ and $x_2 \in \{x_1\}^c$. Similarly $\{x_2\}^c$ is gg -open, $x_2 \notin \{x_2\}^c$ and $x_1 \in \{x_2\}^c$. Hence X is $gg-T_1$ space.

Theorem 3.11 The following results holds

- i) If g is gg -continuous, injective, Y is T_1 space, imply X is $gg-T_1$.
- ii) If g is gg -continuous, injective, Y is T_2 space, imply X is $gg-T_2$ space.
- iii) If g is gg -irresolute, injective, Y is $gg-T_2$ space, imply X is $gg-T_2$ space.

Proof. i) Consider two points x_1, x_2 of X with $x_2 \neq x_1$, then $g(x_1) = y_1$ and $g(x_2) = y_2$. Also, $g(x_1) \neq g(x_2)$. Since (Y, σ) is T_1 then $y_1 \in M, y_2 \notin M$ and $y_1 \notin N, y_2 \in N$. Then $x_1 \in g^{-1}(M), x_1 \notin g^{-1}(N)$ and $x_2 \in g^{-1}(N), x_2 \notin g^{-1}(M)$. By the definition of gg -continuity, $g^{-1}(M)$ and $g^{-1}(N) \in ggO(X)$. For $x_1, x_2 \in X$ with $x_1 \neq x_2, x_1 \in g^{-1}(M), x_1 \notin g^{-1}(N)$ and $x_2 \notin g^{-1}(M), x_2 \in g^{-1}(N)$. Therefore (X, T_1) is $gg-T_1$ space.

Similarly we can prove ii) and iii)

Theorem 3.12 The statements given below are equivalent.

- 1) X is $gg-T_2$
- 2) If $x_1 \in X$, then $x_2 \neq x_1, \exists W$ containing $x_1 \ni x_2 \notin ggcl(W)$.

Proof. (1) \Rightarrow (2)

Take $x_1 \in X$ and $x_2 \in X$ with $x_1 \neq x_2, \exists$ disjoint sets $W, V \in ggO(X) \ni x_1 \in W$ and $x_2 \in V$. Then $x_1 \in W \subseteq V^c$ and $V^c \in ggC(X)$ and $x_2 \notin V^c$. Thus $x_2 \notin ggcl(W)$.

(2) \Rightarrow (1)

Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. By (ii), \exists gg -open W containing $x_1 \ni x_2 \notin ggcl(W)$ implies $x_2 \in (X - (ggcl(W)))$. $(X - (ggcl(W))) \in ggO(X)$ and $x_1 \notin (X - (ggcl(W)))$. Also $W \cap (X - ggcl(W)) = \varphi$. Thus X is $gg-T_2$.

4. gg-Regular Space

Definition 4.1 Let X is known as gg -regular if $\forall F \in ggC(X)$ and $x \notin F, \exists$ disjoint open sets G and $H \ni F \subseteq G, x \in H$.

We have the following relationship between gg -regularity and regularity.

Theorem 4.2 Every gg -regular space is regular.

Proof. Given that X is gg -regular. Take $F \in ggC(X)$ and $x \notin F$. As X is gg -regular, \exists a pair of disjoint open sets U and $V \ni F \subseteq U$ and $x \in V$. Hence X is a regular.

Remark 4.3 Every regular space is not a gg -regular space.

Example 4.4 Let $T_1 = \{\varphi, \{k\}, \{m, l\}, X\}$ be topology on $X = \{m, l, k\}$. Here (X, T_1) is regular but not a gg -regular space. For the gg -closed set $\{1\}$ with $m \notin \{1\}$, no two disjoint open sets U, H exist $\ni \{1\} \subseteq U, m \in H$.

Theorem 4.5 Every regular with $ggTc$ space is gg -regular.

Proof. Given that X is regular, $ggTc$. Let $F \in ggC(X)$ and $x \in X \ni x \notin F$. Now X is $ggTc$ space, F is closed, $x \notin F$. As X is regular, \exists pair of disjoint open sets M and $N \ni F \subseteq M$ and $x \in N$. Therefore X is gg regular.

Theorem 4.6 If X is gg -regular then it is g -regular space.

Proof. Based on the fact, every g -closed belongs to $ggC(X)$.

Theorem 4.7 If X is gg -regular, then it is w regular space.

Proof. Based on fact, each w closed belongs to $ggC(X)$.

Theorem 4.8 The statements given below are equivalent.

- 1) X is gg -regular
- 2) $\forall x \in X$ and each gg -open nbhd $U \ni x$ an open nbhd N of $X \ni x$ $cl(N) \subseteq U$.

Proof.

(1) \Rightarrow (2)

Let U be gg -neighbourhood of x , \exists a $G \in ggO(X) \ni x \in G \subseteq U$. Now $G^c \in ggC(X)$ and $x \notin G^c$. From (1), $\exists P, Q \ni G^c \subseteq P, x \in Q, P \cap Q = \emptyset$. So $Q \subseteq M^c$. Now $cl(Q) \subseteq cl(P^c) = G^c$ and $G^c \subseteq P$. This implies $(P^c \subseteq G \subseteq U)$. Therefore $cl(Q) \subseteq U$.

(2) \Rightarrow (1)

Let gg -closed F in X and $x \notin F$ or $x \in F^c$ and X is gg -open and so F^c is a gg -nbhd of x . By hypothesis, \exists an open nbhd $N \ni x \in N, cl(N) \subseteq F^c$. This implies $F \subseteq \{X - cl(N)\}$ and $N \cap \{X - cl(N)\} = \emptyset$. Thus X is gg -regular.

Theorem 4.9 Let X is gg -regular iff for every $G \in ggC(X)$ and point $p \in (X - G)$ then $x \in U, G \subseteq N$ and $cl(N) \cap cl(U) = \emptyset$ where N and U are open sets.

Proof. Given that X is gg -regular. Let $G \in ggC(X)$ and $x \notin G$. Then $p \in M$ and $G \subseteq N$ and $M \cap N = \emptyset$ where M and N are open sets. This implies $M \cap cl(N) = \emptyset$. Since (X, T_1) is gg -regular, $p \in P$ and $cl(N) \subseteq Q, P \cap N = \emptyset$ where P, Q are open. Also $cl(P) \cap Q = \emptyset$. Let $U = M \cap P$ then $p \in U, G \subseteq N$ and $cl(N) \cap cl(U) = \emptyset$ where N, U are open in X .

Conversely, suppose $\forall G \in ggC(X)$ and $p \in (X - G)$, we have $p \in U, G \subseteq N$ and $cl(N) \cap cl(U) = \emptyset$ where N and U are open sets. This implies $p \in U, G \subseteq N$ and $U \cap N = \emptyset$. Therefore (X, T_1) is gg -regular.

Theorem 4.10 A subspace Y of gg -regular (X, T_1) is gg -regular.

Proof. Obvious.

Theorem 4.11 Let g is bijective, gg -irresolute and open map from gg -regular X in to Y , then Y is gg -regular.

Proof Take $y \in Y$ and $F \in ggC(X)$ with $y \notin F$. As g is gg -irresolute then $g^{-1}(F) \in ggC(X)$. Let $y = g(x)$ then $g^{-1}(y) = x$ and $x \notin g^{-1}(F)$. Again X is gg -regular, \exists open sets P and $Q \ni x \in P$ and $g^{-1}(F) \subseteq Q, P \cap Q = \emptyset$. Given g is open and bijective implies $y \in g(P), F \subseteq g(Q)$ and $g(Q \cap P) = g(\emptyset) = \emptyset$. Thus Y is gg -regular.

Theorem 4.12 Let g be bijective, gg -closed map from X into gg -regular Y . If X is $ggTc$, then it is gg -regular

Proof. Consider $x \in X, F \in ggC(X)$ with $x \notin F$. Then F is closed. Also g is gg -closed implies $g(F) \in ggC(Y)$ with $g(x) \notin g(F)$ in Y . As Y is gg -regular, $\exists P, Q \ni g(x) \in P$ and $g(F) \subseteq Q, P \cap Q = \emptyset$. Therefore $x \in g^{-1}(P)$ and $F \subseteq g^{-1}(Q)$. Hence X is gg -regular space.

5. gg -Normal Space

Definition 5.1 Let X is known as gg -normal if for each pair $D, C \in ggC(X)$, \exists disjoint open sets P, Q in $X \ni C \subseteq P$ and $D \subseteq Q$.

Theorem 5.2 Every gg -normal is normal

Proof. Given that X is a gg -normal. Consider closed D, C which are disjoint in X . Then $D, C \in ggC(X)$. Since X is gg -normal, \exists a pair $G, H \ni C \subseteq G, D \subseteq H$. Hence X is normal.

Example 5.3 Let $T_1 = \{\emptyset, \{1\}, \{k\}, \{1, k\}, \{k, m\}, X\}$ be a topology on $X = \{m, k, 1\}$. Here X is normal and not gg -normal. For the disjoint gg -closed sets $\{m\}$ and $\{k, 1\}$ does not exist disjoint open sets P and Q .

Theorem 5.4 If Y is normal, $ggTc$ space, then Y is gg -normal.

Proof. Given Y is normal. Let a pair of disjoint sets $C, D \in ggC(Y)$. Since $ggTc$ space, then C and D are closed. As Y is normal, \exists a pair of disjoint open sets P, Q in $Y \ni C \subseteq P$ and $D \subseteq Q$. Hence Y is gg -normal.

Theorem 5.5 Every gg -normal is g -normal.

Proof. Given X is gg -normal. Let pair of disjoint sets $C, D \in ggC(Y)$. Then \exists a pair of disjoint $P, Q \ni C \subseteq P$ and $D \subseteq Q$. Hence X is g -normal.

Remark 5.6 Every g -normal is not a gg -normal

Example 5.7 Let $T_1 = \{\emptyset, \{1\}, \{k\}, \{k, 1\}, \{k, m\}, X\}$ be a topology on $X = \{m, 1, k\}$. Here (X, T_1) is g normal and not a gg -normal space. For the disjoint gg -closed sets $\{m\}$ and $\{k, 1\}$ does not exist P, Q .

Theorem 5.8 Every gg -normal is w -normal.

Proof. Similar as Theorem 5.5

Remark 5.9 In general, converse is untrue

Example 5.10 Let $T_1 = \{\emptyset, \{1\}, \{k\}, \{k, 1\}, \{m, k\}, X\}$ be a topology on $X = \{m, 1, k\}$. Here (X, T_1) is w normal and not a gg -normal space. For the disjoint gg -closed sets $\{m\}$ and $\{k, 1\}$ does not exist P, Q .

Theorem 5.11 If Y is gg -closed subspace of gg -normal X , then Y is gg -normal.

Proof. Let X be gg -normal and Y is gg -closed subspace. Let a pair of disjoint sets $C, D \in ggC(Y)$. Then $\exists G, H \in X \ni C \subseteq G$ and $D \subseteq H$. Then $G \cap Y$ and $H \cap Y$ are open in Y . Also we have $C \subseteq G$ and $D \subseteq H$ implies $C \cap Y \subseteq Y \cap G$ and $Y \cap D \subseteq Y \cap H$ and $(G \cap Y) \cap (Y \cap H) = Y \cap (G \cap H) = \emptyset$. Hence Y is gg -normal.

Theorem 5.12 Statements given below are equivalent in (X, T_1) .

- 1) X is gg -normal
- 2) For each $C \in ggC(X)$ and each $M \in ggO(X)$ with $C \subseteq M$ then $C \subseteq N \subseteq cl(N) \subseteq M$ where N is an open set.
- 3) For any disjoint sets $C, D \in ggC(X)$, \exists an open set $N \ni C \subseteq N$ and $cl(N) \cap D = \emptyset$
- 4) For each disjoint sets $C, D \in ggC(X)$, \exists open sets $M, N \ni C \subseteq M, D \subseteq N, cl(M) \cap cl(N) = \emptyset$.

Proof. (1) \Rightarrow (2)

Take $C \in ggC(X), M \in ggO(X) \ni C \subseteq M$. This implies C and $(X - M)$ are two disjoint sets of $ggC(X)$. By the definition of gg -normal, $C \subseteq N$ and $(X - M) \subseteq O$ where N and O are disjoint open sets. Thus $N \subseteq (X - O)$ and $O \cap N = \emptyset$. This implies $N \subseteq (X - O)$ and $cl(N) \subseteq (X - O) \subseteq M$ and so $cl(N) \subseteq M$. Therefore $C \subseteq N \subseteq cl(N) \subseteq M$.

(2) \Rightarrow (3)

Let a pair of disjoint sets $D, C \in ggC(X), C \subseteq (X - D)$ where C and $(X - D)$ are gg -closed and gg -open sets in X respectively. By (2) there exist an open set N with $C \subseteq N \subseteq cl(N) \subseteq (X - D)$. But $cl(N) \subseteq (X - D)$ implies $D \cap cl(N) = \emptyset$. Therefore $C \subseteq N$ and $cl(N) \subseteq D = \emptyset$.

(3) \Rightarrow (4)

Consider two disjoint gg -closed sets C and D in X . Then from (3), \exists an open set $M \ni C \subseteq M$ and $cl(M) \cap D = \emptyset$. But $cl(M)$ is closed and hence gg -closed set in X . Now $cl(M)$, D belongs to $ggC(X)$. By (3), we have $D \subseteq N$ and $cl(M) \cap cl(N) = \emptyset$ where N is open set exist in X .

(4) \Rightarrow (1)

Consider two disjoint gg -closed C, D in X . By hypothesis \exists open sets N, M with $C \subseteq M, D \subseteq N, cl(M) \cap cl(N) = \emptyset$. This implies that $C \subseteq M, D \subseteq N$. Therefore X is gg -normal.

Theorem 5.13 A map g is bijective open, gg -irresolute from gg -normal X on to Y , then Y is gg -normal.

Proof. Take A, B in $ggC(X)$, which are disjoint. As g is gg -irresolute then $g^{-1}(A)$ and $g^{-1}(B)$ are in $ggC(X)$. As X is gg -normal $g^{-1}(A) \subseteq M$ and $g^{-1}(B) \subseteq N$ where M and N are open sets exist in X . Further, as g is bijective and open, $g(M)$ and $g(N)$ are open with $A \subseteq g(M), B \subseteq g(N)$. Therefore Y is gg -normal.

Theorem 5.14 The statements given below are equivalent in X

- (1) X is g -normal
- (2) For each disjoint closed sets C, D, \exists disjoint sets $M, N \in ggO(X) \ni C \subseteq M, D \subseteq N$.

Proof. (i) \Rightarrow (ii)

Suppose X is g -normal. Let disjoint closed C, D of X . As (X, T_1) is g -normal then \exists disjoint g -open sets $M, N \ni C \subseteq M, D \subseteq N$. Then $M, N \in ggO(X)$ with $C \subseteq M, D \subseteq N$ and $M \cap N = \emptyset$.

(ii) \Rightarrow (i)

Consider two disjoint gg -closed sets $C, D \in ggC(X)$. By hypothesis $C \subseteq M, D \subseteq N$ and $N \cap M = \emptyset$ where N, M are disjoint sets in $ggO(X)$. We know that $C \subseteq gint(M), D \subseteq gint(N)$ and $gint(M) \cap gint(N) = \emptyset$. Therefore (X, T_1) is g -normal.

6. Conclusion

A new class of separation axioms called gg -separation axioms are introduced and studied in (X, T) . The relations between gg -separation axioms with existing separation axioms are discussed. The useful results on gg -regular and gg -normal spaces are also presented.

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