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A model for waiting times for non-stationary queueing systems

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ABSTRACT:

Queueing models are stochastic models that represent the probability that a queueing system will be found in a particular configuration or state. Several interesting stationary queueing systems have been solved analytically; on the other hand, non-stationary queueing systems are relatively unexplored. The present study analyses the waiting times of a non-stationary M/M/1 queueing system using simulation methods.

Keywords: queueing models, non-stationary M/M/1 queueing system, simulation .

INTRODUCTION

Queueing models are generally constructed to represent the steady state of a queueing system, that is, the typical, long run or average state of the system. As a consequence, queueing models are stochastic models that represent the probability that a queueing system will be found in a particular configuration or state. Several interesting stationary queueing systems have been solved analytically; on the other hand, non-stationary queueing systems are relatively unexplored (Gross and Harris, 1998).

Bailey (1957, 1964) had solved the M/M/1 system analytically, for both stationary and non-stationary cases. However, his solution in the case of non-stationarity was complex, involving the evaluation of complicated integrals and transcendental functions.

Simulation techniques have been used extensively to study non-stationary queueing systems. They have been used in several practical applications (Bobillier et al, 1976; Nozari et al, 1984). In particular, Brandão (2006) and Brandão and Porto Nova (2009, 2012) have used time series analysis to analyse simulation output from non-stationary queueuing systems. They applied these techniques to study periodicity in traffic systems.

The present study analyses the waiting times of a non-stationary M/M/1 queueing system using simulation methods.

1. DATA & METHODOLOGY

The objective of the study was to understand transient behaviour in the M/M/1:∞/∞/FIFO queueing system; and in particular to analyse the waiting times in the queue and in the system as a function of the arrival times.

The data for the study was generated using Monte Carlo simulation of a simple M/M/1:∞/∞/FIFO queueing system. The arrival and service rates were randomly generated, with $\lambda \geq \mu$, and the queueing system simulated for one thousand iterations. This process was used to generate one hundred simulations of the queueing system, which were subsequently subjected to analysis.

Preliminary simulation of the queueing system yielded some interesting insights. The graph in Figure 1 shows a typical sample plot of waiting times in the queue and system against the arrival time in the system.

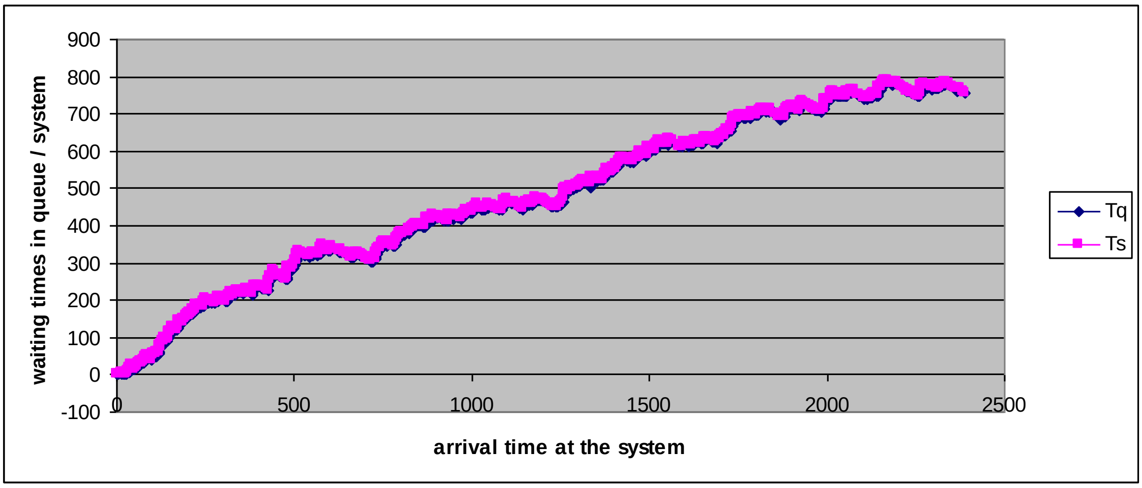

Figure 1. sample plot of waiting time in queue and system against arrival time in the system (with λ = 0.423004 and μ = 0.322325)

It may be observed that the sample plots for both waiting times steadily grow with arrival times; and several iterations of simulation confirmed the randomness of these plots. The study considers a linear model for waiting time as a function of arrival time, of the form: , where Wi represents the waiting time of the ith sample unit in the queue/system, xi represents the arrival time of the ith sample unit in the system, εi represents the error term, and a and b are the intercept and slope parameters, respectively. As usual, the error terms are assumed to be independently and identically normally distributed, with zero mean, and constant variance σ2.

In particular, an important parameter of concern is the slope parameter b, which represents the incremental waiting time per unit delayed arrival in the system. Clearly, this quantity should depend only on the underlying queueing system parameters λ and μ. Another parameter of concern is the variability of errors σ, which should also depend only on the underlying queueing system parameters λ and μ.

To estimate these quantities, the waiting times in the queue and in the system were regressed linearly on the arrival times for each simulation, resulting in sample regression coefficients (βq and βs) and standard errors (σq and σs). These sample estimates were in turn regressed on the ratios (λ-μ)/μ and λ/μ, respectively: the former quantity was of particular interest as it represented the average rate of accumulation in the queueing system (when λ ≥ μ).

2. FINDINGS

The regression results of βq (the regression coefficient of the waiting time in the queue on the arrival time) on (λ-μ)/μ are summarized in Table 1 below.

Table 1. regression results of βq

	linear	quadratic	cubic
((λ-μ)/μ)	0.988441**	1.001004**	0.955467**
((λ-μ)/μ)2**		-0.000383	0.003321**
((λ-μ)/μ)3**			-0.000058**
R^2	99.8%	99.8%	99.9%
adjusted R^2	99.8%	99.8%	99.9%
F stat	61831.867	31311.738	22845.217
p-value	0.000**	0.000**	0.000**

It was found that βq was dependent only on the rate (λ-μ)/μ. Linear regression of βq on (λ-μ)/μ was found to explain 99.8% of the variation in βq, and was found to be highly statistically significant. In order to investigate the possibility of non-linearity in the drift term with respect to (λ-μ)/μ, some higher order regression models were also tried. It was found that cubic regression of βq on (λ-μ)/μ also yielded statistically significant results; of course, the higher order terms in (λ-μ)/μ were found to be small compared to the linear term, so that a linear approximation would suffice for most applications.

The regression results of σq (the standard error of the regression of the waiting time in the queue on the arrival time) on (λ-μ)/μ are summarized in Table 2 below.

Table 2. regression results of σq

	linear	quadratic	cubic
(λ/μ)	17.184233**	26.912189**	20.747924**
(λ/μ)2**		-0.300729**	0.221170
(λ/μ)3**			-0.008192*
R^2	75.0%	81.0%	81.9%
adjusted R^2	74.8%	80.6%	81.4%
F stat	297.129	209.271	146.708
p-value	0.000**	0.000**	0.000**

It was found that σq was dependent only on the rate λ/μ. Linear regression of σq on λ/μ was found to explain only 75.0% of the variation in σq, and was found to be statistically significant. Quadratic and cubic regression of σq on λ/μ were also found to yield statistically significant results, explaining 81.0% and 81.9% of the variation in σq, respectively.

The regression results of βs (the regression coefficient of the waiting time in the system on the arrival time) on (λ-μ)/μ are summarized in Table 3 below.

Table 1: regression results of βs

	linear	quadratic	cubic
((λ-μ)/μ)	0.988468**	1.000901**	0.955447**
((λ-μ)/μ)2**		-0.000379	0.003319**
((λ-μ)/μ)3**			-0.000058**
R^2	99.8%	99.8%	99.9%
adjusted R^2	99.8%	99.8%	99.9%
F stat	61858.297	31310.544	22835.336
p-value	0.000**	0.000**	0.000**

It was found that βs was dependent only on the rate (λ-μ)/μ. Linear regression of βs on (λ-μ)/μ was found to explain 99.8% of the variation in βq, and was found to be highly statistically significant. In order to investigate the possibility of non-linearity in the drift term with respect to (λ-μ)/μ, some higher order regression models were also tried. It was found that cubic regression of βs on (λ-μ)/μ also yielded statistically significant results; of course, the higher order terms in (λ-μ)/μ were found to be small compared to the linear term, so that a linear approximation would suffice for most applications. In fact, the results for βq and βs were found to be almost identical, indicating similarity of the drift components of the waiting times in the queue and in the system.

The regression results of σs (the standard error of the regression of the waiting time in the system on the arrival time) on (λ-μ)/μ are summarized in Table 4 below.

Table 2: regression results of σs

	Linear	quadratic	cubic
(λ/μ)	44.484349**	57.290485**	52.874982**
(λ/μ)2**		-0.395887**	-0.022048
(λ/μ)3**			-0.005868
R^2	85.5%	87.2%	87.3%
adjusted R^2	85.3%	87.0%	86.9%
F stat	581.978	334.884	222.569
p-value	0.000**	0.000**	0.000**

It was found that σs was dependent only on the rate λ/μ. Linear regression of σs on λ/μ was found to explain only 85.5% of the variation in σs, and was found to be statistically significant. Quadratic and cubic regression of σq on λ/μ were also found to yield statistically significant results, explaining 87.2% and 87.3% of the variation in σs, respectively. Again, the results for σq and σs were found to be quite similar, indicating similarity of the volatility components of the waiting times in the queue and in the system.

3. DISCUSSION

Place The study examines a model for waiting times of the form: $W_i = a + bx_i + \epsilon_i$, i.e. as a linear function of the arrival time of the unit in the system, with slope parameter b and error variance parameter σ2. This formulation was suggested by the nature of the sample paths of the waiting times.

The above formulation is equivalent to a generalized Wiener model for waiting times of the form $dW_t = b(\lambda, \mu, t)dt + \sigma(\lambda, \mu, t)dz$, in which the drift and volatility components are time-independent, i.e. $b(\lambda, \mu, t) = b(\lambda, \mu)$, and $\sigma(\lambda, \mu, t) = \sigma(\lambda, \mu)$. In fact, the results of the study do suggest that the drift component of the waiting times $b(\lambda, \mu, t)$ is time-independent, and is of the form $b((\lambda-\mu)/\mu)$. Unfortunately, the results of the study for the volatility component $\sigma(\lambda, \mu, t)$ were not very conclusive.

The present study has some limitations. The analysis assumes a constant drift, i.e. $b(\lambda, \mu)$, and a constant volatility, i.e. $\sigma(\lambda, \mu)$, i.e. stationarity of the underlying stochastic process. There is scope to develop some stochastic volatility models in this context. Also, the results of the study may indicate heteroskedasticity or even the presence of a unit root. Thus, as suggested by Brandão (2006) and Brandão and Porta Nova (2009, 2012), the time series characteristics of non-stationary queueing processes should be further investigated.

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