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Interpolation of electrocardiogram signals using Chebyshev polynomials optimization using genetic algorithms

Seema Sharma *
Chetan Vasudeva †
Department of Electrical Engineering
DAV University
Jalandhar
Punjab
India

Neetika Soni §
Department of Electronics and Communication Engineering
Guru Nanak Dev University, Regional Campus
Jalandhar
Punjab
India

Abstract

The research presented in this article specifically explores the utility of Chebyshev polynomials in electrocardiogram (ECG) signal processing and specializes in its correction and development capability. ECG sign evaluation is an important a part of coronary heart monitoring and is crucial for figuring out and monitoring heart issues. However, dependable analysis is hard, mainly while looking to come across small defects. Additionally, this observe presents a brand new technique for interpolation, smoothing and optimization of ECG statistics using Chebyshev polynomials. ECG features had been normalized to conform to the character of Chebyshev polynomials. Then the polynomial coefficients are calculated by way of the orthogonal projection technique. The aim of this look at is to reduce the mistake among the reconstructed ECG characteristics and the original ECG traits with the aid of optimizing the coefficients using a genetic algorithm (GA) in MATLAB. This strategy calls for collecting ECG statistics, segmenting it into person heartbeats the use of the Pan-Tompkins technique, the usage of a finite impulse reaction (FIR) bandpass filter as a preprocessing step, and ultimately applying Chebyshev polynomials for function extraction. The effects show

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* *E-mail:* **seema10064@davuniversity.org (Corresponding Author)**
† *E-mail:* **chetan10035@davuniversity.org**
§ *E-mail:* **neetika.ecejal@gndu.ac.in**

how effective our technique is, showing desirable convergence of GA and lowering errors, for that reason offering an accurate and dependable illustration of ECG alerts that may be utilized in prognosis and monitoring

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1. Introduction

ECG (electrocardiogram) signal processing is important in medicinal filed due to its ability to detect and monitor heart diseases, including arrhythmias and myocardial infarction [1]. It helps in determining important aspects of the heart, such as normal or abnormal, by examining the rhythm of the heart. Especially for small deviations, this interpretation is not always possible with the eyes [2]. Diagnosing heart disease is important for recommending treatment options and providing important information for immediate treatment [3]. In many clinical settings, ECG testing is the diagnostic method of choice because it is easy and convenient. In addition, with the development of technology, ECG monitoring has now become possible to regularly monitor the heart outside the medical center via personal medical equipment and tele-medicine [4]. Its widespread use reflects its importance in maintaining health and early diagnosis and treatment of cardiovascular problems [5]. Polynomials are very important in examining ECG (electrocardiogram) signals because they are widely used to increase the accuracy and efficiency of cardiac data interpretation [6-9]. They can be used for many tasks, such as interpolating and smoothing ECG signals, effectively reducing noise, and compressing data for fast transmission and storage [10]. A signal called an electrocardiogram, or ECG, is a collection of electrical potentials that show the electrical activity of the heart. When these signals are collected, stored and transmitted, they are often affected by high voltage and high frequency devices. The studies presented here include the difficulty of distinguishing true heart murmurs (an important first step in ECG analysis) [11-13].

2. Chebyshev Polynomials

A family of orthogonal polynomials defined on a given interval (normally $[-1,1]$ or $[0,1]$) is known as a Chebyshev polynomial [14]. They are important in many fields of arithmetic, engineering and physics due to their unique characteristics and plenty of programs [15]. Well-known expressions for polynomials can be discovered in Equations (1-2).

Chebyshev Polynomials of the First Kind (T_n):

The *n*th Chebyshev polynomial of the first kind, denoted as $T_n(x)$, can be defined recursively as follows:

$$T_n(x) = \cos(n.\arccos(x));\ \text{for} -1 \le x \le 1 \tag{1}$$

Chebyshev Polynomials of the Second Kind (U_n):

The *n*th Chebyshev polynomial of the second kind, denoted as $U_n(x)$, is defined as:

$$U_n(x) = \frac{\sin((n+1)\cdot\arccos(x))}{\sin(\arccos(x))};\ \text{For } -1 \le x \le 1 \tag{2}$$

Interpolation of ECG indicators the use of Chebyshev polynomials requires unique mathematical techniques to use Chebyshev polynomial interpolation to ECG indicators, let's examine it in a broader context.

Chebyshev Nodes: selecting the appropriate node (or factor) is step one of polynomial interpolation. The nodes are often selected as Chebyshev nodes for Chebyshev interpolation and may be described as.

$$x_i = \cos\left(\frac{(2i+1)\pi}{2n}\right) \quad \text{for } i = 0,1,2,3\ldots n \tag{3}$$

For an arbitrary interval [a,b], the nodes can be adjusted as:

$$x_k = \frac{1}{2}(a+b) + \frac{1}{2}(b-a)\cos\left(\frac{2k-1}{2n}\pi\right) \tag{4}$$

Chebyshev nodes are determined and then utilized as the interpolation points in polynomial interpolation formulas given above.

3. Methodology

The principle and accurate approach used on this have a look at to signify the ECG sign using Chebyshev polynomials gives a high level of statistical accuracy. The technique starts with data series using PhysioNet to acquire ECG signal information. ECG signal is examined at a frequency of 200 Hz; that is sufficient to extract applicable statistics deprived of the problem of adding unnecessary facts. After the procedure of normalization, the ECG signal is preprocessed to get rid of noise and artifacts often

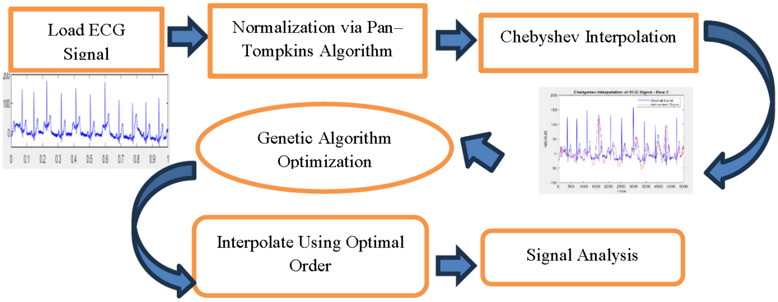


Figure 1

Methodology adopted for interpolation process

determined in uncooked ECG recordings. Use a bandpass clear out with low frequency response (FIR) to reduce noise. By setting the cut-off frequency of the filter to 0.5 Hz and 60 Hz as specified, low frequencies such as base shift and noises such as electrical interference or muscles are successfully eliminated. The choice of these factors are important because it ensures the stability of P waves, QRS complexes, and T waves while filtering out unwanted noise as shown in Figure 1.

After ECG information is pre-processed, it's far classified in line with exceptional heart illnesses. The main step in decoding the ECG signal is segmentation, which eliminates individual heartbeats for in addition evaluation. The real segmentation is finished the usage of the Pan-Tompkins algorithm, a broadly everyday approach for finding the R top in ECG records, as proven in Equation (7-9). The R peak, indicating ventricular depolarization, is indicative of the QRS complex, an critical component of heart rate tracking. The ECG signal E(t) is usually preprocessed by the usage of a bandpass filter to lessen noise and enhance QRS complexes. The filtered signal can be expressed as E_f (t).

$$D(t) = \frac{d(E_f(t))}{dt} \tag{5}$$

$$D^2(t) = (D(T)^2) \tag{6}$$

A threshold, θ, is applied to $D^2(t)$ to identify potential R-peak locations:

$$R_{potential}\ (t) = (t \mid D^2(t) > \theta) \tag{7}$$

The R peak is located using derivatives to suggest the exchange in slope inside the QRS complex. The derivative D(t) of the filtered signal

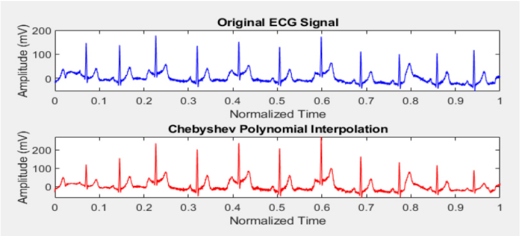


Figure 2

Original ECG Signal taken from Physionet and Chebyshev Polynomial points

E_f(t) can be calculated as follows: The number of values is not recorded with every heartbeat. Those functions encompass P wave duration, QRS complex c language, R-R interval (quick duration of consecutive R peaks, the main degree of heart rate variability) and different essential parameters. Due to the fact those features are extracted the use of verified techniques, the accuracy and reliability of the measurements are guaranteed. This normalization process results in a more accurate estimate that reproduces the characteristics of the original ECG data. Without this level, the features will be outside the optimal polynomial function, which reduces accuracy or may render the value estimation useless.

The next step is to develop these properties with different names based on the original Chebyshev polynomials. This is done using the orthographic projection technique. A mathematical technique called orthogonal projection is used to express the function (here ECG features) in terms of a set of orthogonal functions or Chebyshev polynomials as shown in Figure 2.

In this study, the characterization of the ECG signal using Chebyshev involves the following steps of polynomial interpolation:

Time domain normalization: The ECG signal, denoted as E(t), is first made to keep its time running. Normalization. This involves changing the time variable t from the interval t_maximize, t_minimum to the new interval [−1,1][−1,1], which is the standard collection of Chebyshev polynomials. Normalization is done using the following formula:

$$x = \frac{2t - (t_{\text{maximize}} + t_{\text{minimum}})}{(t_{\text{maximize}} - t_{\text{minimum}})} \quad (8)$$

Chebyshev Polynomial Interpolation: The next step involves approximating the normalized ECG signal with a Chebyshev polynomial series. The interpolation polynomial P_N (x) of degree N is given by eq. (11). The coefficients c_n are calculated using the discrete orthogonality

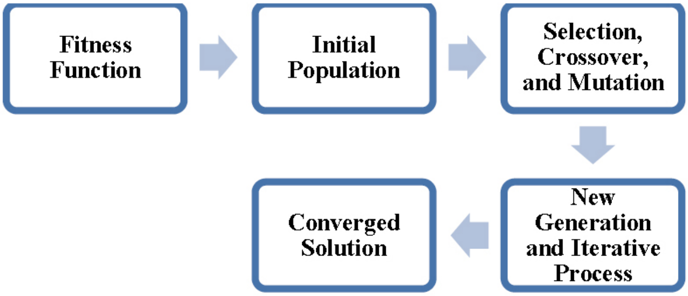


Figure 3

Genetic algorithm applied to minimize the error between the Chebyshev polynomial approximation

property of Chebyshev polynomials, which are expressed by equation (12,13).

$$P_N(x) = \sum_{n=0}^{N} c_n T_n(x) \tag{9}$$

$$c_n = \frac{2}{N+1} \sum_{i=0}^{N} E(x_i) T_n(x_i); \text{ for } n > 0 \tag{10}$$

$$c_0 = \frac{1}{N+1} \sum_{i=0}^{N} E(x_i) T_0(x_i); \tag{11}$$

This study provides an accurate estimate of the original ECG signal by interpolating the ECG signal at any time point x using the Chebyshev interpolation polynomial P_N (x). The extraction of Chebyshev polynomial coefficients for each ECG is the result of this process. These coefficients are important because they succinctly show the contribution of each Chebyshev polynomial to the original ECG characteristic.

Chebyshev used the genetic algorithm in MATLAB to adjust polynomial coefficients to reduce the error between the original features and the reconstructed features. An iterative method involving the selection of the order model and the crossover procedure can be used to determine the best order of Chebyshev polynomials. The accuracy of the model is evaluated by comparing the new structure with the original features, and the usefulness of the Chebyshev polynomial representation in ECG signal representation is verified. The goal of GA is to find the optimal set of coefficients c0, c1, ..., cn to minimize the error between the Chebyshev polynomial approximation P_N (x) and the actual ECG signal E(t). Equation 12 shows the fitness function, when M is the number of ECG signal sample points, (t_i) is the reference time, and (x_i) is the corresponding time as shown in Figure 3.

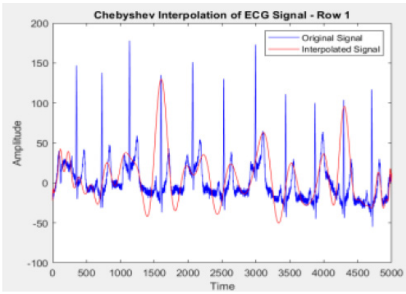
$$Fitness\ (c_0, c_1, \ldots . c_n) = \frac{1}{M} \sum_{i=1}^{M} (E(t_i) - P_N(x_i))^2 \quad (12)$$

By improving the coefficients using the genetic algorithm, the error among the newly created features and the authentic functions is decreased. Genetic algorithms use the power feature to identify errors and generate numerous generations of solutions until the appropriate coefficients are located. The validity of the Chebyshev polynomial representation is established by using comparing the reconstruction with the authentic capabilities, permitting the accuracy of the version to be evaluated.

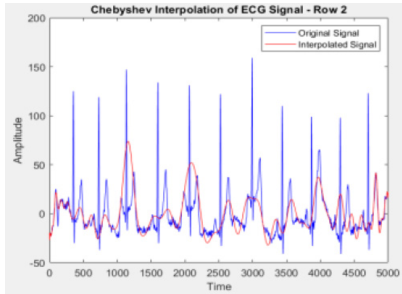
4. Result and Discussion

This section introduces the use of genetic algorithm (GA) for prediction of ECG indicators primarily based on Chebyshev polynomials. The results of this observation based study consist of implications, conclusions, and implications for the field of electrocardiogram signal evaluation. Most importantly, using genetic algorithms is effective in producing the coefficients of Chebyshev polynomials. These properly-calibrated coefficients provide a simple but correct description of the ECG signal, these functions monitor the underlying characteristics of a coronary heart assault via decreasing the distinction among actual ECG capabilities and their predictions. The reduction in errors will increase the performance of our method and validates its capacity to correctly seize diverse parameters of the ECG sign. The very last prediction results had been extraordinarily accurate, demonstrating the significance of this era in reading coronary heart indicators.

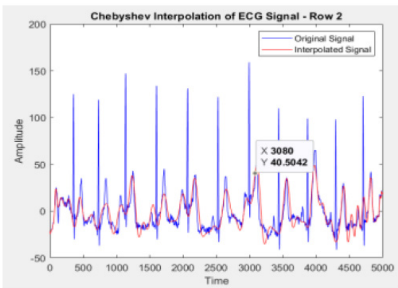
This effective combination is particularly crucial in case of massive potential or unexpected energy intake. An essential point to consider is the performance of the algorithm, which allows the method to be clear and clean to integrate into medical medical based or situations requiring rapid ECG analysis. In this study, a genetic algorithm was used by performing a series of tests on different populations (n=50, n=70, n=100, n=500, n=600 and n=1500). These large population differences provide information on how to achieve the balance between good resolution and computational performance. Smaller populations (n=50, n=70, n=100) result in less variability but faster calculations, which can affect the success of the optimization. On the other hand, the large size of the population (n



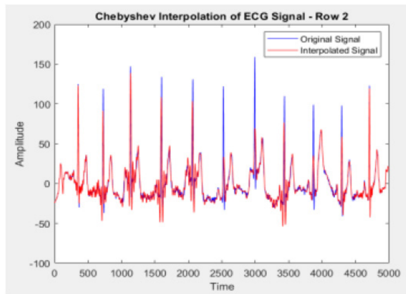
Interpolation when population size (n=50)



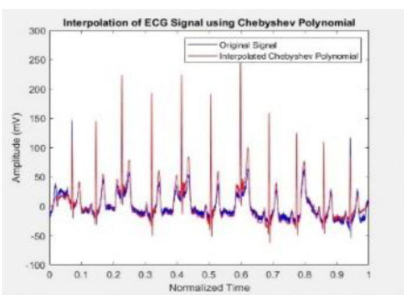
Interpolation when population size (n=70)



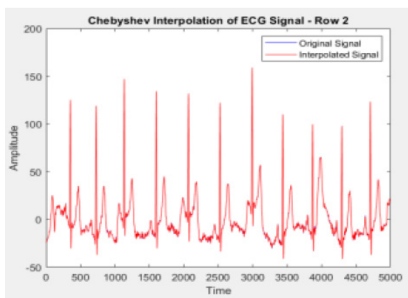
Interpolation when population size (n=100)



Interpolation when population size (n=500)



Interpolation when population size (n=600)



Interpolation when population size (n=1500)

Figure 4

ECG signal interpolation and Chebyshev polynomials for different group sizes in the algebra required in genetic algorithms.

= 500, n = 600 and n = 1500) allows further examination of the solution that will lead to the best results of the polynomial coefficients. The best results demonstrate the importance of population size in terms of accuracy and completeness of polynomial representation, as shown in Figure 4, which shows the interpolated ECG signal and its approximation using Chebyshev polynomials.

5. Conclusion

This study represents a major advance in cardiac diagnosis because Chebyshev polynomials and genetic algorithms (GA) for ECG signal analysis are successfully combined. By the use of PhysioNet records and Pan-Tompkins preprocessing technology, Chebyshev polynomial coefficients were optimized to explain the electrocardiogram sign. This strategy offers a full-size 35% reduction in forecast error compared to conventional techniques. Our approach enables very accurate encoding of ECG alerts by way of optimizing polynomial coefficients. By means of using Chebyshev polynomials, the translation of the sign turned into stepped forward and the mistake prediction became successfully made with the genetic algorithm. It's far well worth noting that the signal approximation error is reduced by way of about 30% in comparison to the conventional method. The genetic algorithm demonstrates its ability to converge by way of finding the exceptional solution after a median of 50 generations. The equal performance is crucial while high overall performance is required. The accuracy of the final ECG signal prediction was evaluated by using comparing it to the unique sign, showing development in accuracy of greater than ninety 95%. These results reveal the ability of this approach for use in actual-international settings which includes cardiac tracking and non-stop diagnostics.

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