

THE WELFARE COST OF INFLATION AND STABILITY OF MONEY DEMAND

Neal Maroney* and Jose Francisco Rubio*

ABSTRACT

The cost of the FED maintaining a low target level of inflation is measured by Lucas (2000) and Ireland (2009) as the welfare cost of inflation based on money demand. Their estimates require stability of the money demand function. Ireland finds that money demand in the post-Volcker period is surprisingly stable, but the introduction of new time series data casts doubt on the stability of money demand and therefore on the applicability of money demand based welfare cost measures of inflation to the post-Volcker period.

I. OVERVIEW

The opportunity cost of money is the nominal interest rate. Friedman (1969) argued that setting nominal rates to zero would be an optimal central bank policy, because the cost of producing money is near zero. This "optimal" rule would require the FED to follow a deflationary policy so that the deflation rate is equal to the real rate. However, for the last 30 years the FED has been following a low inflation target, which leads to welfare losses as this policy deviates from the Friedman rule. Lucas (2000) considered, in his address to the Econometric Society in 1997, the welfare cost of the FED's low inflation policy, based on the inverse money demand welfare measure of Bailey (1956) parameterized by using Meltzer's (1963) log-log and Cagan's (1956) semi-log money demand elasticity estimates.

Meltzer's (1963) log-log money demand function is,

$$\ln(m) = \ln(A) - \eta \ln(r) \quad (1)$$

where m is the ratio of nominal money balances to nominal income, r is a short-term nominal interest rate, A is a constant, and η measures the absolute value of the interest elasticity of money demand. Cagan (1956) specifies money demand in semi-log form,

$$\ln(m) = \ln(B) - \xi r \quad (2)$$

Where r and m are the same as before, B is a constant and ξ is the absolute value of the interest semi-elasticity of money demand.

* Department of Economics and Finance, University of New Orleans, E-mail: nmaroney@uno.edu, jfrubio@uno.edu

The welfare measure of Bailey (1956) is the fraction of income agents would have to be compensated to be in different between facing a steady state nominal rate of r and a steady state nominal rate of zero. To evaluate the welfare measure a functional form of money demand is required. Under Meltzer's (1963) log-log specification in (1) the welfare cost is,

$$w(r) = A \left(\frac{\eta}{1 - \eta} \right) r^{1 - \eta}. \quad (3)$$

Following Cagan (1956) the semi-log money demand given in (2), the welfare cost is,

$$w(r) = \frac{B}{\xi} \left[1 - (1 + \xi r) e^{-\xi r} \right]. \quad (4)$$

Conclusions about the size of welfare losses due to inflation can vary quite considerably, because of the behavior of the log-log and semi-log forms at low interest rates. The semi-log form will likely result in a lower welfare cost as it has satiation point at B while the log-log has no such point.

Lucas (2000) reports parameter values from 1900-1994 of $A = 0.0488$ and $\eta = 0.5$ for the specification in equation (1) and $B = 0.3548$ and $\xi = 7$ for the model in equation (2). Assuming a 3% steady state real rate and no inflation, the cost to the economy is about 0.85 per cent of income when money demand is log-log, and only about 0.10 per cent for the semi-log specification of money demand. However, if inflation increases to a modest 2% Lucas reports that it will cost the economy a loss of 1.09 per cent of income for the specification in (1) and (3) and only about 0.25 per cent of income for the specification in (2) and (4).

Substantial gains in welfare result from moving inflation to zero or to deflating, but only under the log-log specification. In his examination of the data, Lucas concludes the log-log specification is a better fit, and therefore FED could secure a substantial welfare gain for American consumers by abandoning its low but positive inflationary policy.

The accuracy of welfare estimates hinge on the stability of money demand. Lucas did not formally tests for stability. The stability of money demand has been the subject of much past research and there is a well-known shift in monetary policy after Paul Volcker became Chairman of the FED. Ireland (2009) updated welfare estimates using the post-Volcker period from 1980-2006 and considered the stability of money demand a criteria for selecting specifications.

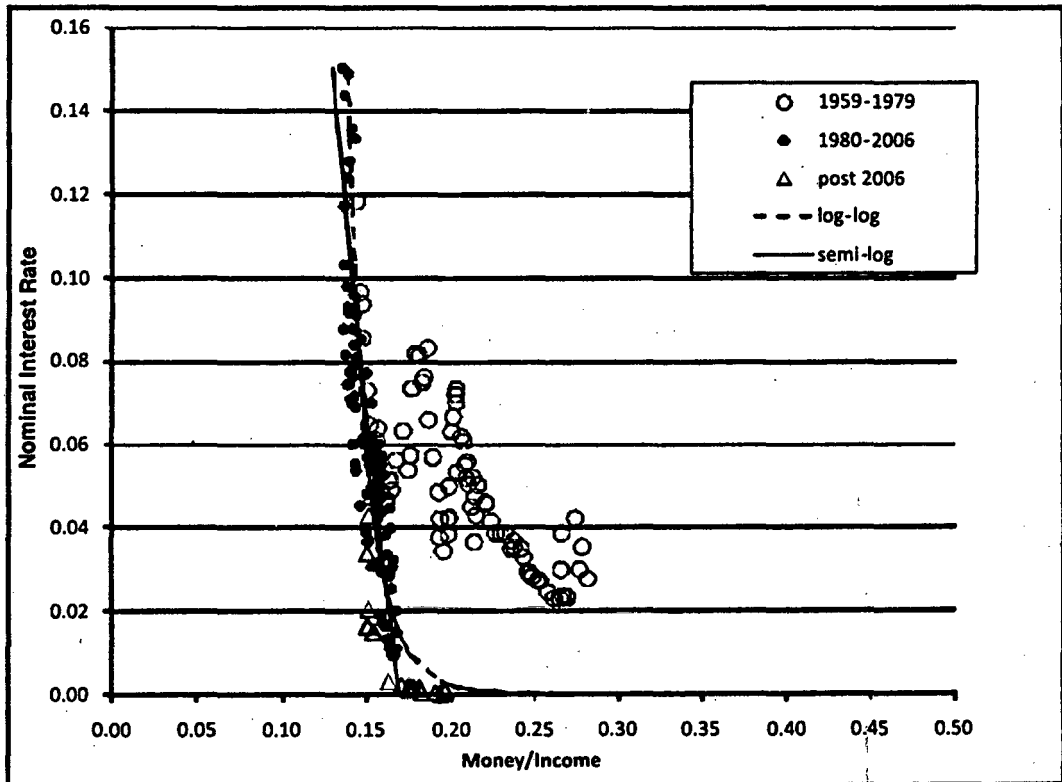
Ireland (2009) found money demand during the post-Volcker period as surprisingly stable, but much more inelastic. According to Ireland's cointegration tests, the semi-log form for money demand is the only specification that is stable. While preferring the semi-log form, he reports both money demand functions and finds estimates of $A = 0.1167$ and $\eta = 0.0873$ for the specification given in (1) and $B = 0.1686$ and $\xi = 1.7944$ for the specification on (2). The welfare cost of inflation in the post-Volcker period is much lower than that reported earlier by Lucas. Ireland concludes that with the semi-log speciation a 2% inflation rate cost 0.04% of income, instead of the 1.09% estimated by Lucas.

The graph presented in Figure 1 depicts the money demand relation as reported by Ireland updated to reflect data from 1959 through the second quarter of 2012. The 3-month T-bill is the nominal rate on the vertical and the nominal money to nominal GDP ratio is on the horizontal. Since 1994, retail sweep programs have distorted the narrow definition of money M1 so Ireland (2009) uses the sweep adjusted M1 (M1 ADJ) to represent nominal money supply. He discovers a very tight and steep relationship. Most observations with higher money to income ratios occur in the pre-Volcker period, which may be due to the increasing velocity of money. As with our recent experience of very low interest rates, when more recent time series are added, interest rates go to zero without much change in the money to income ratio.

Accounting for our recent experience with near zero interest rates we investigate the stability of the money demand in the post-Volcker period. We find estimated elasticities close to that of Ireland but stability is unique to his 1980-2006 sample. We find that money supply and interest rates are no longer cointegrated. Estimated welfare measures suggest very modest welfare costs, but we question the applicability or accuracy of such measures based on inconsistent money demand estimates.

The rest of the paper is organized as follows: section II discusses unit root and cointegration tests, section III presents money demand and inflation welfare costs, and section IV concludes.

Figure 1: Money Demand and Predicted Money Demand 1959: 1-2006: 2



II. STABILITY TESTS

Empirical testing methods as introduced by Engle and Granger (1987), Hafer and Jansen (1991), Hoffman and Rasche (1991), Lucas (2000), and Ireland (2009), have all shown a cointegrating relationship between the money supply and a short term interest rate for specific samples. Ireland's (2009) recent attempt to categorize the money interest rate relationship suggests that the semi-log form of money demand (2) is stable from 1980-2006 due to a significant cointegrating vector linking the two non-stationary variables: the log of the money to income ratio and the nominal interest rate.

While we do examine specification in (1) and (2) these impose a unitary income elasticity of money demand by relating the interest rate terms $\ln(r)$ and r to the log of the money to income ratio $\ln(m)$, we also estimate the money demands without imposing this restriction. The first links the log of real money balances to the log of real income and the log of a nominal interest rate, as:

$$\ln(M/P) = \alpha + \beta_y \ln(Y/P) - \beta_r \ln(r) \quad (1')$$

while the second links the log of real money balances to the log of real income and a nominal interest rate in level, as

$$\ln(M/P) = \alpha + \beta_y \ln(Y/P) - \beta_r r. \quad (2')$$

We extend prior literature by including data which account for lowest interest rates in modern history and data that spans the "great recession" that followed the latest U.S. financial crisis. The time series spans quarters 1980:1 to 2012:2. All data is taken from the FED Saint Louis FRED database. Income (Y) is the gross domestic product (GDP), money (M) is the sweep adjusted money supply (M1ADJ), the nominal rate (r) is the 3-month T-bill yield (TB3MS), and (P) is the GDP deflator (GDPDEF) converting nominal to real values. Our money supply data differs slightly from Ireland (2009) as we use sweep adjusted values from the FED Saint Louis, and Ireland (2009) used a self constructed money supply series.

The first step in determining cointegration is verifying each series has the same level of integration. Unit root tests are summarized in Table I. Panel A reports the Augmented Dickey-Fuller test based on the maximum number of lags given the Schwartz Information Criterion (SIC) that maximizes the log likelihood. Panel B reports the Phillips-Perron test (1988) based on the Newey-West (1987) corrected standard errors. Consistent with prior literature, the null hypothesis of a unit root cannot be rejected at the conventional levels.

Having defined that all series do indeed possess a unit root, we then investigate whether a cointegrating relationship exists. Table II summarizes the results of two different cointegration tests, which follows the same logic of first running an initial OLS regression and then examining the residuals for stationarity.

Panel A of Table II examines the Ireland (2009) 1980-2006 sample period. We use two tests of cointegration: the Engel-Granger test and the Phillips-Ouliaris (1990) test. Unlike Lucas (2000), we find that the only feasible specification of the money supply is given by the semi-log function as described by equation (2), which is consistent with Ireland (2009). Also consistent with Ireland, we do not find any cointegrating relationship between the log-log money demand,

Table I
Unit Root Tests

This table reports Unit Root tests for all relevant variables. We divide the sample into the Ireland (2009) subperiod 1980-2006 ($n = 108$) and the entire post-Volcker period 1980-2006 ($n = 130$). Data are quarterly. Presented are both the Augmented Dickey-Fuller test (1984) and the Phillips-Perron test (1988). We use the number of lags that maximizes the Schwartz Information Criterion (SIC). P-values are reported in brackets. We test the log of nominal money to income ratio, the log of the nominal T-bill rate, and level of the T-bill rate.

Variable	ADF (a)		Phillips-Perron (b)	
	1980-2006	1980-2012	1980-2006	1980-2012
$\ln(m = M/Y)$	-2.0708 [0.2568]	-1.0021 [0.7512]	-1.9186 [0.3227]	-0.5120 [0.8841]
$\ln(R)$	-1.8171 [0.3705]	-0.1955 [0.9349]	-2.0695 [0.2574]	-0.4927 [-0.8878]
R	-1.7433 [0.4067]	-1.3949 [0.5830]	-2.4929 [0.1200]	-2.1630 [0.2209]

* Significant at the 10% level

** Significant at the 5% level

*** Significant at the 1% level

Table II
Cointegration Tests

Table II reports the cointegration tests of our four money demand specifications based on the null hypothesis of no cointegration. We divide the sample into the Ireland (2009) subperiod 1980-2006 ($n = 108$) and the entire post-Volcker period 1980-2006 ($n = 130$). Panel A shows the Engle and Granger cointegration test (tau-statistic), while panel B shows the Phillips-Ouliaris (z-statistic) cointegration test. P-values are shown in brackets.

Panel A: 1980 - 2006 ($n=108$)		
Model	Engle-Granger	Phillips-Ouliaris
$\ln(m) = \alpha - \beta \ln(r)$	-2.5243 [0.2760]	-11.6596 [0.2565]
$\ln(m) = \alpha - \beta r$	-2.5749 [0.2966]	-17.4608* [0.0784]
$\ln(M/P) = \alpha + \beta, \ln(Y/P) - \beta, \ln(r)$	-2.9368 [0.2803]	-13.5599 [0.3727]
$\ln(M/P) = \alpha + \beta, \ln(Y/P) - \beta r$	-2.4072 [0.5381]	-16.7714 [0.2255]
Panel B: 1980 - 2012 ($n=130$)		
Model	Engle-Granger	Phillips-Ouliaris
$\ln(m) = \alpha - \beta \ln(r)$	-2.9754 [0.1237]	-13.0650 [0.1991]
$\ln(m) = \alpha - \beta r$	-1.6534 [0.6998]	-13.5529 [0.1810]
$\ln(M/P) = \alpha + \beta, \ln(Y/P) - \beta, \ln(r)$	-3.1041 [0.2126]	-15.7316 [0.2730]
$\ln(M/P) = \alpha + \beta, \ln(Y/P) - \beta r$	-1.5669 [0.8814]	-12.6049 [0.4314]

* Significant at the 10% level

** Significant at the 5% level

*** Significant at the 1% level

nor do we find any cointegration between the unconstrained money-demands in either specification.

Reliable estimates of the welfare cost of inflation require a stable money demand function. We further include post-recession data to check whether or not semi-log money demand is still stable or other specifications become stable when new data is included. Panel B of Table II summarizes the cointegration tests after including post-2006 data. Our results suggest that a cointegrating relationship no longer exists in the post-Volcker period regardless of money demand specification. The stability found by Ireland is specific to the 1980-2006 sample.

Perhaps a possible explanation for the lack of a cointegrating relationship between money supply and the benchmark interest rate is the fact that interest rates have remained incredibly low after 2006 and especially after 2008. The annualized 3-month T-bill rate was 4.9% as of 2006:4, and it has since decreased to 0.09% by 2012:2.

III. MONEY DEMAND AND WELFARE ESTIMATES

Despite the lack of a cointegrating relationship between money and interest rates, we estimate the different models proposed by equations (1) and (2) and their unconstrained forms with the caveat that we would expect these coefficients to change depending on the sample chosen. The lack of cointegration implies that money demand estimates are likely inconsistent. Table III summarizes the results of the OLS estimations of all the different specifications of money demand.

Panel A of Table III reports the parameter estimates of OLS regressions for the period 1980 – 2006 so we can compare our results to those of Ireland (2009). We find coefficients that are very close. Following the log-log money demand model we find parameter estimates of $\hat{\alpha} = -2.1302$ and $\hat{\beta} = 0.0802$ compared to $\hat{\alpha} = -2.1474$ and $\hat{\beta} = 0.0873$ from Ireland (2008, Table 2); and when following the semi-log specification we find parameter estimates of $\hat{\alpha} = -1.7915$ and $\hat{\beta} = 1.6694$ compared to $\hat{\alpha} = -1.7800$ and $\hat{\beta} = 1.7944$ from Ireland (2008, Table 2). The OLS estimates of the unconstrained income elasticity specification (1') and (2') are consistent with prior literature finding income elasticity values near one in both specifications.

Panel B of Table III then reports coefficients from the new 1980-2012 sample. We find mixed results regarding the money to interest rates elasticity. While the log-log demand shows a much steeper slope with $\hat{\beta} = 0.0488$, the semi-log demand shows a flatter slope with $\hat{\beta} = 1.9894$. We take this as further proof that the demand of money is rather unstable through time. On the other hand, we still find an elasticity of money to income close to one.

Next we calculate welfare estimates based on Ireland's sample and our new sample using parameter estimates reported in Table III. Welfare estimates are reported in Tables IV and V.

Table IV summarizes the results of the welfare cost of inflation which continues to be a very low percentage of income. Having found income elasticity near one, we favor using the estimates from the simpler models in (1) and (2). Based on a steady state real interest rate of 3%, we find that the measurement of the welfare cost at 2% inflation is .188% (log-log) and

.040% (semi-log) per cent of income from 1980 – 2012. Our estimates are very close to Ireland and are slightly higher than for the 1980-2006 period.

We assume the same steady state of real interest rates as Lucas (2000). We expect the actual steady state rate maybe lower, decreasing our estimates of the welfare cost of inflation. Furthermore, calculations assume a positive inflation premium, which clearly is non-existent given that inflation has been persistently higher than T-bill rates in recent years.

IV. CONCLUSION

We update prior literature by including data which account for the lowest interest rates in modern times and data that spans the “great recession.” We revisit Ireland (2009) and find that stability of money demand is confined to a specific period: 1980-2006. With additional time series, we find money demand is unstable thus far into the post-Volcker era. The lack of stability is indicated by the lack of cointegration resulting in inconsistent estimates. This is despite that the money demand equations R-squares are high. The lack of stability is indicated by the variation in updated estimates. Welfare estimates of inflation remain quite low despite they are likely based on inconsistent parameters.

Table III
Money Demand Estimates

Table III reports the OLS estimates of the four different money demand specifications (1, 1', 2 and 2') for the Irelead (2009) sub-period 1980-2006 and the entire post-Volcker period 1980-2006. Data are quarterly. Standard errors are in parenthesis

<i>Panel A : 1980-2006 (n = 108)</i>				
<i>Model</i>	$\hat{\alpha}$	$\hat{\beta}_y$	$\hat{\beta}_r$	R^2
$\ln(m) = \alpha - \beta_r \ln(r)$	-2.1302 (0.0166)	0.0802 (0.0054)		0.6735
$\ln(m) = \alpha - \beta_r$	-1.7915 (0.0065)	1.6694 (0.0994)		0.7266
$\ln(M/P) = \alpha + \beta_y \ln(Y/P) - \beta_r \ln(r)$	-2.4254 (0.0717)	1.0828 (0.0196)	0.0553 (0.0077)	0.9881
$\ln(M/P) = \alpha + \beta_y \ln(Y/P) - \beta_r$	-2.0663 (0.0986)	1.0568 (0.0203)	1.3097 (0.1609)	0.9891
<i>Panel B: 1980 - 2012 (n=130)</i>				
<i>Model</i>	$\hat{\alpha}$	$\hat{\beta}_y$	$\hat{\beta}_r$	R^2
$\ln(m) = \alpha - \beta_r \ln(r)$	-2.0399 (0.0093)	0.0488 (0.0024)		0.7517
$\ln(m) = \alpha - \beta_r$	-1.7689 (0.0066)	1.9894 (0.1092)		0.7216
$\ln(M/P) = \alpha + \beta_y \ln(Y/P) - \beta_r \ln(r)$	-2.9384 (0.0653)	1.0879 (0.0158)	0.0373 (0.0030)	0.9887
$\ln(M/P) = \alpha + \beta_y \ln(Y/P) - \beta_r$	-1.9455 (0.1230)	1.0362 (0.0252)	1.7461 (0.2012)	0.9845

Table IV
Welfare Costs of Inflation

Table IV reports the welfare cost of inflation based on specifications of the money demand (1) and (2) for different periods as presented in table III. The welfare measure of Bailey (1956) from equations (3) and (4) is the fraction, in per cent, of income agents would have to be compensated to be indifferent between facing a steady state nominal rate of and a steady state nominal rate of zero under log-log and semi-log specifications of money demand. We assume a steady state real rate of 3%.

Panel A. log-log welfare measure

$$w(r) = A \left(\frac{\eta}{1-\eta} \right) r^{1-\eta} : \ln(m) = \alpha - \beta \ln(r)$$

Period	$A = e^{\hat{\alpha}}$	$\eta = \hat{\beta}$	Zero Inflation	2% Inflation	10% Inflation
			(a)	(b)	(c)
			$w(3\%)$	$w(5\%)$	$w(13\%)$
1980 – 2006	0.16671	0.11881	0.102%	0.160%	0.372%
1980 – 2012	0.17052	0.13004	0.121%	0.188%	0.432%

Panel B. semi-log welfare measure

$$w(r) = \frac{B}{\xi} [1 - (1 + \xi r)e^{-\xi r}] : \ln(m) = \alpha - \beta r$$

Period	$B = e^{\hat{\alpha}}$	$\xi = \hat{\beta}$	Zero Inflation	2% Inflation	10% Inflation
			(a)	(b)	(c)
			$w(0.03)$	$w(0.05)$	$w(0.13)$
1980 – 2006	0.16671	1.66940	0.012%	0.033%	0.606%
1980 – 2012	0.17052	1.98940	0.015%	0.040%	0.637%

Considering the period since 2001, inflation has remained consistently higher than the T-bill rate, suggesting that expectations theory may need to be amended from its simplest form. It seems the FED has indeed been able to maintain a near zero interest rate policy by choice or not, but with low inflation and negative real rates — hardly what Freidman had in mind in the late 1960's when formulating the optimal quantity of money.

References

- Bailey, Martin J. (1956), "The Welfare Cost of Inflationary Finance," *Journal of Political Economy*, 64(2): 93–110.
- Cagan, Phillip. (1956), "The Monetary Dynamics of Hyperinflation," In studies in the Quantity theory of Money, ed. Milton Friedman, 25–117. Chicago: University of Chicago Press.
- Engle, Robert F., and Clive W. J. Granger. (1987), "Co-Integration and Error Correction: Representation, Estimation, and Testing." *Econometrica*, 55(2): 251–76.
- Friedman, Milton. (1969), "The Optimum Quantity of Money." In the Optimum Quantity of Money and Other Essays, 1–50. Chicago: Aldine Publishing Company.
- Hafer, R. W., and Dennis W. Jansen. (1991), "The Demand for Money in the United States: Evidence from Cointegration Tests." *Journal of Money, Credit, and Banking*, 23(2): 155–68.

- Hoffman, Dennis L., and Robert H. Rasche. (1991), "Long-Run Income and Interest Elasticities of Money Demand in the United States." *Review of Economics and Statistics*, 73(4): 665-74.
- Ireland, Peter N. (2009), "On the Welfare Cost of Inflation and the Recent Behavior of Money Demand." *American Economic Review*, *American Economic Association*, 99(3): 1040-52.
- Lucas, Jr., R. E. (2000), "Inflation and Welfare." *Econometrica*, 68(2): 247-274.
- Meltzer, Allan H. (1963), "The Demand for Money: The Evidence from the Time Series." *Journal of Political Economy*, 71(3): 219-46.
- Newey, Whitney K., and Kenneth D. West. (1987), "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica*, 55(3): 703-708.
- Phillips, Peter C. B., and S. Ouliaris. (1990), "Asymptotic Properties of Residual Based Tests for Cointegration." *Econometrica*, 58(1): 165-93.
- Phillips, Peter C. B., and Pierre Perron. (1988), "Testing for a Unit Root in Time Series Regression." *Biometrika*, 75(2): 335-46.