Modeling Interaction Effects in Latent Growth Curve Models

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This article presents an extension of Jöreskog and Yang's (1996) method of estimating interaction effects among latent variables to latent growth curve models. Models involving a product of 2 latent factors of either static or dynamic variables (parental monitoring and rule-setting) predicting an outcome growth variable (adolescents' initial status and rate of change in antisocial behavior) are used as substantive illustrations. The results are discussed in terms of practical and statistical problems associated with interaction analyses in latent curve models, and in structural equation models with latent variables in general.

A recent methodological focus in the structural equation modeling (SEM) literature has been the extension and application of standard linear models to situations involving nonlinear relations between latent variables (see Jaccard & Wan, 1996; Schumacker & Marcoulides, 1998). Two techniques now available for SEM application are (a) multisample analysis and (b) indicant product analysis (Rigdon, Schumacker, & Wothke, 1998). Generally, in situations where one or both of the interacting variables are discrete or categorical, the multiple sample approach is the analysis of choice. The indicant product approach, on the other hand, is used when both interacting variables are continuous in nature. Within the indicant product approach, several procedures are available, including methods proposed by Jaccard and Wan (1995), Jöreskog and Yang (1996), and Ping

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(1995, 1996). Each of these relies on the pioneering work of Kenny and Judd (1984) but provides variations that are simpler in their procedural implementation.

Recently, the analysis of intra- and inter-individual trajectories in the context of developmental models of growth and change has been of growing interest. These growth trajectory models enable more integrated and flexible approaches for the study of group change and individual differences in change. Growth modeling techniques in the context of latent variable modeling (McArdle, 1988; Meredith & Tisak, 1990; Muthén, 1991) have been well demonstrated (e.g., Duncan & Duncan, 1995; McArdle & Epstein, 1987; Raykov, 1996; Stoolmiller, 1995; Willett & Sayer, 1994). Although it is clear that latent curve analysis can be a useful tool for analyzing patterns of change and predictors of change, the method can be extended to the analysis of more complex dynamic models (e.g., McArdle, in press; Muthén, in press). In the case of this article, it involves interactive relations between growth parameters. Modeling the interaction among change scores (growth factors) may be of substantive interest in longitudinal studies involving hypotheses testing of how changes in latent attributes interact to produce a joint effect on growth of an outcome variable.

Although there are various techniques for estimating latent interaction effects (see Schumacker & Marcoulides, 1998, for a review of currently available procedures), among the full information-based methods, the Jöreskog and Yang (1996) procedure is the most relevant for applications within the latent growth curve modeling framework. This is because the Jöreskog and Yang approach involves specification of both first and second moments (i.e., the mean vector and covariance matrix) in the model estimation.[1] Other procedures (e.g., Jaccard & Wan, 1995; Ping, 1995, 1996) require centering of the raw scores (Jaccard & Wan, 1996) and are, therefore, not directly applicable to latent curve analysis that utilizes both covariance and mean structures.

In this article, the potential utility of Jöreskog and Yang's (1996) procedure to the analysis of latent growth models is explored. To facilitate the presentation, a two-factor linear growth curve model is briefly presented. This is followed by a model incorporating an interaction effect between two static (time-invariant) latent predictors on growth of an outcome variable (i.e., predictors of the initial level and rate of change). This model is further expanded to include estimation of an in-

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[1]Bollen (1995) proposed a two-stage least squares analysis of latent variable interactions, which relies on a limited information rather than a full information method of estimating parameters. Although the procedure involves the use of the mean vector in addition to the covariance matrix, we chose to use Jöreskog and Yang's (1996) procedure in this article because latent curve analysis to date has relied heavily on the use of the full information approach (e.g., fitting the moment matrix implied by the model to the corresponding sample moment matrix by minimizing a fit function with respect to all parameters simultaneously).

teraction term between two latent growth, or shape, factors describing changes in the growth form. Each of these models is followed by an example using empirical data. Model parameterizations are presented within the LISREL framework (Jöreskog & Sörbom, 1993).

A TWO-FACTOR LATENT GROWTH MODEL

For presentation purposes, an unconditional linear growth model, shown in Figure 1, consisting of four measurement data points, is briefly discussed (for further details see McArdle & Epstein, 1987; Meredith & Tisak, 1990; Willett & Sayer, 1994). The basic latent growth modeling, using the LISREL Y-measurement model, is specified by the following equation:

$$Y = \tau_y + \Lambda_y \eta + \varepsilon \tag{1}$$

where Y is the $p \times 1$ vector of repeated measures, τ is a $p \times 1$ vector of constant intercept terms that is set to zero (Equation 5; Willett & Sayer, 1994, p. 369), Λ is the $p \times m$ parameter matrix of sequential known values of the growth curve records, η is an $m \times 1$ vector of latent growth parameters, and ε is a $p \times 1$ vector of unique variances in Y, where it is assumed that the covariance matrix Θ_ε is a diagonal matrix ($\Theta_\varepsilon = Cov(\varepsilon) = \{\sigma^2_{\varepsilon_1}, \sigma^2_{\varepsilon_2}, \sigma^2_{\varepsilon_3}, \sigma^2_{\varepsilon_4}\}$). Equation 1 is expressed in the following matrix form to represent the model depicted in Figure 1:

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & t_1 \\ 1 & t_2 \\ 1 & t_3 \\ 1 & t_4 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{pmatrix} \tag{2}$$

The Λ parameter matrix to the right of the equal sign is a product matrix representing known times of measurement (t_1 through t_4) and constraints (the values of t should reflect the spacing between measurement occasions), and a latent growth vector containing the initial level (η_1; intercept) and rate of change (η_2; slope). The two-factor linear growth model in this case is specified so that the intercept factor (constrained to a constant value of 1) serves as the starting point (i.e., initial status) for any change (growth) across time and the slope factor captures the rate of change of the trajectory over time. The scaling of the slope can be specified by using either fixed value restrictions (e.g., 0, 1, 2, 3) representing a straight-line growth, or unspecified value restrictions (where the first two loadings are fixed, i.e., $t_1 = 0$, $t_2 = 1$, for model identification purposes and the remaining two, t_3 and t_4, are freely estimated) allowing estimation of

an optimal pattern of change over measurement occasions (Meredith & Tisak, 1990).

The equation for the latent growth parameters (intercept, slope) can be written as

$$\eta = \alpha + B\eta + \zeta \tag{3}$$

where α is an $m \times 1$ vector of the population averages of the latent individual growth parameters, B is a $m \times m$ matrix of coefficients expressing the structural relations between the η variables, and ζ is an $m \times 1$ vector of equation residuals or random disturbances. With the B matrix constrained to zero (Equation 10; Willett & Sayer, 1994, p. 370), the simplified equation becomes

$$\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix} \tag{4}$$

where α is the latent mean vector (where $\alpha_1 =$ intercept, $\alpha_2 =$ slope), and the ζ vector contains the deviations of the latent growth scores from their respective population means with their covariance, $Cov(\zeta_2,\zeta_1)$, as indicated by ψ_{21} in Figure 1, parameterized in the LISREL ψ matrix.
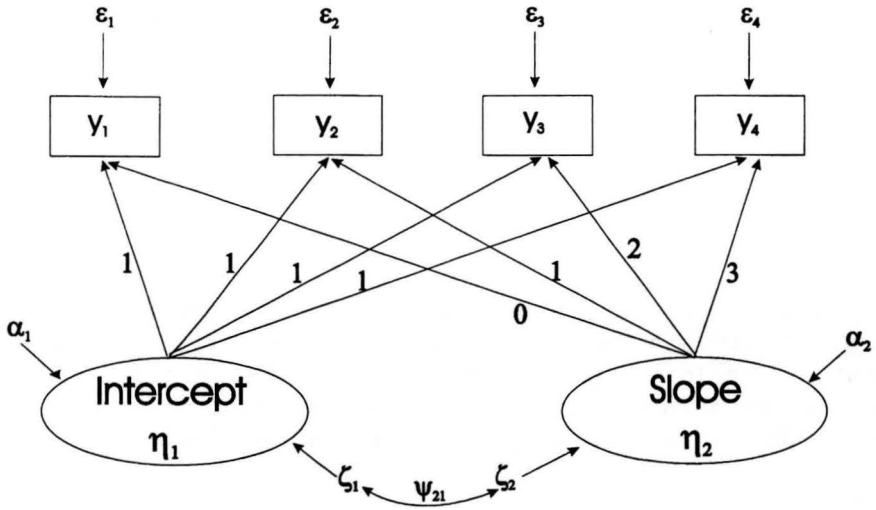

FIGURE 1   A linear trend growth curve model.

LATENT GROWTH CURVE MODELS WITH
INTERACTION EFFECTS

The latent growth model presented in the preceding section can be quite flexible in addressing various questions related to growth or change (e.g., Duncan, Duncan, Strycker, Li, & Alpert, 1999; MacCallum, Kim, Malarkey, & Kiecolt-Glaser, 1997; Muthén, 1997; Muthén & Curran, 1997; Willett & Sayer, 1994, 1996). In addition to evaluating hypotheses about differences in initial levels of an attribute and intra-individual variability in the rate of change, predictors may be specified in the models to account for the variation in these growth parameters. For example, questions and hypotheses about determinants and outcomes of growth can be parameterized by focusing on predictors of slope scores and/or slope scores as predictors (e.g., Stoolmiller, Duncan, Bank, & Patterson, 1993; Walker, Acock, Bowman, & Li, 1996). In the following sections, an extension of SEM interaction techniques to latent growth curve models is provided. Specifically, two models are illustrated: (a) a cross-sectional model incorporating the effect of an interaction between two static factors on change in an outcome, and (b) a longitudinal model incorporating the effect of an interaction between two growth factors on change in an outcome.

Latent Growth Model With Time-Invariant Latent Predictors

In this section, a model incorporating an interaction effect between two time-invariant static latent predictors is illustrated using the method outlined by Jöreskog and Yang (1996). This static model is tested prior to creation of a dynamic model to determine whether an interaction effect exists on an outcome variable of interest.

In longitudinal research, a major focus with regard to individual development is the evaluation of predictors of variation in change over time. What variables predict why some individuals have a dramatic change and other individuals have little or no change? In a cross-sectional regression model with two static variables, X and Z, for predicting a latent growth outcome variable, Y, consisting of initial status and rate of change, one asks the question, "Does the influence of predictor X on the initial status of Y depend on the level of Z predictor (a moderator)?" Longitudinally, one asks, "Does the influence of predictor X on individual differences in change of Y depend on the level of Z predictor?" These questions involve moderation (in the sense defined by Baron & Kenny, 1986) and/or the joint effect of X and Z in predicting the dynamic variable Y. Here, the interaction is conceptualized in terms of the effect of the product XZ on Y, where XZ is a latent multiplicative term as described by Kenny and Judd's (1984) work.

To illustrate this, the hypothetical model in Figure 1 is expanded to include two latent exogenous traits (ξ_1 and ξ_2), each measured by two observed indicators (x_1 and x_2 for ξ_1, and x_3 and x_4 for ξ_2). The third factor, denoted by $\xi_1\xi_2$, represents a la-

tent product term of ξ_1 and ξ_2, with its cross-product indicators formed by multiplying each of observed indicators of ξ_1 and ξ_2. Details on the formulation of these product terms are presented in Appendix A (see also Jöreskog & Yang, 1996). There is a two-factor linear latent endogenous growth outcome variable (symbolized by η_1 [intercept] and η_2 [slope]), measured by four observed indicators taken at four measurement time points. The initial status factor (intercept), η_1, specifies the initial levels of individual growth whereas the linear growth factor (slope), η_2, specifies the rate of change over the four-measurement period. The model described is shown in Figure 2 with model parameterization in LISREL notation presented in Appendix B.

The central focus of the presentation here is (a) the interaction between the two latent exogenous static factors (ξ_1 and ξ_2) on the latent endogenous outcome, consisting of the initial status and growth factors, and (b) the interpretation of the interaction. Substantively, it examines the composite hypothesis that the influence of ξ_1 on η_1 (initial status) and η_2 (rate of change) is moderated by the level of ξ_2. This hypothesis is examined in terms of the effect of $\xi_1\xi_2$, the latent product term, on the latent outcome growth factors, η_1 and η_2. This multiplicative interaction effect can be observed in the value of γ_{13} and γ_{23} in each of the following structural regression equations:

$$\eta_1 = \alpha_1 + \gamma_{11}\xi_1 + \gamma_{12}\xi_2 + \gamma_{13}\xi_1\xi_2 + \zeta_1 \tag{5}$$

$$\eta_2 = \alpha_2 + \gamma_{21}\xi_1 + \gamma_{22}\xi_2 + \gamma_{23}\xi_1\xi_2 + \zeta_2 \tag{6}$$

The regression coefficients, γ_{11}, γ_{12}, and γ_{13}, in Equation 5, can be interpreted as putative, synchronous effects on η_1, estimated with all variables measured cross-sectionally, because the operationalization of ξ_1, ξ_2, and η_1 is with respect to a single point in time (assuming the initial status, η_1, is defined as $t_1 = 0$). If γ_{13} is significant, then the effect of ξ_1 on the initial level of Y is contingent on the level of ξ_2, indicating the interaction effect. Moreover, the influence of either ξ_1 or ξ_2 on η_2, expressed in Equation 6, may be viewed as a moderator affecting the relation between repeated measures Y variable ($t=0$) and time (the slope of the Y variable on $T = 4$ measurements). That is, γ_{21} and γ_{22} represent the effect of ξ_1 and ξ_2, respectively, on the rate of change in Y as expressed by the Y slope factor, η_2. If both of these values are positive, then people who are high on ξ_1 and ξ_2 change more on Y than people who are low. The exogenous product factor, $\xi_1\xi_2$, tests for an interaction between ξ_1 and ξ_2. The value of γ_{23} measures the extent to which the influence of ξ_1 is contingent on the level of ξ_2. A positive value of γ_{23} signifies that the higher the score a person has on ξ_2, the stronger the effect of ξ_1 on the rate of change in η_2.
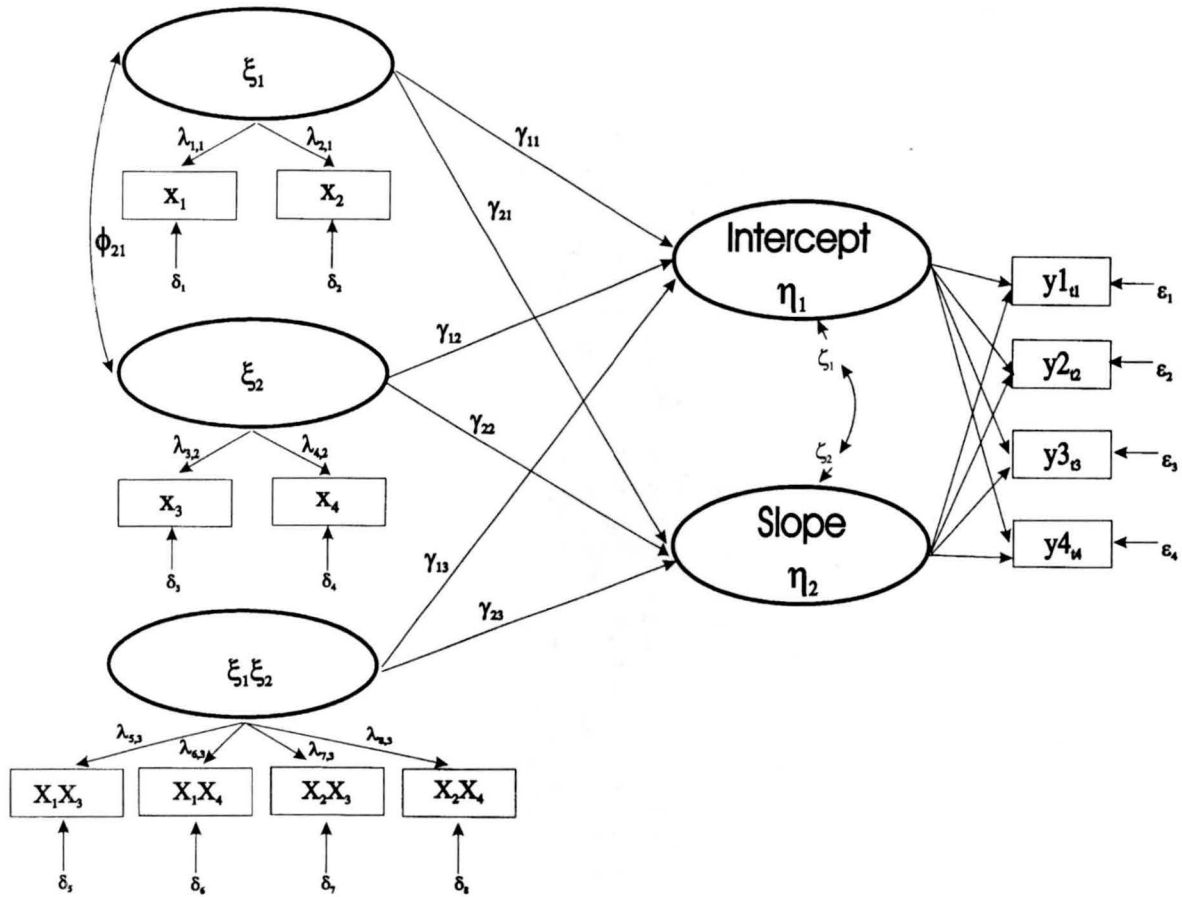
FIGURE 2   A latent variable model including two static variables and one latent growth outcome variable.

Selection of product indicators for the interaction term.   A relevant issue in testing the cross-sectional interaction effect is the selection of product indicators for defining the latent product variable $\xi_1\xi_2$. With two observed variables for each latent factor, four observed product variables (i.e., x_1x_3, x_1x_4, x_2x_3, x_2x_4) can be formed as indicators of the latent product variable $\xi_1\xi_2$ (Kenny & Judd, 1984). However, assuming measurement of the latent variables is reliable, Jöreskog and Yang (1996) showed that only one observed product indicator (e.g., x_1x_3) is sufficient to identify all the parameters of the model. Thus, the exact number of product indicators to be used in the model is a decision to be made by the substantive researcher on the basis of either statistical and/or empirical justifications.

AN ILLUSTRATION OF LATENT GROWTH MODELING
WITH STATIC PREDICTORS OF CHANGE

The following section provides an empirical example of the latent growth curve model involving an interaction term between the latent time-invariant (static) predictors shown in Figure 2. The data used in this example came from a longitudinal study of the prevention of problem behavior among middle school students (Metzler, Biglan, Ary, & Li, 1998). Participants ($N = 221$) were asked to provide self-report data about parenting practices, family interactions, youth problem behavior, and peer behavior every 3 months over a 4-year period. Only the data collected during the first year were used in this article. Of all the students, 36% were in fifth grade, 35% were in sixth grade, and 29% were in seventh grade at the initial assessment.

Measures of adolescents' antisocial behavior, parental monitoring, and parental rule-setting were used in this example. They were youth-report measures assessed on four occasions (t_1, t_2, t_3, and t_4) with 12 observed variables (i.e., y_1, y_2, y_3, and y_4 for antisocial behavior; x_1, x_2, x_3, and x_4 for monitoring; x_5, x_6, x_7, and x_8 for rule-setting). The parental monitoring variable was defined by five items, each anchored with a 5-point scale ranging from 1 (*never*) to 5 (*always;* see Metzler et al., 1998). These items reflected the degree to which parents monitored their children's activities, whereabouts, and plans. An average of these items was taken as a measure of parental monitoring with a high score indicative of high levels of parental monitoring. For the variable of rule-setting, there were four items reflecting rules or expectations that students reported their parents had (e.g., do homework everyday, don't smoke/chew, etc.). For each item, students indicated the degree to which their parents had a clear rule or expectation on a 4-point scale ranging from 1 (*My parents don't have a rule or expectation*) to 4 (*My parents have a clear rule or expectation*). An average of these items was taken as a measure of parental rule-setting with a high score indicative of

high levels of rule-setting. Finally, seven items such as stealing, skipping school, staying out all night without permission, destroying property, and hitting others were used to assess students' antisocial behavior. These items were adapted from Elliott, Ageton, Huizinga, Knowles, and Canter (1983). Students reported the number of times they had engaged in these behaviors in the past month on a 6-point scale. Scores were summed to form a composite score and used as an outcome variable in the interaction model analysis.

Preliminary analyses indicated that there were no differences in the variables of interest across the three grades in the sample. For tests of the model presented in Figure 2, baseline measures of parental monitoring and rule-setting taken from Wave 1 ($t = 1$, two observed indicators for each construct via random splitting of the measure) and time series measures of antisocial behavior taken from the four repeated measurement points ($t = 1,2,3,4$) were used. Therefore, in this first example, both parental monitoring and rule-setting constructs were treated as static predictors (at $t = 1$) of change in adolescents' antisocial behavior.

As is typical in longitudinal studies, there are missing data for many participants at several of the four time points. Assuming implicitly that the data were missing at random as defined by Little and Rubin (1987), missing values were imputed using NORM (Schafer, 1997; Schafer & Olsen, 1998), a program for the analysis of incomplete multivariate data.

Theory Underlying the Empirical Data

Evidence suggests that parenting practices are highly influential in the development of adolescent problem behavior (Dishion, French, & Patterson, 1995; Loeber & Dishion, 1983; Patterson, Reid, & Dishion, 1992). Although it is beyond the scope of this article to develop a sound theoretical rationale for studying the relations, we note that the parenting practices variables are central to the Patterson et al. (1992) coercion theory of the development of antisocial behavior. Coercion theory postulates that early antisocial behavior is the product of unskilled and coercive parenting. Although yet to be empirically examined, the effectiveness of parental monitoring and rule-setting was hypothesized to interactively influence adolescents' disruptive behavior. The hypothesized interaction between the two parenting practices is such that the relation between parental monitoring and adolescents' antisocial behavior becomes more negative as the level of parental rule-setting increases. In other words, parental monitoring helps the most when there is a high level of parental rule-setting. This hypothesized interaction effect is examined both cross-sectionally (i.e., monitoring and rule-setting defined as latent time-variant predictors) and longitudinally (i.e., monitoring and rule-setting defined as latent growth predictors). The following section presents the cross-sectional results, where monitoring and rule-setting were treated as static predictors.

Results

As a preliminary analysis examining change on the dependent variable of adolescent antisocial behavior over the four measurement occasions, a univariate growth model was first tested. A two-factor growth model consisting of intercept and slope for the four repeated measures of antisocial behavior found a significant positive mean for the slope factor ($\mu_{slope} = .052$, $p < .05$). This showed that the overall group reported increases in adolescent antisocial behavior over time (i.e., an increase in the rate of change of the trajectory over time on antisocial behavior). The equally spaced factor loadings (0, 1, 2, 3) reflected that this increase was linear. A significant variance component in both the intercept ($\psi_{intercept} = .06$, $p < .05$) and the slope ($\psi_{slope} = .004$, $p < .05$) factors indicated that there were significant individual differences in both initial levels and growth in adolescent antisocial behavior over time.

The estimated interaction model resulted in a chi-square value of $\chi^2(62, N = 221) = 215.29$, $p < .001$, and goodness-of-fit indexes of Nonnormed Fit Index (NNFI) = .97, Comparative Fit Index (CFI) = .97, and Root Mean Squared Error of Approximation (RMSEA) = .10, indicating a marginal fit of the model to the data. The maximum likelihood (ML) estimates of gammas (γs), standard errors, and z statistics for Equations 5 and 6 are presented later, where ξ_1 represents parental monitoring and ξ_2 represents rule-setting:

$$\eta_1 = 1.280 + (-.176)(\xi_1) + (-.086)(\xi_2) + .252(\xi_1\xi_2)$$
$$\quad\quad\quad\quad (.035)\quad\quad\quad (.065)\quad\quad\quad (.060)$$
$$\quad\quad\quad\quad -5.097\quad\quad\quad -1.337\quad\quad\quad 4.175$$

$$\eta_2 = -.007 + .024(\xi_1) + (-.025)(\xi_2) + (-.053)(\xi_1\xi_2)$$
$$\quad\quad\quad\quad (.015)\quad\quad\quad (.029)\quad\quad\quad (.027)$$
$$\quad\quad\quad\quad 1.561\quad\quad\quad -.869\quad\quad\quad -1.986$$

Results indicated a statistically significant interaction effect for the estimate of γ_{13} and γ_{23} ($p < .05$), showing a positive interaction ($\gamma_{13} = .252$ unstandardized and .436 standardized) between $\xi_1\xi_2$ and η_1 and a negative interaction ($\gamma_{23} = -.053$ unstandardized and −.506 standardized) between $\xi_1\xi_2$ and η_2. Substantively, the γ_{13} in the aforementioned first equation is difficult to interpret. The main effects (γ_{11} and γ_{12}) are negative, showing that both parental monitoring and rule-setting decrease the initial level of antisocial behavior (η_1), although rule-setting is not significant. However, the positive interaction, γ_{13}, partially offsets this effect when both parental monitoring and rule-setting are high. This positive interaction effect is counterintuitive. To further understand the nature of this interaction, we substituted values for parental monitoring and rule-setting that were 1 SD below and 1 SD above their means. For rule-setting we used values of −.48 and .48, and for parental

monitoring we used values of $-.76$ and $.76$.[2] Substituting these values for γ_{11} and γ_{12} in the first equation, we found that the highest level of antisocial behavior occurred when both rule-setting and parental monitoring were low (i.e., 1 SD below their means); however, antisocial behavior was slightly higher when both rule-setting and parental monitoring were high (i.e., 1 SD above their means), than when rule-setting was low and parental monitoring was high.

Because the measures of parents' monitoring and rule-setting were Time 1 variables, the effect of the interaction term, γ_{13}, represents a synchronous effect on the intercept (η_1, initial level). However, the second equation is predicting the longitudinal change in antisocial behavior (η_2, the rate of change). This means that the interaction term found in the second equation, γ_{23}, represents a lagged negative effect on the rate of change in antisocial behavior. The simplest interpretation of the negative value of γ_{23} is that when parents both set rules and monitor their children, there will be a decrease in the rate of change in antisocial behavior (i.e., there will be less antisocial behavior) beyond the additive effects of parental monitoring and rule-setting. Parents who have high scores on both monitoring and rule-setting reduce the growth rate of antisocial behavior for their adolescents. This result appears to conform to coercion theory (Patterson et al., 1992). Because this is an interaction effect, it is in addition to the additive effect of parental monitoring and rule-setting. The combination of close monitoring and rule-setting can reduce the rate of growth of antisocial behavior beyond what the variables do additively.

As a way of further probing this interaction effect, it is possible to substitute various values into the second equation to estimate the effect of particular joint combinations of parental monitoring and rule-setting. For example, if parents are high on monitoring, but low on setting rules, according to our hypothesis, there will be an expected increase in the rate of longitudinal change on antisocial behavior (i.e., more antisocial behavior). Using a value of $.76$ for high monitoring and $-.48$ for low rule-setting, we estimate the slope, η_2, to be $.042$. This shows a positive effect, confirming the preceding supposition. By contrast, using a value of $.76$ for high monitoring and $.48$ for high rule-setting we estimate the slope to be $-.021$. This shows a negative effect indicating lower incidents of antisocial behavior when parents are high on both monitoring and rule-setting. When parents were low in both rule-setting ($-.48$) and monitoring ($-.76$), a negative slope ($-.032$) was also produced. This needs to be understood in the context of the extremely high intercept for this configuration. The interaction model indicates that parents who are low on both monitoring and rule setting may have a reduction in the growth of antisocial behavior on the part of their children, but these children start at a relatively high level of antisocial behavior based on the intercept (the first equation).

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[2]These values are derived from the variance of the latent variables. The variance for the latent variable of parental monitoring (ξ_1) is $.578$ and the variance for the latent variable of rule-setting is $.23$ (ξ_2). The square root of the variance is the estimated latent variable standard deviation.

The model just described accounted for approximately 70% of the variance in initial level and approximately 40% of the variance in rate of change in adolescents' antisocial behavior. The effect size of the interaction, defined as the increment in squared multiple correlation (R^2) beyond a main effect, was assessed by the difference in R^2 between the interactive model and the "individual effect only" model (result not shown). The interactive model was defined as one in which the parameters γ_{13} and γ_{23} were freely estimated, whereas the individual effect model was defined as one in which the interaction effects were fixed at zero ($\gamma_{13} = \gamma_{23} = 0$). The chi-square difference test, $\chi^2_{\text{diff}(2)} = 17.878, p < .001$, indicated that the model without the interaction effect had a poorer fit, $\chi^2(64, N = 221) = 233.166, p < .001$. The difference in R^2 was .122 (.697 to .575) for the initial level and .179 (.403 to .224) for the rate of change, indicating a noticeable increment in variance accounted for over the "individual effect only" model. The LISREL program used to test the model shown in Figure 2 is provided in Appendix C (the sample data are available on request from the authors).

Latent Growth Models With Interaction Between Latent Growth (Shape) Factors

Findings from the previous model provide useful information about the static joint effect of parental monitoring and rule-setting practices on the development of adolescent antisocial behavior. It is possible that this effect unfolds over time. Extending the previous model to a full multivariate latent growth model context, however, further tests the hypothesis about the influence of X and Z on Y by operationalizing X and Z as time-changing latent growth predictors. In this extended hypothetical model, the researcher is able to examine the extent to which the impact of joint changes in X and Z influences changes in Y. More specifically, the model allows for an examination of whether the effect of changes in X on Y is influenced or moderated by the level of simultaneous change in Z, and tests the interactive relation between the two growth parameters (i.e., shape factors) on the slope of Y. As such, the researcher examines whether there is an interaction between two dynamic predictors, X and Z. In the following section, the time-invariant predictor model (Figure 2) is expanded to accommodate the situation where two latent exogenous variables, consisting of growth curve factors, are hypothesized to interact with each other to influence an endogenous growth factor. This model is shown in Figure 3.

Although the model shown in Figure 3 is acknowledged to be complex and the data analysis is an extension of existing procedures, this approach is central to basic questions that are asked by many social science researchers: Does the change in predictor variables influence the rate of change in an outcome variable? Does the simultaneous change in predictor variables (interaction effect) influence the rate of change in an outcome variable? Social science and clinical research have a long standing interest in how change in one variable or set of variables is related to
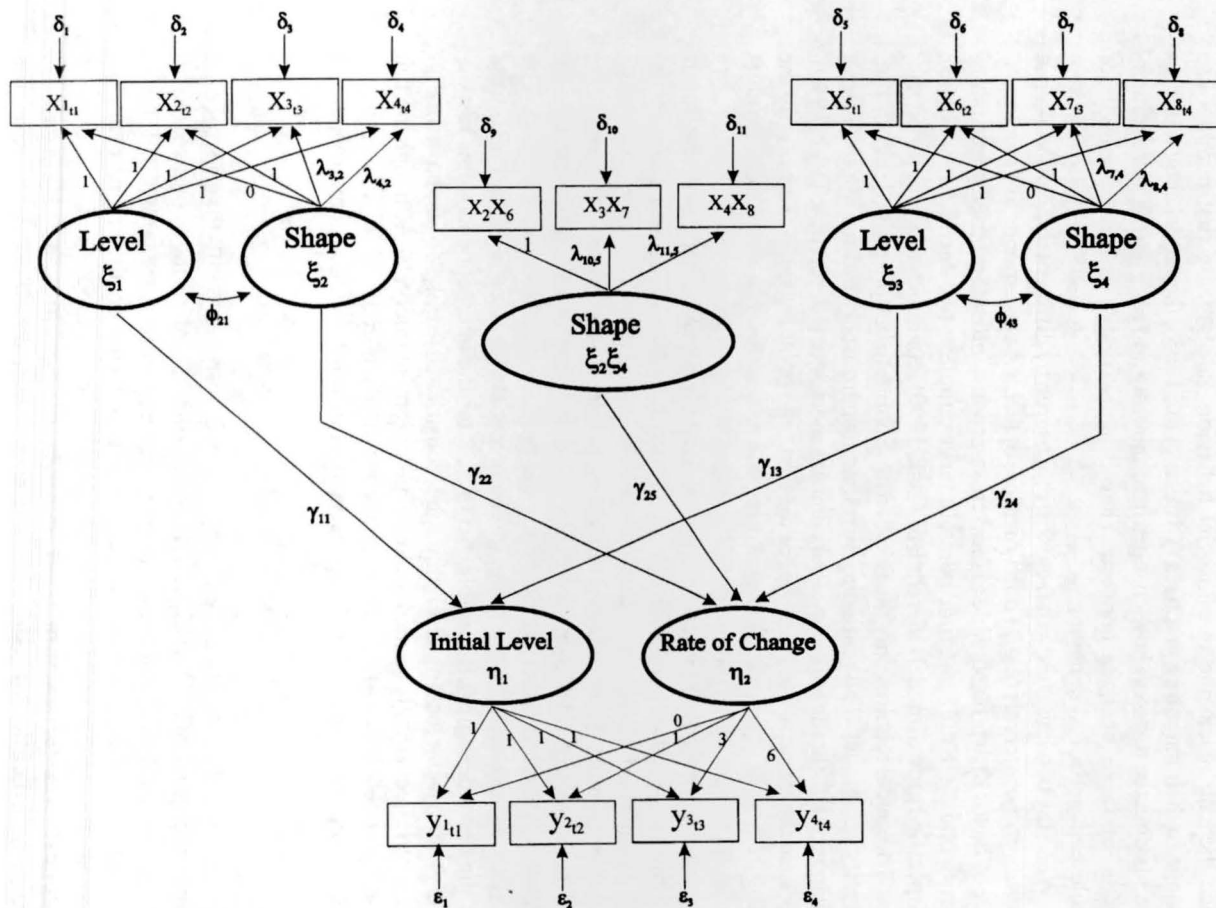
FIGURE 3   A latent growth curve model involving interaction between two latent growth curve factors.

change in another variable. The model shown in Figure 2 only allows us to examine how initial monitoring and rule-setting and their interaction at Time 1 influence the rate of change in antisocial behavior. Recognizing that families can change in their parenting practices (Dishion et al., 1995), the model shown in Figure 3 is directed to assess how changes in monitoring and rule-setting and the interaction of these changes between Time 1 and Time 4 influence growth of adolescent antisocial behavior.

When the two latent growth factors are measured by different indicators taken at the same time intervals (e.g., Time 1 measure of factor X corresponds to Time 1 measure of factor Z), it is reasonable to form cross-product indicators based on measures corresponding to the same time point. In this example, a two-factor (level-shape), unspecified growth model (Meredith & Tisak, 1990) is parameterized for the predictors. The model requires only the first two time points to be constrained (i.e., 0, 1) to identify the model. The remaining two loadings are freely estimated providing a "shape model" that approximates curvilinear trajectories (Meredith & Tisak, 1990). Because the shape factor loadings for the first time point are fixed to zero, only the measurement of t_2, t_3, and t_4 for monitoring and rule-setting are used to form product variables to be used as indicators of the latent product variable.

Measures of the latent variables were selected from t_2 and subsequent time points to form cross-product terms of x_2x_6, x_3x_7, and x_4x_8, as shown in Figure 3. Note that the x_1x_5 product variable was not used in the model because the two original nonproduct variables (x_1 and x_5) were constrained to zero for identification purposes (see Figure 3). Therefore, instead of using the x_1x_5 product indicator as a reference variable for scaling the latent product variable $\xi_2\xi_4$, the product indicator x_2x_6 was used. The LISREL specification for the Y-measurement model expressed in Equation 2 is used for the dependent growth variable, η_i ($i = 1$, 2 for the intercept and slope, respectively), with factor loadings for t_1 through t_4 constrained to 0, 1, 3, 6, as shown in Figure 3, to reflect equally spaced 3-month measurement intervals. Although the full model gives estimates of all model parameters, for the purpose of demonstrating the interaction effect, we restricted the presentation to the following structural equation:

$$\eta_2 = \alpha_2 + \gamma_{22}\xi_2 + \gamma_{24}\xi_4 + \gamma_{25}\xi_2\xi_4 + \zeta_2 \tag{7}$$

In Equation 7, the prediction of either $\xi_2 \rightarrow \eta_2$ or $\xi_4 \rightarrow \eta_2$ represents the effect of a ξ's slope on the slope of η_2. A negative value means that as monitoring or rule-setting increase between Time 1 and Time 4, there is a decrease in the slope (growth) of antisocial behavior. In other words, an increase in monitoring or rule-setting over time leads to a decrease in adolescent antisocial behavior over time. The $\xi_2\xi_4$ interaction signifies that the regression of η_2 on ξ_2 depends on the specific value of ξ_4 (a moderator) at which the effect of ξ_2 slope on the η_2 slope is measured. For example, to examine the regression of η_2 on ξ_2 at the particular value of ξ_4, Equation 7 can be rearranged as

$$\eta_2 = (\gamma_{22} + \gamma_{25}\xi_4)\xi_2 + (\alpha_2 + \gamma_{24}\xi_4) + \zeta_2 \tag{8}$$

The term $[(\gamma_{22} + \gamma_{25}\xi_4)\xi_2]$ is often referred to as the simple slope of the regression of η_2 on ξ_2 for a given ξ_4 (Aiken & West, 1991; Jöreskog, 1998). Because of the slope-to-slope relation in the growth model depicted in Figure 3, this effect reflects the dynamic influence of the exogenous latent variable ξ (operationalizing the slope of X) on the endogenous latent variable η (operationalizing the slope of Y). Therefore, Equation 7 represents a complex form of the dynamic interaction portion of the growth model shown in Figure 3.

The LISREL specification for the X-measurement model of exogenous latent variables, ξs, is outlined in matrix form here:

$$
x = \tau_x + \Lambda_x \quad \xi + \delta \tag{9}
$$

$$
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_2x_6 \\ x_3x_7 \\ x_4x_8 \end{pmatrix}
=
\begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \\ \tau_7 \\ \tau_8 \\ \tau_2\tau_6 \\ \tau_3\tau_7 \\ \tau_4\tau_8 \end{pmatrix}
+
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
1 & \lambda_{32} & 0 & 0 & 0 \\
1 & \lambda_{42} & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & \lambda_{74} & 0 \\
0 & 0 & 1 & \lambda_{84} & 0 \\
0 & \tau_6 & 0 & \tau_2 & 1 \\
0 & \tau_7\lambda_{32} & 0 & \tau_3\lambda_{74} & \lambda_{32}\lambda_{74} \\
0 & \tau_8\lambda_{42} & 0 & \tau_4\lambda_{84} & \lambda_{42}\lambda_{84}
\end{pmatrix}
\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_2\xi_4 \end{pmatrix}
+
\begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \\ \delta_9 \\ \delta_{10} \\ \delta_{11} \end{pmatrix}
$$

Cross-products of the observed variables for the latent product variable (a set of nonlinear constraints necessary for model estimation) in Equation 9 can be derived using the same method shown previously (see Appendixes A and B). These constraints are necessary for model estimation. The latent exogenous mean vector (κ) and variance/covariance matrix (Φ) of ξs (ξ_1, ξ_2, ξ_3, ξ_4, $\xi_2\xi_4$) are

$$
\kappa = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \phi_{42} \end{pmatrix}, \quad
\Phi = \begin{pmatrix}
\phi_{11} & & & & \\
\phi_{21} & \phi_{22} & & & \\
\phi_{31} & \phi_{32} & \phi_{33} & & \\
\phi_{41} & \phi_{42} & \phi_{43} & \phi_{44} & \\
0 & 0 & 0 & 0 & \phi_{22}\phi_{44} + \phi_{42}^2
\end{pmatrix} \tag{10}
$$

Note that, because the observed intercepts (the X-intercept vector τ in Equation 9) are allowed to be freely estimated, the nonproduct exogenous latent variable means are constrained to zero ($\kappa_1 = \kappa_2 = \kappa_3 = \kappa_4 = 0$) to impose model identification restrictions. This specification implies that the means of κ have been recast into the vector of τs. Alternatively, τ can be set at zero while estimating the means of κs. The mean of the product variable $\xi_2\xi_4$ equals $Cov(\xi_2,\xi_4) = \phi_{42}$. The only nonlinear constraint in the Φ matrix is the variance of the latent product variable that must equal $Var(\xi_2)Var(\xi_4) + Cov(\xi_2\xi_4)^2$.

The variance/covariance matrix of uniquenesses, Θ_δ, is

$$\Theta_\delta = \begin{pmatrix} \theta_1 & & & & & & & & & & \\ 0 & \theta_2 & & & & & & & & & \\ 0 & 0 & \theta_3 & & & & & & & & \\ 0 & 0 & 0 & \theta_4 & & & & & & & \\ 0 & 0 & 0 & 0 & \theta_5 & & & & & & \\ 0 & 0 & 0 & 0 & 0 & \theta_6 & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \theta_7 & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_8 & & & \\ 0 & \theta_{92} & 0 & 0 & 0 & \theta_{96} & 0 & 0 & \theta_9 & & \\ 0 & 0 & \theta_{10,3} & 0 & 0 & 0 & \theta_{10,7} & 0 & 0 & \theta_{10} & \\ 0 & 0 & 0 & \theta_{11,4} & 0 & 0 & 0 & \theta_{11,8} & 0 & 0 & \theta_{11} \end{pmatrix} \quad (11)$$

Readers are referred to Appendix A for the derivation of uniqueness terms in Equation 11 (i.e., θ_9, θ_{10}, θ_{11}, θ_{92}, θ_{96}, $\theta_{10,3}$, $\theta_{10,7}$, $\theta_{11,4}$, $\theta_{11,8}$).

AN ILLUSTRATION OF LATENT GROWTH MODELING WITH GROWTH PREDICTORS OF CHANGE

The model presented in Figure 3 is estimated using the same empirical data used in the previous example but extended to include multiple time points (t_1 through t_4) for the two exogenous latent growth variables (monitoring and rule-setting).

Results

This model test resulted in $\chi^2(91, N = 221) = 776.75$, $p < .001$, NNFI = .88, CFI = .89, RMSEA = .18, indicating a marginal fit of the model to the data. The ML estimates for the ξs and $\xi_2\xi_4$ on the growth factor (η_2) and their corresponding test statistics are as follows:

$$\eta_2 = -.003 + (-.02)(\xi_2) + .002(\xi_4) + (-.006)(\xi_2\xi_4)$$
$$(.004)(.012)(.003)$$
$$-3.7361.292-2.240$$

The interaction effect ($\gamma_{25} = -.006$ unstandardized and $-.298$ standardized coefficient) in the aforementioned regression equation was statistically significant, as indicated by the z value of -2.240 ($p < .05$). This significance term indicates an interaction between the two dynamic latent shape variables (ξ_2 and ξ_4), suggesting that a joint effect of longitudinal changes in parental monitoring and rule-setting over time was predictive of longitudinal changes in adolescent antisocial behavior.[3] Substantively, the relation between longitudinal change in parental monitoring (in the sense of increasing monitoring) and longitudinal change in adolescent antisocial behavior becomes more negative over time as parents increase their rate of change in rule-setting (i.e., setting more rules for their children). Thus, to reduce the rate of growth in antisocial behavior by their children, parents must simultaneously increase their monitoring and rule-setting practices over time.

Another way of interpreting this interaction involves considering separate slopes for longitudinal change in antisocial behavior on growth in parental monitoring (i.e., at 1 *SD* above its mean, 1.334). One slope applies when the rate of change in rule setting is increasing (i.e., 1 *SD* above its mean, .502) and the other slope applies when there is a decrease in the rate of change in rule-setting (i.e., 1 *SD* below its mean, $-.502$).[4] Substituting the values into the earlier predicted equation shows that both of these slopes are negative, indicating that the more parental monitoring increases over time the more negative the slope for antisocial behavior (i.e., a main effect of parental monitoring). However, there is also an interaction effect. The negative slope relating slopes on parental monitoring and slopes on antisocial behavior is steeper at higher levels of slopes on rule-setting (i.e., when rule-setting is at 1 *SD* above its mean). This indicates that the relations between longitudinal change in parental monitoring and longitudinal change in antisocial

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[3]To further confirm this interaction effect, a multisample method (see Rigdon, Schumacker, & Wothke, 1998) was used via chi-square difference tests. First, the sample was split into low- and high-rule-making groups based on levels of individual slopes (on the $t = 4$ measurements). A model that allowed the slope of monitoring to predict the slope of antisocial behavior was specified for each level condition. There were two models estimated. The first model specified all parameters, including the monitoring slope–antisocial behavior slope relation, to be constrained to be equal across low- and high-rule-setting groups, whereas the second model estimated the interaction by allowing the monitoring slope–antisocial behavior slope relation path to differ across groups. The chi-square statistic was $\chi^2(65) = 213.031$, for the first model, and $\chi^2(64) = 207.243$, for the second model, respectively, with a chi-square difference of $\chi^2_{\text{diff}}(1) = 5.788, p < .025$, suggesting the constraint in the second model was not consistent with the data, and thus an interaction effect existed.

[4]The variance for the latent shape variable of parental monitoring (ξ_2) is 1.780 and the variance for the latent shape variable of rule-setting is .252 (ξ_4). Taking the square root of these values gives the standard deviation of 1.334 for parental monitoring and .502 for rule-setting.

behavior is more negative when the increase in parental rule-setting is high. Thus, parents who increase their rule-setting over time while increasing their monitoring will reduce the growth in antisocial behavior the most.

This model involving growth shape predictors accounted for approximately 96% of the variance in rate of change in adolescents' antisocial behavior. The chi-square difference test between the interactive model and the "individual effect only" model was statistically significant, $\chi^2_{\text{diff}(1)} = 73.965, p < .001$. The difference in R^2 was .05 (.960 to .910), indicating a small increment in the variance accounted for over the "individual effect only" model. Although the increment was small, this needs to be understood in the context of being close to the upper limit of 1.0. The LISREL program specifications used for this example can be found in Appendix D.

DISCUSSION

The search for the best methods to address complex issues in behavioral development has been a major concern of developmental research (e.g., Collins & Horn, 1991; Nesselroade & Baltes, 1979; Collins & Sayer, in press). Using standard SEM-based procedures (i.e., Jöreskog & Yang, 1996), this article presented an extension of interaction effects in latent growth curve models including interactions between latent growth (shape) parameters.

Using an empirical data set on parenting practices, two examples of latent growth models were given. The first example illustrated tests of interactions between two static latent variables in predicting a growth outcome variable, whereas the second example illustrated an interaction between two dynamic latent growth variables. In both examples, the interaction effect in latent growth models showed the dynamic influence of parental monitoring and rule-setting on the development of adolescent antisocial behavior. When the latent interaction variable was operationalized as a time-invariant static predictor in a cross-sectional model, the interaction between parental monitoring and rule-setting was related to a lower rate of change in adolescents' problem behavior. In a longitudinal model, the interaction between the two slopes of parenting practices variables had a significant effect on development of problem behavior, suggesting that simultaneous increases in parental monitoring and rule-setting reduced the rate of change in adolescent antisocial behavior.

SOME PRACTICAL ISSUES IN THE INTERACTION
ANALYSIS

Methodologically, the modeling techniques illustrated in this article show that interaction analysis with latent variables can be extended to the analysis of more com-

plex, dynamic models involving interactive relations between growth parameters. Applications of modeling interactions among change scores (growth factors) are likely to expand theoretical models in which structural relations among a set of variables have been conventionally specified as either direct or indirect. These analytic tools will also allow researchers to more vigorously test hypotheses relating interactions between dynamic latent attributes to development of important behavioral outcomes. In the remaining sections, we discuss practical issues that researchers need to consider when conducting interaction analysis in the context of longitudinal data. These include (a) using multiple indicators versus single indicators for latent product variables, (b) interpretation of interaction effects in latent growth models, (c) data normality, and (d) issues related to the application of interaction analysis in latent growth modeling (for an excellent discussion on several related practical issues with SEM-based analyses of interaction effects, see Jaccard & Wan, 1996).

Multiple Indicators Versus Single Product Indicator

One of the fundamental issues in applying the method of Jöreskog and Yang (1996) used in this article and those of others (e.g., Jaccard & Wan, 1995; Ping, 1996) concerns the use of either multiple or single product indicators in operationalizing the latent product variable. In the first example in this article (the model in Figure 2), we used multiple product indicators of constituent observed variables because of the uncertainty about the consequence of using one or two product indicators in regard to the validity of the interactions. Although the use of multiple indicators permits the estimation of parameters with correction of measurement error in regression equations involving interactions (a distinct advantage of the SEM-based approach; Aiken & West, 1991; Jaccard & Wan, 1995, 1996), these multiple indicators create estimation problems in interaction models (e.g., nonconvergence, improper solutions). That is, increases in the number of product indicators necessarily increase the size of the covariance matrix, and also the likelihood of making the matrix analyzed nonpositive definite, a problem that can occur as a result of nonnormality, empirical underidentification, and model misspecification (see Wothke, 1993).

From a model estimation perspective, one product indicator is sufficient to identify the model using Jöreskog and Yang's (1996) method. This procedure, therefore, represents a much simpler method to that outlined by Kenny and Judd (1984). However, a simulated study by Yang-Jonsson (1997) showed that parameter estimates with three or four product variables in the model provided less bias and a smaller mean square error, but severely underestimated standard errors compared to models with one product variable. Although no specific recommendation was provided by Yang-Jonsson, a number of methodologists have adopted and/or

proposed specific strategies for the use of product indicators. For example, rather than using all product indicators, Jaccard and Wan (1995) adopted the approach of using a subset of them. Ping (1995, 1998) proposed a single-indicator specification to interaction modeling. Yang-Jonsson (1998) also used two product indicators in her illustration of the Jöreskog and Yang (1996) method.

Substantively, however, the extent to which the use of one or more product indicators in a model impairs the content validity of the interaction remains unknown. As noted by Ping (1998), it would be practically useful to understand the conditions under which product indicators in the model could safely be dropped without impinging on the content validity of the resulting latent variable interaction. Although there is no clear guidance in this regard in the literature, a conservative recommendation would be to use at least two indicators in simple to moderately complex models, based strictly on the assumption that the observed indicators and product variables have high levels of content validity and reliability.

In the level-shape growth interaction model shown in this article, the formation of observed product variables was based on corresponding occasions of measurement (e.g., x_2 at t_2 on construct X and z_2 at t_2 on construct Z), although the multiple indicators (e.g., x_2z_3, x_2z_4) approach discussed earlier could have been considered. Therefore, only one product indicator was used at each time point to specify the underlying latent variable representing the interaction of rate of change in X and rate of change in Z. Additionally, it should be noted that because identification restriction in latent growth modeling requires that one of the loadings be fixed to zero (in this case the Time 1 measure), the product of x_1z_1 was not used as one of the indicators for the latent product variable in the model. Although the choice of selecting a reference is largely arbitrary, the selection of what is excluded from the product terms might have substantive consequences.

Interpreting the Interaction Effects Within Latent Growth Models

Although it was shown in this article that the specification of interactions between latent growth parameters is possible using Jöreskog and Yang's (1996) method, a substantive question is how these interaction effects within the context of latent growth models can be interpreted meaningfully from a theory testing perspective, an issue that is not often discussed in the literature (Wood & Erickson, 1998).

The interpretation of the model interaction depicted in Figure 2 is straightforward. The interaction effect of $\xi_1\xi_2$ on η_1 (intercept) represents a synchronous effect. This is the same interpretation as in any cross-sectional data. However, because the η_2 slope factor is operationalized as Y's slope over time, the individual effect of ξ_1 or ξ_2 may serve as a moderator between $Y(t-1)$ and Y slope in the context of latent growth models. Given this, the joint effect of $\xi_1\xi_2$ may be viewed as a

higher order interaction in the $\xi_1\xi_2 \rightarrow \eta_2$ relation, representing a longitudinal prospective prediction or lagged effect over time. Alternatively, it can be said that the effect of $\xi_1\xi_2$ on η_2 actually represents a three-way interaction effect; that is, that the moderator effect that ξ_1 has on the relation between Y and time is itself moderated by values on ξ_2.[5]

The interpretation of the interaction effect in the model depicted in Figure 3 is not as straightforward as the model of Figure 2 because it involves simultaneous, dynamic change scores of the latent continuous predictors. In multiple regression involving interactions, values of change in η_2 are predicted by a regression coefficient (slope) that gives the increase in η_2 resulting from a one-unit increase in $\xi_2\xi_4$. In the context of latent growth modeling, this would indicate change in η_2 that is predicted by simultaneous and joint change in ξ_2 and ξ_4. So the interaction in this context represents a joint and concurrent change between two latent growth shape factors, ξ_2 and ξ_4. In the example of parental practices, this would mean that when there are concurrent changes in monitoring and rule-setting practices (assuming in the same direction) their joint effect is enhanced (i.e., lowering developmental trajectories of adolescent problem behavior).

Normality Issue

It is well known that the ML fitting function, as used in this article, assumes the observed variables are multivariately normally distributed to produce reliable standard errors and a chi-square test of overall model fit (Bollen, 1989; Jöreskog, 1993). Even though the observed variables are distributed normally, the product variables in the structural equation are known to be nonnormal (Jöreskog & Yang, 1996). However, there are several alternatives for remedying the nonnormality situation.

One way to deal with data nonnormality is to use corrected test statistics. This involves the use of a scaled Satorra–Bentler chi-square statistic (Satorra & Bentler, 1988) implemented in EQS (Bentler & Wu, 1995) and LISREL (8.30; Jöreskog, Sörbom, du Toit, & du Toit, 1999). The scaled statistic has been shown to perform better than the standard statistics (Chou & Bentler, 1995). The idea of this scaled statistic is to modify the standard test statistics to make them more appropriately chi-square distributed. Use of this scaled statistic in the context of latent variable interaction analysis has been demonstrated by Li et al. (1998) and Ping (1998).

Some researchers have proposed methods that take into account the problems of nonnormality. For example, Schermelleh-Engel, Klein, and Moosbrugger (1998) described a method called latent moderated structural equations that takes

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[5]We thank an anonymous referee for providing insightful comments on the overall interpretation of the interaction effect in the latent curve analyses conducted in this article.

the nonnormality of the variables explicitly into account. However, as indicated by Jöreskog (1998), its use remains limited in practice. Bollen (1995) proposed a two-stage least squares (2SLS) procedure to estimate nonlinear effects that makes no assumptions regarding the underlying distribution of the unobserved variables. The procedure uses a limited information approach and is known to be simple for implementation (Bollen & Paxton, 1998; Jöreskog et al., 1999). Although viewed as an alternative, the 2SLS approach has not been applied to latent curve analysis in which both fixed constants and freely estimated factor loadings are necessary on the growth parameters (i.e., intercept, slope). Recent work by Arminger and Muthén (1998) presents another way of dealing with the problems of nonnormality and a general specification of nonlinear latent variable models as well as with the problem of small sample sizes. Arminger and Muthén proposed use of a Bayesian framework that employs two Markov Chain Monte Carlo procedures (the Gibbs sampler and Metropolis–Hastings) to estimate the posterior distributions of the parameters and the latent variables given the data. Both the simulated and empirical data show promising results in recovering model (true) parameters. In short, although these are promising developments, more research is needed to explore the relative merits of each approach in applied settings.

Finally, one frequently advocated approach for problems of nonnormality is the use of an asymptotic distribution free (ADF) method (Browne, 1984). However, to obtain asymptotically unbiased estimators of the chi-square statistic test, parameter estimates, and standard errors, the ADF estimator requires sample sizes that may be in the range of 1,000 to 5,000 observations (West, Finch, & Curran, 1995). An ADF-based weighted least squares (WLS) approach using an augmented moment matrix (Jöreskog & Yang, 1996; Yang-Jonsson, 1997) also requires large sample sizes for establishing the asymptotic properties. Clearly, it may not be suitable for most social science research, in particular longitudinal studies of small or medium sample sizes.

Application Issues

Several additional issues evolved from this article that deserve consideration when applying interactions within the latent growth modeling framework. First, we presented a common model of interest in latent growth models: the level-shape model. This model contains a complex model specification involving latent intercepts of the growth parameters that may complicate model estimation as well as interpretation. A much simpler specification of the level-shape model is a shape-only model that involves only a growth shape factor. However, the highly restrictive set of model expectations (i.e., zero mean and zero variance in the intercept) often makes justification of these assumptions in practice unlikely. Thus, in practice the choice between the level-shape and shape-only models must be made based on substantive

theory and empirical data. Therefore, if it is reasonable to assume that the intercept factor can be eliminated, or if it is strictly proportional to the slope, the predictive model with the fewer parameters (in this case the shape model) represents the model of choice (see Duncan et al., 1999).

A second concern is model convergence and the appropriateness of obtained estimates. There are estimation problems associated with the inclusion of latent product terms. Techniques such as those proposed by Jöreskog and Yang (1996) and Jaccard and Wan (1995) frequently encounter the problem of nonconvergence and improper solutions (e.g., Jaccard & Wan, 1996; Li et al., 1998; Ping, 1995, 1998; Wood & Erickson, 1998), particularly with the inclusion of all possible cross-products of observed variables as indicators for a latent interaction model. Nonconvergence may be compounded by factors such as nonnormality of the data being analyzed, and noncentering of the product indicators, which results in model estimation that is more sensitive to starting values than usual (Rigdon et al., 1998). One possible remedy to such improper or inadmissable solutions, as noted by Ping (1998), may be to include user-specified starting values for the latent variable variances and covariances, the structural coefficients, and the variances of the structural disturbances. This is because better starting values imply faster model convergence. In our examples, we encountered a number of unacceptable solutions. Through a trial-and-error process, starting values can be chosen so that the model converges to an acceptable solution. This may be less likely when simpler methods of estimation procedures such as 2SLS (Bollen, 1995) are considered.

A third issue involves sample sizes. The sample in this article was small in light of the results on sample sizes reported by Yang-Jonsson (1997). Yang-Jonsson indicated that large sample sizes (at least 400) are often necessary to avoid estimation problems such as nonconvergence and nonadmissible solutions. We selected the sample size for our illustrations because it is typical of longitudinal studies. However, because most fitting functions in SEM such as ML rely heavily on asymptotic properties, and these properties may only be satisfied with the use of relative large sample sizes (Rigdon et al., 1998), results from all models illustrated in this article may be limited in terms of their generalizability. Further work is necessary to assess the extent of these small sample size problems.

Finally, although it is a useful tool for studying nonlinear dynamic relations, the extension of Jöreskog and Yang's (1996) procedure requires careful model parameterizations because as in standard structural equation models with interactions, model parameterizations in latent growth models are quite complex. In addition to its complexity, the procedure is also a tedious and error-prone task of writing the nonlinear equations for the covariances of cross-products as functions of the model parameters. This is a common concern that has been echoed by a number of researchers (Jaccard & Wan, 1996; Jöreskog & Yang, 1996; LaPlante, Sabourin, Cournoyer, & Wright, 1998; Li et al., 1998; Neale, 1998; Ping, 1998) and as such "utmost care must be taken to specify the constraints in the model cor-

rectly" (Jöreskog & Yang, 1996, p. 85). One solution to this is to simplify the estimating procedure. For example, Jöreskog et al. (1999) have shown a simple way of estimating the nonlinear models by means of latent variable scores. This method involves estimating a measurement model using observed variables and the latent variables scores are saved. In a follow-up step, the structural model of interest is estimated using these latent variable scores as observed variables. As illustrated by Jöreskog et al. (1999) and Yang-Jonsson (1998), the method may represent a simpler way of modeling interaction compared to the one proposed by Jöreskog and Yang (1996).

Summary

Although there has been an increasing application of latent growth models in the social sciences, models with interactions representing different rates of change that test complex hypotheses for both lagged and longitudinal designs have not been widely applied in practice. The method presented in this article is considered preliminary and requires further work to establish its viability. It can, however, be a useful method for detecting whether inter-individual differences in change (growth parameters) in one domain interact with change in another domain to jointly predict time-varying outcomes.

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APPENDIX A

The following demonstrates how the cross-product indicators of the latent product variable ($\xi_1\xi_2$) are formulated. Readers are referred to Jöreskog and Yang (1996) and Yang-Jonsson (1997) for more detailed descriptions. For ease of presentation, let a_i be x_1 and x_2 for the latent factor ξ_1, and d_j be x_3 and x_4 for the latent factor ξ_2. The measurement equation for the observed product indicators using the symbol a_i ($i = 1,2$) and d_j ($j = 1,2$) can be expressed as

$$a_i d_j = (a_i)(d_j) = (\tau_{a_i} + \lambda_{a_i}\xi_1 + \delta_{a_i})(\tau_{d_j} + \lambda_{d_j}\xi_2 + \delta_{d_j}) \quad (12)$$
$$= \tau_{a_i}\tau_{d_j} + \tau_{a_i}\lambda_{d_j}\xi_2 + \tau_{a_i}\delta_{d_j} + \tau_{d_j}\lambda_{a_i}\xi_1 + \lambda_{a_i}\lambda_{d_j}\xi_1\xi_2 +$$
$$\lambda_{a_i}\xi_1\delta_{d_j} + \tau_{d_j}\delta_{a_i} + \lambda_{d_j}\xi_2\delta_{a_i} + \theta_{a_i d_j}$$

Substituting the four cross-product terms (a_1d_1, a_1d_2, a_2d_1, and a_2d_2) into the preceding equation, the following measurement equations are obtained:

$$a_1 d_1 = (\tau_{a_1} + \lambda_{a_1}\xi_1 + \delta_{a_1})(\tau_{d_1} + \lambda_{d_1}\xi_2 + \delta_{d_1}) \quad (13)$$
$$= \tau_{a_1}\tau_{d_1} + \tau_{a_1}\lambda_{d_1}\xi_2 + \tau_{d_1}\lambda_{a_1}\xi_1 + \lambda_{a_1}\lambda_{d_1}\xi_1\xi_2 + \theta_{a_1 d_1}$$

where $\theta_{a_1 d_1} = \tau_{a_1}\delta_{d_1} + \tau_{d_1}\delta_{a_1} + \lambda_{a_1}\xi_1\delta_{d_1} + \lambda_{d_1}\xi_2\delta_{a_1} + \delta_{a_1}\delta_{d_1}$;

$$a_1 d_2 = (\tau_{a_1} + \lambda_{a_1}\xi_1 + \delta_{a_1})(\tau_{d_2} + \lambda_{d_2}\xi_2 + \delta_{d_2}) \quad (14)$$
$$= \tau_{a_1}\tau_{d_2} + \tau_{a_1}\lambda_{d_2}\xi_2 + \tau_{d_2}\lambda_{a_1}\xi_1 + \lambda_{a_1}\lambda_{d_2}\xi_1\xi_2 + \theta_{a_1 d_2}$$

where $\theta_{a_1 d_2} = \tau_{a_1}\delta_{d_2} + \tau_{d_2}\delta_{a_1} + \lambda_{a_1}\xi_1\delta_{d_2} + \lambda_{d_2}\xi_2\delta_{a_1} + \delta_{a_1}\delta_{d_2}$;

$$a_2 d_1 = (\tau_{a_2} + \lambda_{a_2}\xi_1 + \delta_{a_2})(\tau_{d_1} + \lambda_{d_1}\xi_2 + \delta_{d_1}) \quad (15)$$
$$= \tau_{a_2}\tau_{d_1} + \tau_{a_2}\lambda_{d_1}\xi_2 + \tau_{d_1}\lambda_{a_2}\xi_1 + \lambda_{a_2}\lambda_{d_1}\xi_1\xi_2 + \theta_{a_2 d_1}$$

where $\theta_{a_2 d_1} = \tau_{a_2}\delta_{d_1} + \tau_{d_1}\delta_{a_2} + \lambda_{a_2}\xi_1\delta_{d_1} + \lambda_{d_1}\xi_2\delta_{a_2} + \delta_{a_2}\delta_{d_1}$;

$$a_2 d_2 = (\tau_{a_2} + \lambda_{a_2}\xi_2 + \delta_{a_2})(\tau_{d_2} + \lambda_{d_2}\xi_2 + \delta_{d_2}) \quad (16)$$
$$= \tau_{a_2}\tau_{d_2} + \tau_{a_2}\lambda_{d_2}\xi_2 + \tau_{d_2}\lambda_{a_2}\xi_1 + \lambda_{a_2}\lambda_{d_2}\xi_1\xi_2 + \theta_{a_2 d_2}$$

where $\theta_{a_2 d_2} = \tau_{a_2}\delta_{d_2} + \tau_{d_2}\delta_{a_2} + \lambda_{a_2}\xi_1\delta_{d_2} + \lambda_{d_2}\xi_2\delta_{a_2} + \delta_{a_2}\delta_{d_2}$.

Specification of these uniqueness terms can be seen in the LISREL Λ and Θ matrices presented in Appendix B. For example, for the cross-product indicator

a_1d_1 (i.e., x_1x_3 in Equation 24, see Appendix B), the term $\tau_{a_1}\tau_{d_1}$ is the intercept (τ_{x_1}, τ_{x_3}), the term $\tau_{d_1}\lambda_{a_1}\xi_1$ is the fifth loading for the latent variable ξ_1 ($\lambda_{51} = \tau_3$), the term $\tau_{a_1}\lambda_{d_1}\xi_2$ is the fifth loading for the latent variable ξ_2 ($\lambda_{52} = \tau_1$), the term $\lambda_{a_1}\lambda_{d_1}\xi_1\xi_2$ is the first loading for the latent product variable ($\lambda_{53} = 1$, where the constant value of 1 is used for scaling purposes), and, finally, the term $\theta_{a_1d_1}$ is the uniqueness term (θ_5) for a_1d_1 (or x_1x_3) shown as the fifth diagonal element in Equation 25 in Appendix B.

A total of 12 pairs of covariances are specified in the LIRESL Θ matrix. These include the covariances between observed indicators (i.e., θ_{51}, θ_{61}, θ_{72}, θ_{82}, θ_{53}, θ_{73}, θ_{64}, and θ_{84}) and product indicators (θ_{65}, θ_{75}, θ_{86}, and θ_{87}). Note that the covariances between the cross-product terms are those that share the same elements (Jaccard & Wan, 1995; e.g., the covariance between a_1d_1 and a_1d_2 (θ_{65}) both have an a_1 indicator). The covariance between δ_{a_i} and $\delta_{a_id_j}$ (i.e., θ_{51}, θ_{61}, θ_{72}, θ_{82}, θ_{53}, θ_{73}, θ_{64}, and θ_{84}) must equal $\tau_{d_j} Var(\delta_{a_i})$. Similarly, the covariance between δ_{d_j} and $\delta_{a_id_j}$ must equal $\tau_{a_i} Var(\delta_{d_j})$. For example, the covariance between δ_{a_1} and $\delta_{a_1d_1}$ (i.e., θ_{51}) equals $\theta_{51} = \tau_{d_1}\delta_{a_1}$ and the covariance between δ_{d_1} and $\delta_{a_1d_1}$ (i.e., θ_{53}) equals $\theta_{53} = \tau_{a_1}\delta_{d_1}$. The covariances between the product indicators (θ_{65}, θ_{75}, θ_{86}, θ_{87}) are derived by multiplying the corresponding uniqueness component, $\theta_{a_id_j}$, of the measurement equation shown in Equations 13 through 16. For example, the covariance between $\theta_{65} = Cov(\theta_{a_1d_1}, \theta_{a_1d_2})$ is derived by multiplying $\theta_{a_1d_1}$ by $\theta_{a_1d_2}$ in Equations 13 and 14, then taking the expectation using the fact that the ξ's and δ's have a joint multinormal distribution (with the assumptions of δ_i being independent of δ_j for $i \neq j$, and of ξ_k for $i = 1, ...,4; k = 1,2$):

$$\theta_{65} = Cov(\theta_{a_1d_1}, \theta_{a_1d_1}) = Cov[(\tau_{a_1}\delta_{d_1} + \tau_{d_1}\delta_{a_1} + \lambda_{a_1}\xi_1\delta_{d_1} + \lambda_{d_1}\xi_2\delta_{a_1} + \delta_{a_1}\delta_{d_1})$$
$$(\tau_{a_1}\delta_{d_2} + \tau_{d_2}\delta_{a_1} + \lambda_{a_1}\xi_1\delta_{d_2} + \lambda_{d_2}\xi_2\delta_{a_1} + \delta_{a_1}\delta_{d_2})]$$
$$= Cov(\tau_{a_1}\delta_{d_1}, \tau_{a_1}\delta_{d_2}) + Cov(\tau_{d_1}\delta_{a_1}, \tau_{a_1}\delta_{d_2}) + Cov(\lambda_{a_1}\xi_1\delta_{d_1}, \tau_{a_1}\delta_{d_2}) +$$
$$Cov(\lambda_{d_1}\xi_2\delta_{a_1}, \tau_{a_1}\delta_{d_2}) + Cov(\delta_{a_1}\delta_{d_1}, \tau_{a_1}\delta_{d_2}) + Cov(\tau_{a_1}\delta_{d_1}, \tau_{d_2}\delta_{a_1}) +$$
$$Cov(\tau_{d_1}\delta_{a_1}, \tau_{d_2}\delta_{a_1}) + Cov(\lambda_{a_1}\xi_1\delta_{d_1}, \tau_{d_2}\delta_{a_1}) + Cov(\lambda_{d_1}\xi_2\delta_{a_1}, \tau_{d_2}\delta_{a_1}) +$$
$$Cov(\delta_{a_1}\delta_{d_1}, \tau_{d_2}\delta_{a_1}) + Cov(\tau_{a_1}\delta_{d_1}, \lambda_{a_1}\xi_1\delta_{d_2}) + Cov(\tau_{d_1}\delta_{a_1}, \lambda_{a_1}\xi_1\delta_{d_2}) +$$
$$Cov(\lambda_{a_1}\xi_1\delta_{d_1}, \lambda_{a_1}\xi_1\delta_{d_2}) + Cov(\lambda_{d_1}\xi_2\delta_{a_1}, \lambda_{a_1}\xi_1\delta_{d_2}) + Cov(\delta_{a_1}\delta_{d_1}, \lambda_{a_1}\xi_1\delta_{d_2}) +$$
$$Cov(\tau_{a_1}\delta_{d_1}, \lambda_{d_2}\xi_2\delta_{a_1}) + Cov(\tau_{d_1}\delta_{a_1}, \lambda_{d_2}\xi_2\delta_{a_1}) + Cov(\lambda_{a_1}\xi_1\delta_{d_1}, \lambda_{d_2}\xi_2\delta_{a_1}) +$$
$$Cov(\lambda_{d_1}\xi_2\delta_{a_1}, \lambda_{d_2}\xi_2\delta_{a_1}) + Cov(\delta_{a_1}\delta_{d_1}, \lambda_{d_2}\xi_2\delta_{a_1}) + Cov(\tau_{a_1}\delta_{d_1}, \delta_{a_1}\delta_{d_2}) +$$
$$Cov(\tau_{d_1}\delta_{a_1}, \delta_{a_1}\delta_{d_2}) + Cov(\lambda_{a_1}\xi_1\delta_{d_1}, \delta_{a_1}\delta_{d_2}) + Cov(\lambda_{d_1}\xi_2\delta_{a_1}, \delta_{a_1}\delta_{d_2}) + Cov(\delta_{a_1}\delta_{d_1}, \delta_{a_1}\delta_{d_2})$$

Using the earlier assumptions, all covariances that are zero are removed, resulting in $Cov(\theta_{a_1d_1}, \theta_{a_1d_2}) = Cov(\tau_{d_1}\delta_{a_1}, \tau_{d_2}\delta_{a_1}) + Cov(\lambda_{d_1}\xi_2\delta_{a_1}, \lambda_{d_2}\xi_2\delta_{a_1})$, which gives $\theta_{65} = \tau_{d_1}\tau_{d_2}\delta_{a_1} + \lambda_{d_2}\phi_{22}\delta_{a_1}$. Note that there is no covariance between ϕ_{22}

and δ_{a_1} because these two terms are assumed to be independent of each other. All terms are listed as follows:

$$\begin{aligned}
&\theta_{51} = \tau_{d_1}\delta_{a_1}, \quad \theta_{61} = \tau_{d_2}\delta_{d_1}, \quad \theta_{72} = \tau_{d_1}\delta_{a_2}, \quad \theta_{82} = \tau_{d_2}\delta_{a_2}, \\
&\theta_{53} = \tau_{a_1}\delta_{d_1}, \quad \theta_{73} = \tau_{a_2}\delta_{d_1}, \quad \theta_{64} = \tau_{a_2}\delta_{d_2}, \quad \theta_{84} = \tau_{a_2}\delta_{d_2}, \\
&\theta_{65} = \tau_{d_1}\tau_{d_2}\delta_{a_1} + \lambda_{d_2}\phi_{22}\delta_{a_1}, \quad \theta_{75} = \tau_{a_1}\tau_{a_2}\delta_{d_1} + \lambda_{a_2}\phi_{11}\delta_{d_1}, \\
&\theta_{86} = \tau_{a_1}\tau_{a_2}\delta_{d_2} + \lambda_{a_2}\phi_{11}\delta_{d_2}, \quad \theta_{87} = \tau_{d_1}\tau_{d_2}\delta_{a_2} + \lambda_{d_2}\phi_{22}\delta_{a_2}
\end{aligned} \tag{17}$$

Finally, the uniqueness terms of $(\theta_5, \theta_6, \theta_7,$ and $\theta_8)$ are

$$\theta_5 = \tau_{a_1}^2\delta_{d_1} + \tau_{d_1}^2\delta_{a_1} + \phi_{11}\delta_{d_1} + \phi_{22}\delta_{a_1} + \delta_{a_1}\delta_{d_1} \tag{18}$$

$$\theta_6 = \tau_{a_1}^2\delta_{d_2} + \tau_{d_2}^2\delta_{a_1} + \phi_{11}\delta_{d_2} + \lambda_{d_2}^2\phi_{22}\delta_{a_1} + \delta_{a_1}\delta_{d_2} \tag{19}$$

$$\theta_7 = \tau_{a_2}^2\delta_{d_1} + \tau_{d_1}^2\delta_{a_2} + \lambda_{a_2}^2\phi_{11}\delta_{d_1} + \phi_{22}\delta_{a_2} + \delta_{a_2}\delta_{d_1} \tag{20}$$

$$\theta_8 = \tau_{a_2}^2\delta_{d_2} + \tau_{d_2}^2\delta_{a_2} + \lambda_{a_2}^2\phi_{11}\delta_{d_2} + \lambda_{d_2}^2\phi_{22}\delta_{a_2} + \delta_{a_2}\delta_{d_2} \tag{21}$$

It should be noted that these variances and covariances are not estimated independently but are entirely constrained to nonlinear functions of the free parameters in the model. All these constraints are imposed with reference to the Theta Epsilon matrix in LISREL.

APPENDIX B

This appendix provides a brief description of the parameterization of the interaction model presented in Figure 2 using LISREL measurement models for the vectors of endogenous variables Y and exogenous variables X. These include nonlinear constraints in the X measurement model on parameters related to the interaction term that need to be implemented for model estimation. The Y-measurement component of the model has the following matrix form

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & t_1 \\ 1 & t_2 \\ 1 & t_3 \\ 1 & t_4 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{pmatrix} \tag{22}$$

Note τ_y in Equation 22 is a null vector. The matrix form for the structural component of the model is

$$
\begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_1\xi_2 \end{bmatrix} + \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}
\tag{23}
$$

Unlike standard latent growth models (Willett & Sayer, 1994), elements in α and ζ vectors have different interpretations in this regression-type model (i.e., the model with predictors). The αs represent the regression intercepts of the structural equation model, or that part of the endogenous variable mean that is not explained by the additional predictor variables, and the ζs are deviation-from-predicted-value variables, generally referred to as disturbance terms.

The LISREL X-measurement model of Figure 2 is

$$
X = \tau_x + \Lambda_x \xi + \delta
\tag{24}
$$

$$
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_1x_3 \\ x_1x_4 \\ x_2x_3 \\ x_2x_4 \end{pmatrix} = \begin{pmatrix} \tau_{x_1} \\ \tau_{x_2} \\ \tau_{x_3} \\ \tau_{x_4} \\ \tau_{x_1}\tau_{x_3} \\ \tau_{x_1}\tau_{x_4} \\ \tau_{x_2}\tau_{x_3} \\ \tau_{x_2}\tau_{x_4} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \lambda_{42} & 0 \\ \tau_3 & \tau_1 & 1 \\ \tau_4 & \tau_1\lambda_{42} & \lambda_{42} \\ \tau_3\lambda_{21} & \tau_2 & \lambda_{21} \\ \tau_4\lambda_{21} & \tau_2\lambda_{42} & \lambda_{21}\lambda_{42} \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_1\xi_2 \end{pmatrix} + \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \end{pmatrix}
$$

Derivations of cross-product indicators of the latent product variable $\xi_1\xi_2$ are provided in Appendix A. The variance/covariance matrix for uniqueness Θ_k, where k ranges from 1 to 8, is

$$
\Theta_\delta = \begin{pmatrix}
\theta_1 & & & & & & & \\
0 & \theta_2 & & & & & & \\
0 & 0 & \theta_3 & & & & & \\
0 & 0 & 0 & \theta_4 & & & & \\
\theta_{51} & 0 & \theta_{53} & 0 & \theta_5 & & & \\
\theta_{61} & 0 & 0 & \theta_{64} & \theta_{65} & \theta_6 & & \\
0 & \theta_{72} & \theta_{73} & 0 & \theta_{75} & 0 & \theta_7 & \\
0 & \theta_{82} & 0 & \theta_{84} & 0 & \theta_{86} & \theta_{87} & \theta_8
\end{pmatrix}
\tag{25}
$$

Note that correlations among certain uniqueness terms are allowed in the Θ_δ matrix. Derivations of these terms are provided in Appendix A.

For this nonlinear structural equation model, the assumptions specified by Jöreskog and Yang (1996, p. 58) apply:

1. $x_1, \ldots, x_4$ are multivariate normal with zero means.
2. ξ_1 and ξ_2 are bivariate normal with zero means.
3. δ_i is normally distributed with zero means, $i = 1, \ldots, 4$.
4. ζs are normal with zero mean.
5. δ_i is independent of δ_j for $i \neq j$.
6. δ_i is independent of ξ_k for $i = 1, \ldots, 4; k = 1, 2$.
7. ζs are independent of δ_i and ξ_k for $i = 1, \ldots, 4$ and $k = 1, 2$.

Within the LISREL framework, the mean of the exogenous ξ variables are collected in a vector κ. The mean vector, κ, and variance/covariance matrix, Φ, among the ξ latent factors are as follows:

$$\kappa = \begin{pmatrix} 0 \\ 0 \\ \phi_{21} \end{pmatrix}, \quad \Phi = \begin{pmatrix} \phi_{11} & & \\ \phi_{21} & \phi_{22} & \\ 0 & 0 & \phi_{11}\phi_{22} + \phi_{21}^2 \end{pmatrix} \tag{26}$$

Also, note that the latent mean values of ξ_1 and ξ_2 are fixed to zero (i.e., $\kappa_1 = \kappa_2 = 0$). This constraint puts the two latent variables in deviation score form (i.e., mean centered). Following the assumption that the latent variables of ξ_1 and ξ_2 are multivariately normally distributed and mean centered in the population (Jöreskog & Yang, 1996), the covariance of each with the latent product variable ($\xi_1\xi_2$) is set to zero, as shown in the last row of the Φ matrix (i.e., $\phi_{31} = \phi_{32} = 0$).

The variance for the latent product term, $\xi_1\xi_2$, is

$$Var(\xi_1\xi_2) = E(\xi_1^2\xi_2^2) - [E(\xi_1\xi_2)]^2 = \phi_{11}\phi_{22} + 2\phi_{21}^2 - \phi_{21}^2 = \phi_{11}\phi_{22} + \phi_{21}^2 \tag{27}$$

as per Jöreskog & Yang (1996, p. 59) and shown in the third diagonal element in the last row of the Φ matrix.

APPENDIX C

The following LISREL script can be used for estimating the interaction model shown in Figure 2.

Analysis of Interaction Effects Using LISREL 8

```
Da Ni=12 No=221
La
y1 y2 y3 y4 x1 x2 z1 z2 x1z1 x1z2 x2z1 x2z2
Mo Ny=4 Nx=8 Ne=2 Nk=3 Lx=Fu Ly=Fu Td=Sy Te=sy Ph=Sy Ps=Sy,fr Ga=Fu c
ka=fu tx=fu ty=ze al=fu,fr
File = cov.cm
File = mean.me
Lk
Ksi_1 Ksi_2 Ksi_12
Le
Eta_Int Eta_Slp
fi ly(1,1) ly(2,1) ly(3,1) ly(4,1)
va 1 ly(1,1) ly(2,1) ly(3,1) ly(4,1)
fi ly(1,2) ly(2,2) ly(3,2) ly(4,2)
va 0 ly(1,2)
va 1 ly(2,2)
va 3 ly(3,2)
va 6 ly(4,2)
Pa Lx
1 0 0
1 0 0
0 1 0
0 1 0
1 1 1
1 1 1
1 1 1
1 1 1
co lx(5,1)=tx(3)
co lx(6,1)=tx(4)
co lx(7,1)=tx(3)*lx(2,1)
co lx(8,1)=tx(4)*lx(2,1)
co lx(5,2)=tx(1)
co lx(6,2)=tx(1)*lx(4,2)
co lx(5,3)=lx(1,1)*lx(3,2)
co lx(6,3)=lx(4,2)
co lx(7,2)=tx(2)
co lx(7,3)=lx(2,1)
co lx(8,2)=tx(2)*lx(4,2)
co lx(8,3)=lx(2,1)*lx(4,2)
Pa Te
1
0 1
```

```
0 0 1
0 0 0 1
Pa Td
1
0 1
0 0 1
0 0 0 1
1 0 1 0 1
1 0 0 1 1 1
0 1 1 0 1 0 1
0 1 0 1 0 1 1 1
Pa Ph !Variance for the latent variables
1
1 1
0 0 1
Pa Ga
1 1 1 1 1 1
Pa ka
0 0 1
Pa Al
1 1
Pa Tx
1 1 1 1 1 1 1 1
co tx(5)=tx(1)*tx(3)
co tx(6)=tx(1)*tx(4)
co tx(7)=tx(2)*tx(3)
co tx(8)=tx(2)*tx(4)
co ka(3)=ph(2,1)
co ph(3,3)=ph(1,1)*ph(2,2)+ph(2,1)**2
co td(5,5)=tx(1)**2*td(3,3) + tx(3)**2*td(1,1)+ c
  ph(1,1)*td(3,3) + ph(2,2)*td(1,1) + td(1,1)*td(3,3)
co td(6,6)=tx(1)**2*td(4,4) + tx(4)**2*td(1,1) + c
  ph(1,1)*td(4,4)+lx(4,2)**2*ph(2,2)*td(1,1) + td(1,1)*td(4,4)
co td(7,7)=tx(2)**2*td(3,3)+tx(3)**2*td(2,2)+ c
  lx(2,1)**2*ph(1,1)*td(3,3)+ ph(2,2)*td(2,2)+td(2,2)*td(3,3)
co td(8,8)=tx(2)**2*td(4,4)+tx(4)**2*td(2,2) + c
lx(2,1)**2*ph(1,1)*td(4,4) + lx(4,2)*lx(4,2)*ph(2,2)*td(2,2) + c
  td(2,2)*td(4,4)
co td(5,1)=tx(3)*td(1,1)
co td(6,1)=tx(4)*td(1,1)
co td(7,2)=tx(3)*td(2,2)
co td(8,2)=tx(4)*td(2,2)
```

```
co td(5,3)=tx(1)*td(3,3)
co td(7,3)=tx(2)*td(3,3)
co td(6,4)=tx(1)*td(4,4)
co td(8,4)=tx(2)*td(4,4)
co td(6,5)=tx(3)*tx(4)*td(1,1)+lx(4,2)*ph(2,2)*td(1,1)
co td(7,5)=tx(1)*tx(2)*td(3,3)+lx(2,1)*ph(1,1)*td(3,3)
co td(8,6)=tx(1)*tx(2)*td(4,4)+lx(2,1)*ph(1,1)*td(4,4)
co td(8,7)=tx(3)*tx(4)*td(2,2)+lx(4,2)*ph(2,2)*td(2,2)
fi lx(1,1) lx(3,2) lx(5,3)
va 1 lx(1,1) lx(3,2) lx(5,3)
ou Sc it=300 ad=off nd=4 ns ep=.01
```

APPENDIX D

The following LISREL script can be used for estimating the interaction model shown in Figure 3.

```
Interaction model with a LGM
      – an intercept–slope factor model
Da Ni=15 No=221
la
y1 y2 y3 y4 x1 x2 x3 x4 z1 z2 z3 z4 x2x6 x3x7 x4x8
file = cov.cm
file = mean.dat
Mo Ny=4 Nx=11 Ne=2 Nk=5 Td=Sy Te=Di Ps=Sy Ga=Fu ka=fr c
  tx=fu ty=fu
Lk
KSIA_int KSIA_slp KSIB_int KSIB_slp KAB_Int
Le
ETA_int ETA_slp
fi ly(1,1) ly(2,1) ly(3,1) ly(4,1)
va 1 ly(1,1) ly(2,1) ly(3,1) ly(4,1)
fi ly(1,2) ly(2,2) ly(3,2) ly(4,2)
va 0 ly(1,2)
va 1 ly(2,2)
va 4 ly(3,2)
va 8 ly(4,2)
!setting metric for KISA_int
fi lx(1,1) lx(2,1) lx(3,1) lx(4,1)
va 1 lx(1,1) lx(2,1) lx(3,1) lx(4,1)
!setting metric for KISA_slp
```

```
fi lx(1,2) lx(2,2) lx(3,2) lx(4,2)
va 0 lx(1,2)
va 1 lx(2,2)
fr lx(3,2)
fr lx(4,2)
st 1 lx(3,2) lx(4,2)
!setting metric for KSIB_int
fi lx(5,3) lx(6,3) lx(7,3) lx(8,3)
va 1 lx(5,3) lx(6,3) lx(7,3) lx(8,3)
!setting metric for KSIB_slp
fi lx(5,4) lx(6,4) lx(7,4) lx(8,4)
va 0 lx(5,4)
va 1 lx(6,4)
fr lx(7,4)
fr lx(8,4)
st 1 lx(7,4) lx(8,4)
Pa Ph
1
1 1
1 1 1
1 1 1 1
0 0 0 0 1
Pa Ga
1 0 1 0 0
0 1 0 1 1
!constraints for the interaction terms
fi lx(9,5)
va 1 lx(9,5)
co lx(10,5) = lx(3,2)*lx(7,4)
co lx(11,5) = lx(4,2)*lx(8,4)
co lx(9,2)=tx(6)
co lx(9,4)=tx(2)
Co lx(10,2)=tx(7)*lx(3,2)
co lx(10,4)=tx(3)*lx(7,4)
co lx(11,2)=tx(8)*lx(4,2)
Co lx(11,4)=tx(4)*lx(8,4)
Pa Al
1 1
pa tx
1 1 1 1 1 1 1 1 1 1 1
co tx(9)=tx(2)*tx(6)
co tx(10)=tx(3)*tx(7)
```

```
co tx(11)=tx(4)*tx(8)
Co ph(5,5)=ph(2,2)*ph(4,4) + ph(4,2)**2
Co td(9,9)=tx(2)**2*td(6,6)+tx(6)**2*td(2,2)+ph(2,2)*td(6,6)+ c
  ph(4,4)*td(2,2)+td(2,2)*td(6,6)
Co td(10,10)=tx(3)**2*td(7,7)+tx(7)**2*td(3,3)+lx(3,2)**2*ph(2,2)*td(7,7) + c
  lx(7,4)**2*ph(4,4)*td(3,3)+td(3,3)*td(7,7)
Co td(11,11)=tx(4)**2*td(8,8)+tx(8)**2*td(4,4)+lx(4,2)**2*ph(2,2)*td(8,8) + c
  lx(8,4)**2*ph(4,4)*td(4,4)+td(4,4)*td(8,8)
Co td(9,2)=tx(6)*td(2,2)
Co td(9,6)=tx(2)*td(6,6)
Co td(10,3)=tx(7)*td(3,3)
Co td(10,7)=tx(3)*td(7,7)
Co td(11,4)=tx(8)*td(4,4)
Co td(11,8)=tx(4)*td(8,8)
Pa Ka
0 0 0 0 1
Co ph(5,5)=ph(2,2)*ph(4,4)+ ph(4,2)**2
Co ka(5) = ph(4,2)
Ou Sc add=off it=300 ns ep=.001 nd=3 xm
```