

Long memory in PIIGS economies: An application of wavelet analysis

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Abstract

This paper studies long-memory properties of stock prices in PIIGS economies (Portugal, Ireland, Italy, Greece and Spain) and measures the evolution of their long-memory phenomena over time. We use the Hurst exponent as a measure of long-range dependence in stock prices. We apply wavelet analysis (based on Haar, Daubechies-4, Daubechies-12 and Daubechies-20 wavelets) for computation of Hurst exponents. In addition, we utilize the semi-parametric Local Whittle approach to test the

robustness of results obtained from wavelet analysis. Our findings support the dynamic nature of efficiency, characteristics of stock prices in PIIGS economies. Portugal and Greece show long-range dependence properties, whereas Ireland and Italy are weakly inclined towards mean reversion and Spain shows martingale behavior in its stock price.

Keywords: Long-range dependence; Time varying Hurst exponent; Wavelets; Local Whittle.

Introduction

In economics and finance, whether or not asset prices display long-range dependence is still an important area of research because of its importance for capital market theories (see for instance, Mandelbrot (1971), Greene and Fielitz (1977), Cutland et al. (1995)). Analysis related to long-memory property can be realized through estimation of the fractional integration parameter or the Hurst exponent. The subject of detecting long memory in a given time series was first studied by Hurst (1951), an English hydrologist, who proposed the concept of the Hurst exponent based on Einstein's contributions to Brownian motion in physics to deal with the obstacles related to the reservoir control near the Nile river dam. The Hurst exponent has characteristics that reflect facts having a bearing on market efficiency. Market inefficiency refers to the fact that the market does not react immediately as new information flows in, but responds to it gradually over a period of time. A violation of the efficient market hypothesis would support the presence of persistence (long memory) or anti-persistence (mean reversion) in the stock market. The presence of long memory in the evolution of asset prices describes the higher-order correlation structure in the series and supports the possibility of predicting its behavior over time. The analysis of long memory in asset prices is important for practitioners because its presence can significantly impact risk management, portfolio selection and trading strategies.

Two measures of long-range dependence are commonly used in finance literature. The first measure 'H', the Hurst exponent (or the self-similarity parameter), is a dimensionless parameter and diverse methodologies exist to estimate it. The Hurst exponent concept finds its applications in

many research fields, including financial studies, due to the ground-breaking work of Mandelbrot (1963, 1997) and Peters (1991, 1994). The Hurst exponent lies in the range $0 \leq H \leq 1$. If the Hurst exponent is 0.5, then the process is said to follow a random walk. When the Hurst exponent is greater than 0.5, it suggests positive long-range auto correlation in the return series or persistence in stock price series. On the other hand, when the Hurst exponent is smaller than 0.5, it suggests the presence of negative auto correlation in returns or mean reversion in stock price series. The second measure 'd' is the fractional integration parameter, which can be estimated from fitting an ARFIMA (p,d,q) model. The Hurst exponent 'H' and the fractional integration parameter 'd' are related by the formula $H = d + 0.5$. The Hurst exponent and the fractional integration parameter can be estimated through a variety of techniques, two of which have been adopted in this paper, viz., the wavelet analysis and the Local Whittle analysis.

Mandelbrot (1972) finds that rescaled-range (R/S) analysis shows superior properties over auto correlation and variance analysis (because it can work with distributions with infinite variance) and spectral analysis (because it can detect non-periodic cycles). Greene and Fielitz (1977) utilize the Hurst R/S method and provide evidence in support of long memory in daily stock return series. Lo (1991) finds that the classical R/S test used by Mandelbrot and Green & Fielitz suffers from a drawback wherein it is unable to distinguish between long memory and short-range dependence. Lo (1991) proposes a modified test of the R/S statistic which can distinguish between short-term dependence and long memory, and finds that daily stock returns do not show long-range dependence properties. Willinger et. al. (1999)

empirically finds that Lo's modified R/S test leading to the acceptance of the null hypothesis of no long-range dependence for CSRP (Center for Research in Security Prices) data is less conclusive than it appears. This is so because of the conservative nature of the test statistic in rejecting the null hypothesis of no long-range dependence, by attributing what is found in the data to short-term dependence instead. Baillie et. al. (1995) investigated the long-range dependence properties in inflation time series and found positive results. Lo (2000) applies non-parametric tests to investigate the market efficiency of six Asian stock markets and finds that none of the markets are stationary or exhibit random behavior. Corazza and Malliaris (2002) find that the Hurst exponent does not remain fixed, but changes dynamically over time. They also provide evidence that foreign currency returns follow either a fractional Brownian motion or a Pareto-Levy stable distribution. Cajueiro and Tabak (2004) use the rolling sample approach to calculate Hurst exponents over October 1992 to October 1996 and provide evidence of long-range dependence in Asian markets. Cajueiro and Tabak (2005) study the possible sources of long-range dependence in returns of Brazilian stocks and find that firm specific variables can partially explain long-range dependence measures, such as the Hurst exponent. Karuppiah and Los (2005) apply the wavelet multi resolution analysis to examine long-term dependencies in currency markets of Germany, Japan, Hong Kong, Indonesia, Malaysia, Philippines, Singapore, Taiwan, and Thailand and find that the German Mark/Dollar and the Japanese Yen/Dollar rates exhibit anti-persistent characteristics. Kyaw et. al. (2006) examines long-range dependence in returns from Latin American stock and currency markets using Hurst exponents based on the wavelet multi resolution analysis and

finds mixed results for different Latin American markets. Maghyereh (2007) applies the semi-parametric Local Whittle analysis on financial returns and volatility of Middle East and North African markets and finds a strong degree of long-range dependence in their equity returns and volatility. Souza et. al. (2008) studies the evolution of long memory over time in returns and volatilities of British pound futures contracts by using the classic R/S approach, the detrended fluctuation analysis (DFA) approach and the generalized Hurst exponent (GHE) approach and finds a change in long-memory characteristics of the British pound around the time of the European financial crisis. Mabrouk et. al. (2008) investigates the long-range dependence property in stock prices and volatility of various emerging and developed markets by using wavelet analysis and finds that the long-memory property in stock returns is approximately associated with emerging markets in comparison to developed markets. Also found is strong evidence of long memory in all the volatility series. Serletis and Rosenberg (2009) use the detrending moving average (DMA) approach to calculate the Hurst exponent and find evidence in support of anti-persistence (mean reversion) in the US stock market. They also estimate the Local Hurst exponent (on non-overlapping windows of 50 observations) to examine the evolution of efficiency characteristics of index returns over time. Kristoufek (2010) reexamines the results of Serletis and Rosenberg (2009) and finds that there are no signs of anti-persistence in the US stock market.

Techniques like R/S analysis, DFA analysis, DMA analysis and GHE analysis estimate the Hurst exponent in the time domain. On the other hand, wavelet analysis and the Local Whittle analysis are frequency domain approaches to detect the

presence of long memory in a given time series. The computation of the fractional differencing parameter from the Local Whittle estimator is based on the periodogram and the estimation procedure is based on the maximum likelihood approach. Wavelet analysis estimates the Hurst exponent based on varying degree of smoothness, which varies from low to high, in that order, for the Haar wavelet, Daubechies-4, Daubechies-12 and Daubechies-20 wavelets.

The core contribution of this study is twofold. First, we study the long-memory properties in stock prices in PIIGS economies using wavelets analysis and Local Whittle analysis (semi-parametric technique). Wavelet analysis (see Mulligan (2004); Gencay et. al. (2005)) and Local Whittle analysis (see Maghyreh (2007)) have many applications in the field of economics and finance including the analysis of long memory in asset prices. Second, we also study the evolution of market efficiency of their stock prices over time using a non-overlapping rolling sub-sample of 256 observations to estimate Hurst exponents over time, which is the same as in Serletis and Rosenberg (2009). To our knowledge, the estimation of the Hurst exponent using a non-overlapping rolling sub-sample approach has not been implemented previously in stock prices of PIIGS economies. In particular, we would like to note that the time varying Hurst exponent approach is very important to understand the dynamic nature of the evolution of market efficiency. PIIGS economies are at the center of study worldwide due to the current European financial crisis. PIIGS countries have experienced similar economic conditions and financial problems over the past several years. They have common high levels of government debt, recently low GDP, high public-spending, high unemployment, high labor costs, financial sector

problems and an inability to deal with debt. Despite facing identical problems, their equity markets are distinct, which is evident from their varying market capitalizations and style distributions. Hence, it is reasonable to conclude that stock markets of PIIGS economies exhibit diverse efficiency characteristics. Furthermore, the constituents of indices belonging to PIIGS economies are different from country to country and news that hits the market may have different firm specific implications and hence have a diverse impact on different indices. Moreover, Figure 1 also indicates a difference in the behavior of prices and returns for the period under consideration for PIIGS economies.

The remainder of this paper is organized as follows: Section 2 introduces tests we will use in this study. Section 3 discusses the Monte Carlo simulations to examine their small sample properties. Section 4 describes data used in this study and discusses the computational details. Section 5 reports empirical results. Section 6 concludes with a summary of our primary findings.

1. Methodology

1.1. Long memory in a financial time series

Both time and frequency domain measures are available to detect the presence of long memory in time series. In time domain, a hyperbolically decaying auto covariance function characterizes the presence of long memory. Suppose x_t is a stationary process and λ_τ is its auto covariance function at lag τ , then, the asymptotic property of the auto covariance function is given as:

$$\lambda_\tau \approx |\tau|^{2H-2} \quad \text{as } |\tau| \rightarrow \infty \quad (1)$$

Where, $H \in (0,1)$ is a long-memory parameter and called the Hurst exponent.

In frequency domain, the long memory is present when the spectral density function approaches infinity at low frequencies. Suppose $f(\lambda)$ is the spectral density function. The series x_t is said to exhibit long memory if:

$$f(\lambda) \sim C_f |\lambda|^{1-2H} \quad \text{as } \lambda \rightarrow 0 \quad (2)$$

Where, $C_f > 0$ and $H \in (0,1)$.

1.2. Discrete wavelet transform

Wavelets are based on multi-resolution analysis wherein the frequency domain (by the spectral representation theorem), any covariance-stationary process x_t can be represented as a linear combination of sine and cosine functions. The Fourier series of any real-valued function $f(x)$ on the $[0,1]$ interval is expressed as:

$$f(x) = b_0 + \sum_{k=1}^{\infty} [b_k \cos 2\pi kx + a_k \sin 2\pi kx] \quad (3)$$

Where, the parameters b_0 , b_k and a_k , $\forall k$, can be estimated by using least squares.

In the wavelet domain, function $f(x)$ can be expressed as:

$$f(x) = c_0 + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} c_{jk} \Psi(2^j x - k) \quad (4)$$

Where, $\Psi(x)$ is known as the mother wavelet, as it is the mother to all dilations and translations of in equation (4). The functions of $\Psi_{jk}(x) = \Psi(2^j x - k)$ for $j \geq 0$ and $0 \leq k < 2^j$ are orthogonal and they form a basis for square integrable functions L^2 along the $[0, 1]$ interval (Tkacz (2001)). Hence, a wavelet is a function $\Psi \in L^2$ such that $\int \Psi(t) dt = 0$ and $\int |\Psi(t)|^2 dt < \infty$. Haar and Daubechies wavelets are most commonly used for economics and finance applications (Tkacz (2001)).

Daubechies (1988) proposes a system of wavelets where different wavelets represent diverse degrees of smoothing of the step function. In this paper, we use the Haar wavelet and three representative Daubechies wavelets to investigate the robustness of results to varying degrees of smoothing. The Haar wavelet is the least smooth wavelet, followed by Daubechies-4 and Daubechies-12; Daubechies-20 is the smoothest wavelet used in this study. We use multi resolution analysis (Mallat (1989)) to obtain the coefficients corresponding to the wavelet transform of the observed time series.

To identify long-memory properties using wavelet analysis, we apply techniques proposed by Jensen (1999) and Tkacz (2001), which are explained as follows:

Suppose x_t is a random process,

$$(1 - L)^d x_t = \varepsilon_t \quad (5)$$

Where, L is the lag operator and ε_t is independent and identically distributed (i.i.d.) normal with mean zero and variance σ^2 and d is the differencing parameter. Jensen (1999) empirically shows (following Tewfik and Kim (1992) and McCoy and Walden (1996)) that for a fractionally integrated $I(d)$ process x_t with $d < 0.5$, the auto covariance function shows that the detail coefficients c_{jk} are distributed as $N(0, \sigma^2 2^{-2(J-j)d})$, where j is the scaling parameter of wavelets ($j = 1, \dots, J$). The fractional integration parameter 'd' can be estimated by using an ordinary least square regression as follows:

$$\ln \text{Var}(c_{jk}) = \ln \sigma^2 + d \ln 2^{-2(J-j)} \quad (6)$$

Where, $\ln \text{Var}(C_{jk})$ is the logarithmic transformation of the variance of the detail coefficients C_{jk} . The variance of the detail coefficients decomposes the variance of the original series across different scales and helps us investigate the behavior of the time series at each scale. The Hurst exponent can be computed as, $H = (1+d)/2$. Note that $H = 0.5$ for white noise. When the process is persistent (has long memory) then $H > 0.5$ and for an anti-persistent process (with mean reversion) $H < 0.5$.

1.3. Local Whittle method

Robinson (1995) introduces the Local Whittle (LW) estimator which assumes the following behavior of the spectral density $f(\lambda)$ close to the origin, i.e., around $\lambda = 0$,

$$f(\lambda) \sim G(H)|\lambda|^{1-2H} \quad \text{as } \lambda \rightarrow 0 \quad (7)$$

Computation of the Local Whittle estimator based on the periodogram involves an additional parameter m , which is less than $N/2$ and is assumed to satisfy the condition:

$$\frac{1}{m} + \frac{m}{N} \rightarrow 0 \quad \text{as } N \rightarrow \infty$$

For the spectral density satisfying equation (7), the Whittle log-likelihood function is given as:

$$L(G, H) = \frac{1}{m} \sum_{j=1}^m \left(\frac{I(\lambda_j)}{G \lambda_j^{1-2H}} + \log G \lambda_j^{1-2H} \right) \quad (8)$$

Where, $\lambda_j = 2\pi j/T$ and G is a constant. The estimate of G is given as:

$$\hat{G} = \frac{1}{m} \sum_{j=1}^m \frac{I(\lambda_j)}{\lambda_j^{1-2H}}$$

Replace in equation (8) G by its estimate \hat{G} ?,

$$R(H) = L(\hat{G}, H) - 1 = \log \left(\frac{1}{m} \sum_{k=1}^m \lambda_k^{-(1-2H)} I(\lambda_k) \right) - \frac{2H-1}{m} \sum_{k=1}^m \log \lambda_k \quad (9)$$

The value \hat{H} which minimizes $R(H)$ asymptotically converges in probability to the actual value H as $N \rightarrow \infty$. Robinson (1995) shows that

$$\sqrt{m}(\hat{H} - H) \rightarrow_d \text{Normal}(0, 0.25), \quad \text{as } N \rightarrow \infty \quad (10)$$

A major issue on the use of the Local Whittle test is the choice of bandwidth parameter m . Here we assume bandwidth parameter m ($N^{0.5}$, $N^{0.6}$, $N^{0.7}$ and $N^{0.8}$).

2. Data and computational details

In order to test the long-memory property in PIIGS economies, we have used daily price data of five indices associated with respective economies. The indices used are PSI-20 (the Portuguese Stock Index, composed of 20 firms with the largest market capitalization and share turnover), ISEQ Overall index (also known as Irish Stock Exchange Quotient, is a capitalization weighted index of all equities listed on Irish Stock Exchange), ATG index (also known as the Athens Composite Share Price Index, composed of 40 firms with the largest market capitalization), IBEX-35 index (composed of the 35 most liquid securities listed on the stock exchange Interconnection System of the four Spanish Stock Exchanges) and FTSE-MIB index (includes 40 Italian stocks that capture 80% of the total market capitalization). All the data has been obtained from Reuters database. The period of study for all the indices is from August 22, 2003, to June 30, 2011, (2,048 observations for each index).

Table 1: Descriptive statistics of daily stock returns for all indices

	Portugal	Ireland	Italy	Greece	Spain
Mean	0.010	-0.021	-0.011	-0.028	0.018
Median	0.038	0.020	0.050	0.007	0.060
Stdev	1.136	1.594	1.393	1.622	1.417
Min	-10.379	-13.964	-8.599	-10.214	-9.586
Max	10.196	9.733	10.874	9.114	13.484
Quartile 1	-0.414	-0.620	-0.548	-0.756	-0.571
Quartile 3	0.507	0.689	0.612	0.787	0.653
Skewness	-0.035	-0.615	0.057	-0.225	0.208
Kurtosis	13.549	8.185	9.302	4.271	10.724
JB Stat	15703.582#	5861.349#	7404.348#	1579.285#	9852.828#
Shapiro Wilk	0.870#	0.896#	0.888#	0.940#	0.892#
ARCH LM	292.908#	512.301#	388.351#	369.975#	320.284#
N	2048	2048	2048	2048	2048
Q(20)	49.241#	56.291#	80.084#	47.539#	47.906#
ADF	-11.872#	-12.456#	-11.848#	-11.968#	-12.189#
KPSS	0.390†	0.463*	0.287	0.590*	0.247

Note: #, * and † mean significant at 1%, 5% and 10% level of significance, respectively. Stdev represents the standard deviation of the series. ARCH LM indicates the Lagrange multiplier test for conditional heteroskedasticity with 10 lags. JB Stat indicates the Jarque Bera statistics. Q(20) indicates the Box-Pierce statistics for 20 lags.

We have used the country name to represent the index i.e., Portugal for the PSI-20 index, Ireland for the ISEQ Overall index, Italy for the FTSE-MIB index, Greece for the ATG index and Spain for the IBEX-35 index. Table 1 provides the descriptive statistics of the daily returns of all indices under study. The median daily return is higher for Spain and Italy, but indices in Ireland, Italy and Greece exhibit decline in their values over the study period and, hence, we observe negative average daily returns in these indices. ATG index (Greece) seems to be more volatile than the other indices, and PSI-20 (Portugal) shows the least volatility in its index

values. Jarque-Bera and Shapiro-Wilk statistics confirm the significant non-normality in the daily returns of all indices.

The ARCH-LM test provides evidence in support of the presence of conditional heteroscedasticity in the return series. Indices associated with Portugal, Ireland and Greece show significant negative skewness. In addition, there is evidence of excess kurtosis, which confirms the leptokurtosis in the distribution of returns of all indices. Box-Pierce Q-test strongly rejects the presence of non-significant auto correlations in the first 20 lags for all the return

series. Insignificant KPSS statistics for all indices support the non-rejection of the null hypothesis of stationarity of the series except for Portugal, Ireland and Greece, which are showing moderate signs of violation of the null hypothesis of the stationarity. Also, the ADF test rejects the null hypothesis of a unit root in all the return series.

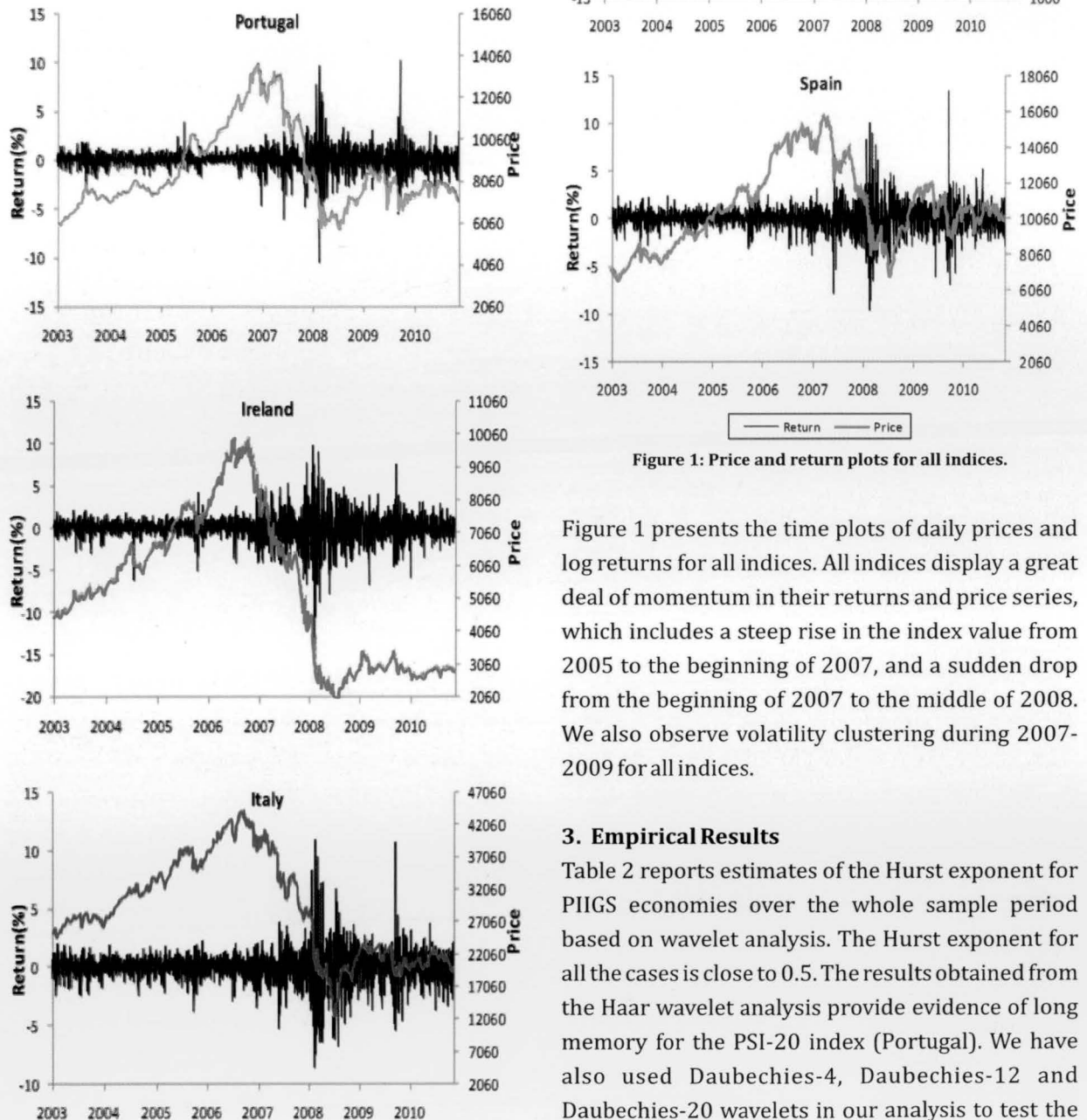


Figure 1: Price and return plots for all indices.

Figure 1 presents the time plots of daily prices and log returns for all indices. All indices display a great deal of momentum in their returns and price series, which includes a steep rise in the index value from 2005 to the beginning of 2007, and a sudden drop from the beginning of 2007 to the middle of 2008. We also observe volatility clustering during 2007-2009 for all indices.

3. Empirical Results

Table 2 reports estimates of the Hurst exponent for PIIGS economies over the whole sample period based on wavelet analysis. The Hurst exponent for all the cases is close to 0.5. The results obtained from the Haar wavelet analysis provide evidence of long memory for the PSI-20 index (Portugal). We have also used Daubechies-4, Daubechies-12 and Daubechies-20 wavelets in our analysis to test the robustness of the results obtained from the Haar wavelet analysis.

Table 2: Estimated Hurst exponents with t-statistics based on wavelet analysis

	Portugal	Ireland	Italy	Greece	Spain
Haar	0.539*	0.508	0.512	0.515	0.500
	(2.221)	(0.485)	(0.796)	(0.800)	(0.010)
Daubechies-4	0.519	0.470#	0.481†	0.538	0.487
	(1.277)	(-2.592)	(-1.676)	(1.468)	(-1.110)
Daubechies-12	0.521	0.500	0.492	0.542	0.495
	(1.403)	(-0.019)	(-0.676)	(1.559)	(-0.351)
Daubechies-20	0.533*	0.530	0.509	0.541	0.503
	(1.988)	(1.362)	(0.614)	(1.541)	(0.244)

Note: t-statistics are in parenthesis.

The Daubechies-20 wavelet supports the same inference of significance of the Hurst exponent as explained by the Haar wavelet analysis. Ireland and Italy show signs of mean reversion (anti-persistence) in their stock prices for the Daubechies-4 wavelet. The Daubechies-12 wavelet does not provide any evidence of a rejection of the null hypothesis of short memory in the time series. Overall, Table 2 provides evidence in support of long memory in Portugal and weak signs of mean reversion in Ireland and Italy and random behavior of stock prices in Greece and Spain.

To test the robustness of the results obtained from wavelets analysis, we apply the semi-parametric Local Whittle technique to estimate the Hurst exponent for PIIGS economies. Here, we have taken bandwidth parameter m to be $N^{0.5}$, $N^{0.6}$, $N^{0.7}$ and $N^{0.8}$. Table 3 presents the estimated values of the Hurst exponent with t-statistic for PIIGS economies over the whole sample period based on Local Whittle analysis.

Table 3: Estimated Hurst exponents with t-statistics based on Local Whittle technique

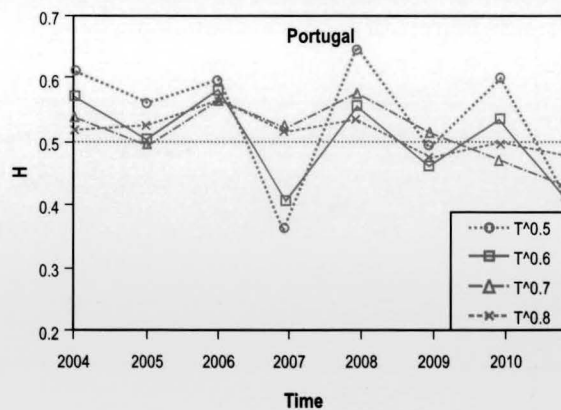
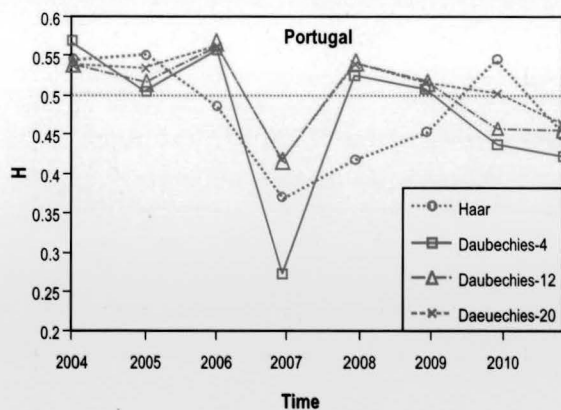
	Portugal	Ireland	Italy	Greece	Spain
$N^{0.5}$	0.566†	0.602#	0.531	0.582*	0.550
	(1.769)	(2.750)	(0.830)	(2.209)	(1.335)
$N^{0.6}$	0.559*	0.520	0.525	0.551*	0.516
	(2.343)	(0.794)	(0.972)	(2.021)	(0.629)
$N^{0.7}$	0.513	0.504	0.510	0.532†	0.483
	(0.751)	(0.205)	(0.586)	(1.856)	(-0.959)
$N^{0.8}$	0.513	0.511	0.503	0.516	0.484
	(1.101)	(0.889)	(0.229)	(1.347)	(-1.391)

Note: t-statistics are in parenthesis.

The results confirm the findings based on the wavelet analysis for Portugal, Italy and Spain. We also find weak evidence of long memory in the Greek stock market. Ireland exhibits signs of long memory when bandwidth parameter is taken to be 0.5. Overall, the Local Whittle semi-parametric technique provides evidence in favor of long memory in Portugal and Greece, and random behavior of stock prices in Italian and Spanish stock markets.

Figure 2 presents the plots of the local Hurst exponent by applying the wavelet approach (first column) and the Local Whittle approach (second column) on non-overlapping subsets for each index under study. The horizontal line in each sub-plot represents $H = 0.5$. In all these plots, the y-axis represents the Hurst exponent and the x-axis represents calendar time. Each non-overlapping subset is created by sliding a window of length 256 (28) observations. The local Hurst exponent is then estimated for each sub-series. The estimated value

of the Hurst exponent for Portugal based on both wavelets and Local Whittle is well above 0.5 for all the periods, except 2007 (which was the beginning of the sub-prime crisis) and 2011. This indicates that the behavior of stock prices in Portugal is more inclined towards persistence than a random walk. This also confirms the results obtained by wavelets analysis and Local Whittle analysis for the whole sample. On the other hand, Hurst exponents over time for Ireland are more tilted towards mean reversion or anti-persistence because the estimated value of the Hurst exponent is below 0.5 for most periods. This also confirms the results obtained by wavelets analysis. In the case of Italy, except for the results of Haar wavelet, other wavelets (Daubechies-wavelets) provide strong evidence of mean reversion for most sub-sample periods. Results from the Local Whittle approach for Italy also support the same findings except for the Hurst exponents estimated based on $N0.5$. This, again, is in alignment with results obtained by wavelets analysis for the whole sample.



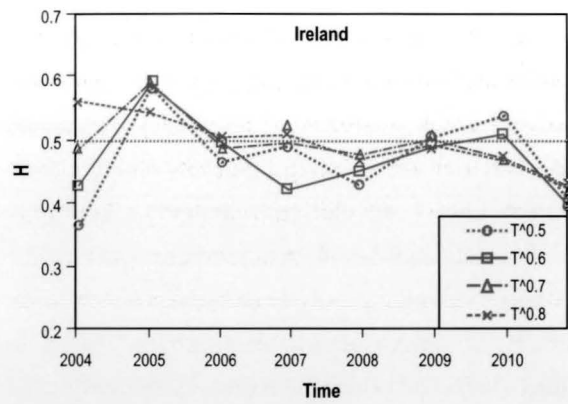
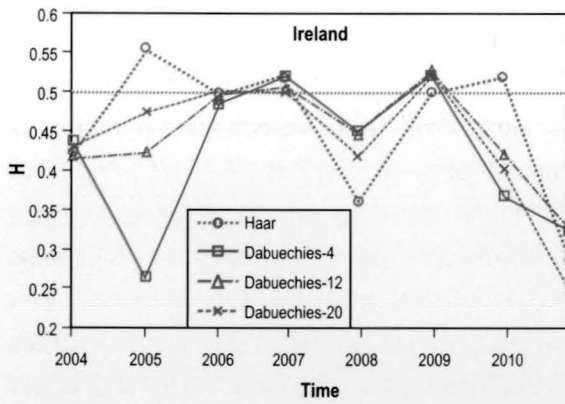


Figure 2: Local Hurst exponents estimated based on wavelets (first column) and the Local Whittle technique (second column) for all indices.

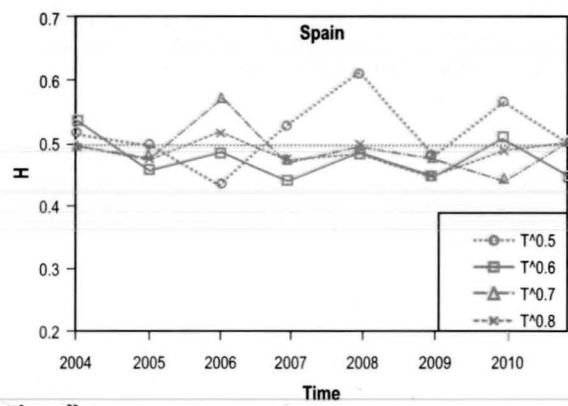
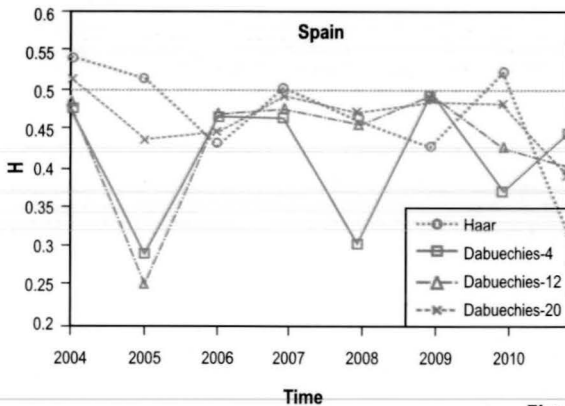
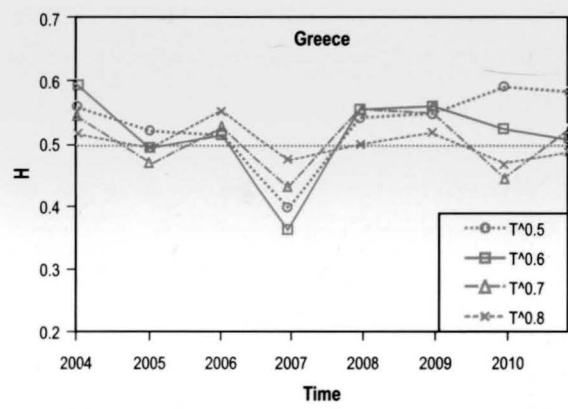
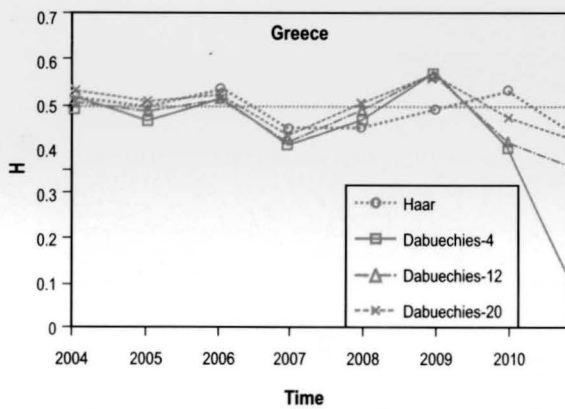
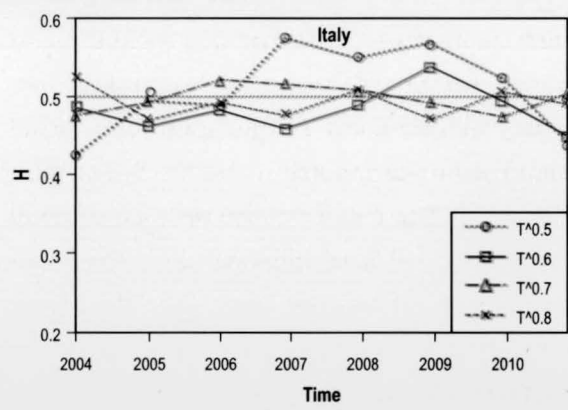
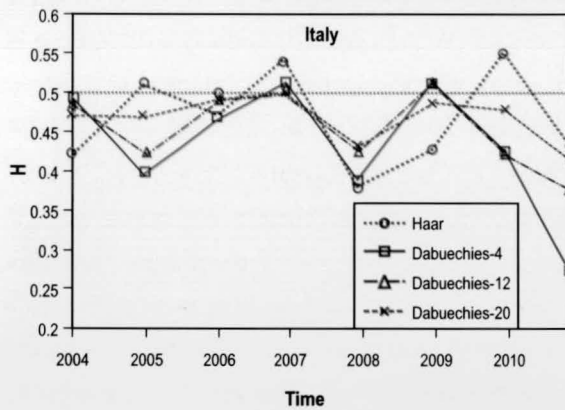


Figure 2 (Continued)

Results for Greece indicate the dynamic change in its stock price behavior between long memory and mean reversion over time, based on both wavelets and the Local Whittle analysis which is in confirmation with the results from the whole sample analysis. The Hurst exponent estimated for Spain by wavelets analysis clearly indicates the absence of long memory in its stock prices. In addition, the Hurst exponents estimated under the Local Whittle approach based on a moving and non-overlapping sub-sample, strongly support the random behavior of stock prices, except for the case of $H=0.5$ which provides some evidence of long memory for 2008 and 2010. Overall, the results from a non-overlapping, moving sub-sample analysis are in confirmation with the results obtained by the same approaches for the whole sample.

4. Conclusion

In this paper, we have examined long-memory properties in stock prices of PIIGS economies using the wavelets approach (based on Haar, Daubechies-4, Daubechies-12 and Daubechies-20 wavelets) and the Local Whittle (semi-parametric) approach. We

have also examined the evolution of long-range dependence in stock prices of PIIGS economies over time, by estimating the local Hurst exponents on non-overlapping sub-samples. Our findings indicate the presence of long memory in stock prices of PSI-20 (Portugal) and ATG-Index (Greece). We also find weak evidence of mean reversion for stock prices in Ireland and Italy using the wavelets analysis and random walk behavior in the IBEX-35 index (Spain). The moving sub-sample approach also confirms the findings. A natural implication of our empirical findings is that wavelet analysis and the Local Whittle analysis can be helpful in developing strategies on different scales with relevance for portfolio management and financial engineering. Trading strategies that seek to exploit short horizon predictability may well result in excessive turnover and, consequently, high transaction costs, when compared to the investment strategies that are based on long-range dependence. Hence, there is ample scope for long horizon investment strategies that can meaningfully create profit opportunities based on our empirical findings.

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