Analysis of laser multimode content on the angle-of-arrival fluctuations in free-space optics access systems

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Abstract. The effects of laser beam multimode content on the angle-of-arrival fluctuations are examined for free-space optics (FSO) access systems. Multimode excitation is represented by coherent addition of Hermite-Gaussian modes. Mean square angle-of-arrival fluctuations are formulated using our previously reported multimode phase structure function. Numerical evaluations are obtained for practical FSO links operating at 1.55- and 0.85-μm wavelengths with link spans of up to 5 km. Mode content is arranged by sequentially grouping all possible mode combinations starting from the single fundamental mode (TEM_{00}) up to a certain higher order (n,m). Angle-of-arrival fluctuations are found to be of the order of several tens of microradians, except for the cases when the mode group terminates with an odd mode or when the extremely higher order modes are present. In these instances, the fluctuations will rise to the radian level. From these results, it is concluded that the performance of a practical FSO receiver, having a field of view of several milliradians, will not be substantially affected by the angle-of-arrival fluctuations due to multimode excitation, provided that the mode content of source excitation is confined to mode indices below 20. [DOI: 10.1117/1.1905383]

Subject terms: atmospheric turbulence; angle-of-arrival fluctuations; multimode; free-space optics.

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1 Introduction

It is well known[1] that the phase of the received field fluctuates in a turbulent atmosphere. Phase fluctuations give rise to fluctuations in the angle of arrival of the mean wavefront that can degrade the performance of an optical receiver. Angle-of-arrival fluctuations can increase the necessary field of view in a direct detection optical receiver while varying the tracking requirements in heterodyne detection. Many researchers[2–10] have investigated in detail both theoretically and experimentally the phase and the angle-of-arrival fluctuations in a turbulent atmosphere. Of particular interest to us is the work reported by Belen'kii and Mironov,[2] where they calculated the phase fluctuations of a multimode laser field in a randomly inhomogeneous medium. In their formulation, Belen'kii and Mironov[2] established the initial spatial-coherence radius of the field as a measure for the multimode content of the laser radiation, and using statistical independence, they expressed the structure function of the total phase fluctuations as the sum of the individual structure functions of the random phase caused by field fluctuations at the laser source aperture and the random phase caused by the turbulent atmospheric inhomogeneities.

In this paper, we strive to understand how the performance of an optical communication system will be influenced (as related to the angle-of-arrival fluctuations) when the source exhibits multimode properties. To this end, we introduce the multimode radiation of the laser beam by coherently adding Hermite-Gaussian modes in ascending order. The mean square angle-of-arrival fluctuations are derived based on our previously reported results of multimode phase structure function due to higher order laser modes,[11] valid in weak atmospheric turbulence. Numerical evaluations are performed for practical free-space optics (FSO) links operating at 1.55- and 0.85-μm wavelengths and for link lengths of up to 5 km. The variation of the angle-of-arrival fluctuations is investigated for various higher order laser mode combinations. Our present findings indicate that the angle-of-arrival fluctuations vary depending on the type of higher order laser mode combinations. The calculated global magnitudes of the angle-of-arrival fluctuations mainly stay in the microradian range except for the extremely high order mode combinations. Therefore, it is stipulated that the performance of a practical optical receiver with a field of view of several milliradians will not be greatly impaired so long as the source does not generate such high-order modes.

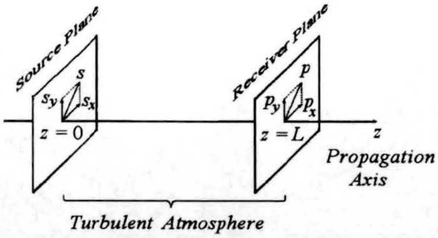

Fig. 1 Propagation geometry.

2 Formulation

The propagation geometry is shown in Fig. 1. Source plane is positioned at $z=0$, with z being the axis of propagation. This plane is coincident with the exit plane of an optical resonator having spherical mirrors. Here the field is written as

$$u(\mathbf{s},z=0)=\sum_{(n,m)} u_{n,m}(\mathbf{s},z=0), \tag{1}$$

where $\mathbf{s}=(s_x,s_y)$ is the transverse coordinates at the source plane, and $u_{n,m}(\mathbf{s},z=0)$ is the field at the source plane due to the (n,m)'th mode, and $\Sigma_{(n,m)}$ corresponds to the summation over all possible generated modes. For the multimode laser incidence, Eq. (1) makes no phase distinction between modes. In other words, the phase differences between the modes are presumed to be zero. Thus, our analysis enables coherent addition of modes.

The source plane field due to the (n,m)'th mode of an optical resonator equipped with spherical mirrors, i.e., $u_{n,m}(\mathbf{s},z=0)$ is described by

$$u_{n,m}(\mathbf{s},z=0)=A_{n,m}H_n(s_x/\alpha_s)H_y(s_y/\alpha_s)\exp(-0.5\mathbf{s}^2/\alpha_s^2). \tag{2}$$

Here $A_{n,m}$ is the amplitude of mode (n,m); H_n and H_m are the Hermite polynomials determining the field distributions in the s_x and s_y directions, respectively; and α_s is the source size common to both the s_x and s_y directions. For all order modes stated in Eq. (2), the radii of curvature of the wavefronts approach infinity.

Employing Eqs. (1) and (2) and resorting to the Huygens-Fresnel principle, the unperturbed field (in the absence of turbulence) is found at the propagation distance $z=\eta$, then utilizing the unperturbed field in the Rytov-method solution, phase fluctuations in the turbulent atmosphere are obtained from which the phase structure function is extracted. During this process, frequencies of operation of all the higher order modes are taken the same. The phase structure function at the receiver plane $z=L$ (L is the link length) of the multimode laser beam in the turbulent atmosphere formulated in the way as outlined above is presented in our earlier work.[11] From there, we import the following phase structure function (D_s) integral (note that the negative sign is missing in the phase structure function expressions in Ref. 11):

$$D_s(\mathbf{p}_1,\mathbf{p}_2,L)=-\pi\,\mathrm{Re}\left\{\int_0^L d\eta\int_0^\infty \kappa\,d\kappa\int_0^{2\pi} d\theta[Y_1(\mathbf{p}_1,\mathbf{p}_1,\kappa)-Y_2(\mathbf{p}_1,\mathbf{p}_1,\kappa)+Y_1(\mathbf{p}_2,\mathbf{p}_2,\kappa)-Y_2(\mathbf{p}_2,\mathbf{p}_2,\kappa)\right.$$
$$\left.-2Y_1(\mathbf{p}_1,\mathbf{p}_2,\kappa)+2Y_2(\mathbf{p}_1,\mathbf{p}_2,\kappa)]\Phi_n(\kappa)\right\}, \tag{3}$$

where $\Phi_n(\kappa)$ is the spectral density of the index-of-refraction fluctuations of the medium, Re means the real part, and

$$Y_1(\mathbf{p}_1,\mathbf{p}_2,\kappa)=\{-k^2/[D(\mathbf{p}_1)D(\mathbf{p}_2)]\}\sum_{(n,m)}\sum_{(n',m')}A_{n,m}A_{n',m'}b_2^{r_1+r_1'}\exp(jP\kappa\cos\theta)\exp(b_3\kappa^2)H_n(b_4p_{x_1}+b_5\kappa\cos\theta)$$
$$\times H_m(b_4p_{y_1}+b_5\kappa\sin\theta)H_{n'}(b_4p_{x_2}-b_5\kappa\cos\theta)H_{m'}(b_4p_{y_2}-b_5\kappa\sin\theta), \tag{4}$$

$$Y_2(\mathbf{p}_1,\mathbf{p}_2,\kappa)=\{k^2/[D(\mathbf{p}_1)D^*(\mathbf{p}_2)]\}\sum_{(n,m)}\sum_{(n',m')}A_{n,m}A_{n',m'}b_2^{r_1}(b_2^{r_1'})^*\exp(jQ\kappa\cos\theta)\exp(b_6\kappa^2)H_n(b_4p_{x_1}+b_5\kappa\cos\theta)$$
$$\times H_m(b_4p_{y_1}+b_5\kappa\sin\theta)H_{n'}(b_4p_{x_2}+b_5\kappa\cos\theta)H_{m'}(b_4p_{y_2}+b_5\kappa\sin\theta), \tag{5}$$

$$D(\mathbf{p})=\sum_{(n,m)}A_{n,m}b_2^{r_1}H_n(b_4\mathbf{p}_x)H_m(b_4\mathbf{p}_y), \tag{6}$$

$r_1=(n+m)/2$, $r_1'=(n'+m')/2$, $\alpha=1/(k\alpha_s^2)$, $b_2=(1-j\alpha L)/(1+j\alpha L)$, $b_3=j\gamma(\eta-L)/k$, $b_4=1/[\alpha_s(1+\alpha^2L^2)^{1/2}]$, $b_5=(\eta-L)b_4/k$, $b_6=-\alpha(L-\eta)^2/[k(1+\alpha^2L^2)]$, $\gamma=(1+j\alpha\eta)/(1+j\alpha L)$, $P=\gamma[(p_{x_1}-p_{x_2})^2+(p_{y_1}-p_{y_2})^2]^{1/2}$, and $Q=[(\gamma p_{x_1}-\gamma^*p_{x_2})^2+(\gamma p_{y_1}-\gamma^*p_{y_2})^2]^{1/2}$. Here $*$ indicates the complex conjugate, $j=(-1)^{1/2}$, $k=2\pi/\lambda$ is the wave number, λ is the wavelength of operation, $\mathbf{p}=(p_x,p_y)$ is the transverse coordinates at the receiver plane such that $\mathbf{p}_1=(p_{x_1},p_{y_1})$ and $\mathbf{p}_2=(p_{x_2},p_{y_2})$. Thus, Y_1 and Y_2 and D of $\mathbf{p}$ arguments other than those stated in Eqs. (4), (5), and (6) can be obtained by suitable replacements of $\mathbf{p}$ indices.

Mean square angle-of-arrival fluctuations $\langle \alpha_a^2 \rangle$ is defined as

$$\langle \alpha_a^2 \rangle = k^{-2} R^{-2} D_s(\mathbf{p}_1=0, |\mathbf{p}_2|=R, L), \tag{7}$$

where R is the radial distance from the origin on the receiver plane at which the angle-of-arrival fluctuations are to be evaluated.

Substituting Eq. (3) in Eq. (7), the mean square angle-of-arrival fluctuations are determined for R much smaller than the inner scale of turbulence, i.e., $R \ll \ell_o$, which practically means that the angle-of-arrival fluctuations are found in the vicinity of origin of the receiver plane, i.e., on-axis angle-of-arrival fluctuations are calculated. In the evaluation of Eq. (7), $R \ll \ell_o$ removes the dependence of $\langle \alpha_a^2 \rangle$ on R since, in this case, R^2 appears both in the numerator and the denominator of $\langle \alpha_a^2 \rangle$. Setting the magnitudes of all the modes to unity, i.e., $A_{n,m}=1$ for all n and m, that is assigning equal magnitude to all modes, using the Kolmogorov spectrum for the spectral density of the index-of-refraction fluctuations of the medium, i.e., $\Phi_n(\kappa)=0.033\,C_n^2 \kappa^{-11/3}$, where C_n is the structure constant, expanding the Hermite polynomials in the series form, and performing a term-by-term integration over θ by utilizing Eq. 3.389.1 of Ref. 12, the phase structure function at the receiver plane $z=L$ (L is the link length) of the multimode laser beam in the turbulent atmosphere as given by Eq. (3) is transformed into

$$\begin{aligned} D_s(\mathbf{p}_1,\mathbf{p}_2,L) = {} & 0.415 C_n^2 k^2 \,\mathrm{Re}\Bigg\{ \sum_{(n,m)} \sum_{(n',m')} 2^{(n+n'+m+m')} \sum_{l_1}^{n/2} \sum_{l_2}^{n'/2} \sum_{l_3}^{m/2} \sum_{l_4}^{m'/2} T_{l_1} T_{l_2} T_{l_3} T_{l_4} \binom{n}{2l_1} \binom{n'}{2l_2} \binom{m}{2l_3} \binom{m'}{2l_4} B(v,r) \\ & \times (-1/2)^{(l_1+l_2+l_3+l_4)} \int_0^L \mathrm{d}\eta \int_0^\infty \mathrm{d}\kappa\, \kappa^{-8/3} (b_5\kappa)^{2v+2r-2} [\exp(b_3\kappa^2) b_2^{r_1+r_1'} (-1)^{n'+m'} (1/D_{y_1})] \\ & \times [1 - {}_1F_2(v,1/2;v+r;-0.25R^2\gamma^2\kappa^2)] + \exp(b_6\kappa^2) b_2^{r_1} (b_2^{r_1'})^* [0.5 + 0.5\,{}_1F_2(v,1/2;v+r;R^2\gamma_i^2\kappa^2) \\ & - {}_1F_2(v,1/2;v+r;-0.25R^2\gamma^{*2}\kappa^2]/D_{y_2}) \Bigg\} \end{aligned} \tag{8}$$

where $D_{y_1}=[\Sigma_{(n,m)} H_n(0) H_m(0) b_2^{r_1}]^2$, $D_{y_2}=[\Sigma_{(n,m)} H_n(0) H_m(0) b_2^{r_1}][\Sigma_{(n,m)} H_n(0) H_m(0) (b_2^{r_1})^*]$, $T_{l_i}=1$ for $l_i=0$, $T_{l_i}=1\times 3\times \ldots (2l_i-1)$, for $l_i \neq 0$, $i=1,\ldots,4$, $v=(n+n'-2l_1-2l_2+1)/2$, $r=(m+m'-2l_3-2l_4+1)/2$, γ_i is the imaginary part of γ, B is the beta function, ${}_1F_2$ is the Hypergeometric function and $\binom{n}{2l_i}=n!/[(n-2l_i)!(2l_i)!]$, $i=1,\ldots,4$.

As the next development in Eq. (8), the integration over κ is to be accomplished. For this, we again benefit from the fact that $\mathbf{p}_2$ is too small (to be more specific, $|\mathbf{p}_2|=R \ll \ell_o$, i.e., $|\mathbf{p}_2|<100\,\mu$m, where ℓ_o is the inner scale of turbulence), hence ${}_1F_2$ can be expanded in series retaining only the first two terms. Using Eq. 3.478.1 of Ref. 12, we are able to reach the final expression for the mean square angle-of-arrival fluctuations. After the change of variable $\eta=Lt$ (here t represents the normalized link length) in Eq. (8) and by substituting Eq. (8) into Eq. (7), the mean square angle-of-arrival fluctuations arising from multimode laser radiation in the turbulent atmosphere will appear in the form of a single integral as

$$\begin{aligned} \langle \alpha_a^2 \rangle = {} & 0.051836 C_n^2 L^{5/6} k^{1/6} \Bigg\{ \sum_{(n,m)} \sum_{(n',m')} 2^{(n+n'+m+m')} \sum_{l_1}^{n/2} \sum_{l_2}^{n'/2} \sum_{l_3}^{m/2} \sum_{l_4}^{m'/2} T_{l_1} T_{l_2} T_{l_3} T_{l_4} \binom{n}{2l_1} \binom{n'}{2l_2} \binom{m}{2l_3} \binom{m'}{2l_4} B(v,r) \\ & \times \Gamma(v+r-5/6)(-1/2)^{(l_1+l_2+l_3+l_4)} \frac{2v}{v+r} (-1)^{2v+2r-2}\, \mathrm{Re}\Bigg\{ \int_0^1 \mathrm{d}t (1-t)^{v+r-7/6} [b_1/(1+b_1^2)]^{v+r-1} \gamma^{-v-r+17/6} \\ & \times b_2^{r_1+r_1'} j^{-(v+r-5/6)} (-1)^{n'+m'}/D_{y_1} + \int_0^1 \mathrm{d}t (1-t)^{-1/3} [b_1/(1+b_1^2)]^{-1/6} [(1+t^2 b_1^2)/(1+b_1^2)] \times b_2^{r_1} (b_2^{r_1'})^*/D_{y_2} \Bigg\} \Bigg\}, \end{aligned} \tag{9}$$

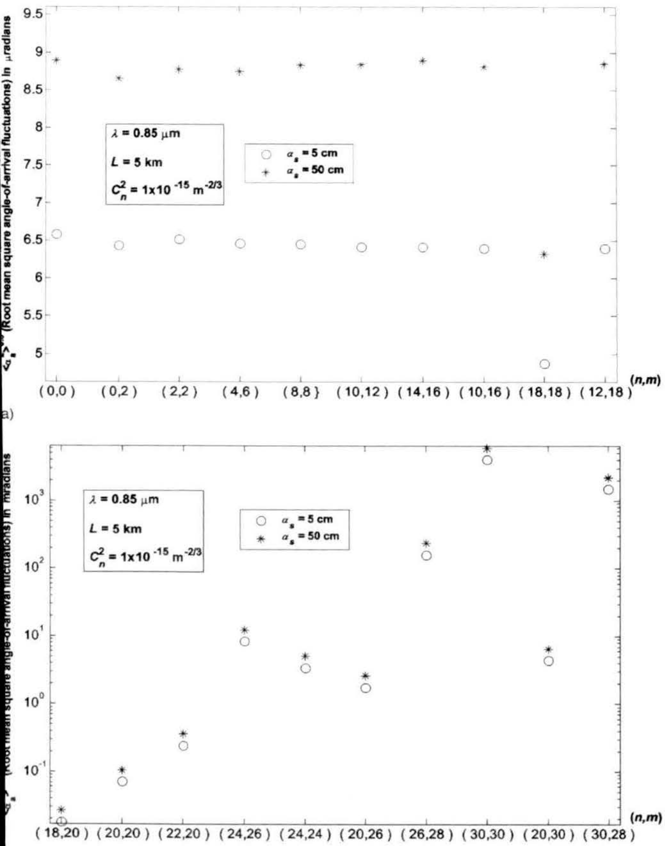

Fig. 2 Root mean square angle-of-arrival fluctuations for single even modes when (a) $(n,m)<20$ and (b) $(n,m)>20$.

where Γ is Gamma function, and $b_1 = L/k\alpha_s^2$.

3 Results and Discussion

Mean square angle-of-arrival fluctuations arising from multimode laser radiation in the turbulent atmosphere are numerically evaluated using Eq. (9). Our results are based on the operational parameters of currently used FSO links. First, Figs. 2(a) and 2(b) provide the variation of the root mean square angle-of-arrival fluctuations, that is, $\langle \alpha_a^2 \rangle^{0.5}$, for certain single even modes including TEM_{00} and higher order modes. In this context, Fig. 2(a) covers the single mode indices, $(n,m)<20$, whereas Fig. 2(b) is for mode indices $(n,m)>20$. Here wavelength, structure constant, and the link length are commonly taken as $\lambda = 0.85$ μm, $C_n^2 = 10^{-15}\,\mathrm{m}^{-2/3}$, and $L = 5$ km. The horizontal axis in these figures shows the order of the single mode. The set of results are according to two different source sizes, i.e., $\alpha_s = 5$ cm and $\alpha_s = 50$ cm. It is seen that when $(n,m)<20$, the root mean square (rms) angle-of-arrival fluctuations are in the order of few microradians and stay nearly the same as the Gaussian beam (TEM_{00}) source case if the laser source size is kept constant. However, angle-of-arrival fluctuations increase with growing source size. Note that the phase correlation function, and thus the phase structure function are functions of the source size, that make the angle-of-arrival fluctuations dependent on the source size, increasing for larger sized sources as in the single fundamental mode

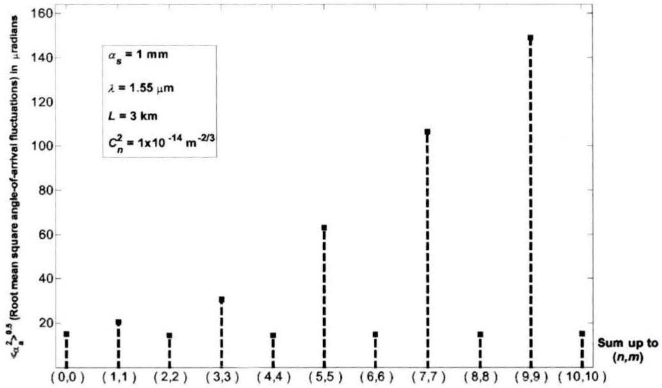

Fig. 3 The rms angle-of-arrival fluctuations for summation of modes from (0,0) up to (10,10).

case.[13] The rms angle-of-arrival fluctuations are determined for R (where R is the radial distance from the origin on the receiver plane at which the angle-of-arrival fluctuations are evaluated) much smaller than the inner scale of turbulence that makes our evaluations valid on the propagation axis, so the receiver aperture dependence on the rms angle-of-arrival fluctuations are not incorporated in our results. To plot Fig. 2(b), we used mode indices of $(n,m)>20$, termed extremely high order modes, and added the single-mode indices of (18,20) for reference. Here we see that angle-of-arrival fluctuations increase dramatically, reaching the radian levels. To better appreciate the disproportionate levels in the angle-of-arrival fluctuations of different single modes, the vertical axis in Fig. 2(b) was converted to the logarithmic scale. From Fig. 2(b), it is obvious that source size dependence of angle-of-arrival fluctuations has diminished; instead mode indices now dominate the magnitude of angle-of-arrival fluctuations. Note that for the single-mode case as displayed in Figs. 2(a) and 2(b), we are unable to calculate $\langle \alpha_a^2 \rangle^{0.5}$ when n or m is odd, since inserting single odd-mode index values in Eq. (9) will lead to the terms D_{y_1} or D_{y_2} [whose definitions appear immediately after Eq. (8)] becoming zero, hence $\langle \alpha_a^2 \rangle$ goes to infinity. This dilemma is resolved, however, when the relevant summation for D_{y_1} or D_{y_2} contains at least one even mode. This is demonstrated in the subsequent plots.

Figure 3 shows the rms angle-of-arrival fluctuations when multimode contents are selected. The (n,m) values placed on the horizontal axis of Fig. 3 indicate the summation of modes from (0,0) up to (n,m) including all the mode combinations that lie in between. For example, (10,10) on the horizontal axis of Fig. 3 represents the multimode formed by $\Sigma_0^{10}\Sigma_0^{10}(n,m)$. The values are obtained for a small size source with $\alpha_s = 1$ mm and the link parameters are chosen as $\lambda = 1.55$ μm, $C_n^2 = 10^{-14}\,\mathrm{m}^{-2/3}$, and $L = 3$ km. We observe that the mean square angle-of-arrival fluctuations remain nearly the same as the fundamental mode when the mode summation in the multimode content ends with a mode that has even numbers for both indices. However, the mean square angle-of-arrival fluctuations become relatively larger when the mode summation in the multimode content terminates with a mode that has odd

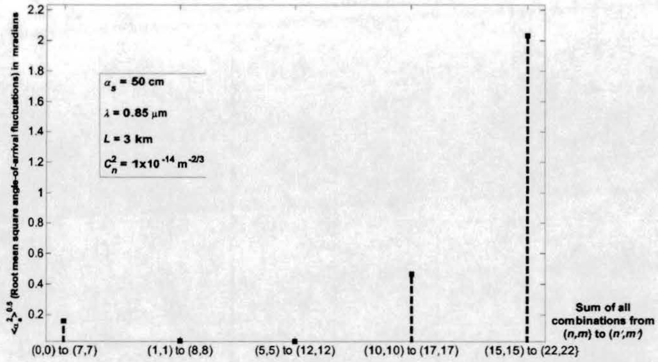

Fig. 4 The rms angle-of-arrival fluctuations for the sum of all combinations of modes starting from the order (n,m) up to order (n',m'), whose numerals are shown on the horizontal axis.

numbers for both indices. This is attributed to the fact that D_{y_1} and D_{y_2} summations will possess one less term when the last mode combination is odd, thus making the numeric magnitude of D_{y_1} and D_{y_2} smaller than the case of mode combination ending with even modes. Consequently $\langle \alpha_a^2 \rangle$ will grow larger for mode combinations ending with odd modes since Eq. (9) entails D_{y_1} and D_{y_2} summations in the denominator. The angle-of-arrival fluctuations in Fig. 3 are in the range of several tens of microradians. Note that fluctuation values of Fig. 3 are acquired using higher turbulence levels than those of Figs. 2(a) and 2(b).

We finally investigate the behavior of mean square angle-of-arrival fluctuations for multimode contents composed of summation of modes with larger indices. These cases are illustrated in Fig. 4 where the horizontal axis represents the sum of all combinations of modes starting from the order (n,m) up to order (n',m'). For example, the point [(15,15) to (22,22)] on the horizontal axis of Fig. 4 refers to the multimode formed by $\Sigma_{15}^{22}\Sigma_{15}^{22}(n,m)$. The values in Fig. 4 are obtained for $\alpha_s = 50\,\text{cm}$ and the link parameters are chosen as $\lambda = 0.85$ μm, $C_n^2 = 10^{-14}\,\text{m}^{-2/3}$, and $L = 3$ km. We observe from Fig. 4 that like the single-mode cases, when the multimode content consists of relatively high order modes, a substantial increase of the mean square angle-of-arrival fluctuations can occur, attaining values beyond few milliradians. Unfortunately, due to excessive computation time required, it is not possible to extend the summation of multimode contents to indices beyond those shown in Fig. 4. Note here that the turbulence-induced angle-of-arrival fluctuations in a multimode beam exhibit wavelength dependence via parameters b_1, b_2, D_{y_1}, and D_{y_2} in Eq. (9). For a Gaussian beam wave (fundamental single mode limit) it is known that the angle-of-arrival fluctuations still exhibit wavelength dependence, however, in the case of plane and spherical wave limits, angle-of-arrival fluctuations become independent of wavelength when the receiver aperture diameter is much larger than the Fresnel zone.[13]

4 Conclusions

We formulated the mean square angle-of-arrival fluctuations of a multimode laser radiation in a turbulent atmosphere. The results we obtained are for the practical operational parameters of currently used FSO links. In this respect, the two widely used wavelengths of 0.85 and 1.55 μm with link spans up to 5 km were covered. The turbulence levels considered are mainly in the weak regime. However, relatively stronger turbulence levels were also considered and it is observed (as seen in Fig. 4) that for multimode contents composed of large number of higher order modes, mean square angle-of-arrival fluctuations can be effective particularly for stronger turbulence levels.

In our formulation, multimode content is represented by the coherent addition of Hermite-Gaussian beams. A laser operating in a single transverse mode exhibits complete spatial coherence. When the laser operates in multimode, the spatial coherence of a beam is seriously affected.[14,15] In this paper, we made an approximation for real sources by assuming that the modes have the same phase at the transmit aperture. This is an easier starting point to introduce the multimode effect. For more realistic sources, random phases should be incorporated for each mode in the total multimode content. The random phase for each mode will also mimic spatial partial source coherence and we intend to investigate the effect of the random phases of the modes on the angle-of-arrival fluctuations in our future work. It seems possible with our formulation to include a partial coherence effect by using computer simulation of random phases and numerically evaluating the angle-of-arrival fluctuations. Although the inclusion of a large number of higher order modes mimics an incoherent source, a proper incoherent representation is possible only by incorporating the random phase property in the formulation starting from source excitation. Also, as noted in the previous section, our formulation fails to account for odd single modes and shows that angle-of-arrival fluctuations tend to increase for mode combinations ending with odd-order modes. Our evaluations for the multimode angle-of-arrival fluctuations are found in general to be of the order of several tens of microradians except for the cases when the mode group terminates with an odd mode or when the extremely higher order modes are present. It is known that the field of view of FSO receivers are of the order of milliradians. Thus, comparing our results of the multimode angle-of-arrival fluctuations (of the order of several tens of microradians) with the field of view of a practical FSO receiver (which is several milliradians), we conclude that the mean square angle-of-arrival fluctuations will not heavily influence the FSO link performance, unless the mode group terminates with an odd mode or when the extremely high order modes are involved in the source excitation. Note that our analysis in this paper considers the angle-of-arrival fluctuations in the vicinity of origin of the receiver plane since the evaluations are made for the R (radial distance from the origin on the receiver plane) much smaller than the inner scale of turbulence, i.e., on-axis angle-of-arrival fluctuations are calculated. For single odd modes, the complex field at the origin of the receiver plane tends to diminish. This gives rise to zero terms in both the numerator and denominator of the phase structure function since odd-order Hermite polynomials evaluated at zero yield a numerical value of zero. To be more specific mathematically, the denominator terms in the integrand of Eq. (9) (i.e., D_{y_1} and D_{y_2}) represent the sum of on-axis $[(p_x,p_y)=0]$ values of Hermite combinations, i.e., Hermite polynomials evaluated at the origin of the receiver plane. Thus, mode combinations ending with an odd mode contributes nothing to the D_{y_1} and D_{y_2} parameters, but a finite positive quantity is added to the numerator parameters via the sums $\Sigma_{(n,m)}$ and $\Sigma_{(n',m')}$. Inevitably, the angle-of-arrival fluctuations tend to rise in such cases. The significance and the physical interpretation of this matter are to yet be explored experimentally. These subjects will further be scrutinized and elaborated in our subsequent publications.

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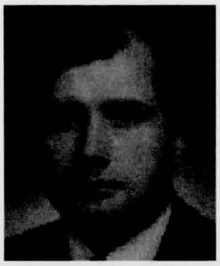

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