RAMSEIAN PARTITIONS AND WEIGHTED STABILITY IN GRAPHS

I. E. ZVEROVICH AND I. I. ZVEROVICH

ABSTRACT. Let $\mathcal{P}$ and $\mathcal{Q}$ be hereditary classes of graphs. A class of graphs $\mathcal{C}$ is called *α-bounded* (respectively, *ω-bounded*) if there exists a constant C such that $\alpha(G) \leq C$ (respectively, $\omega(G) \leq C$) for each graph $G \in \mathcal{C}$. Here $\alpha(G)$ and $\omega(G)$ are the stability number and the clique number of G, respectively.
The ordered pair $(\mathcal{P}, \mathcal{Q})$ is called *Ramseian* if both $\mathcal{P}$ and $\mathcal{Q}$ are polynomially recognizable, $\mathcal{P}$ is α-bounded, and $\mathcal{Q}$ is ω-bounded. Let $(\mathcal{P}, \mathcal{Q})$ be an ordered pair of graph classes. We denote by $\mathcal{P} * \mathcal{Q}$ the class of all graphs G such that there exists a partition $A \cup B = V(G)$ with $G(A) \in \mathcal{P}$ and $G(B) \in \mathcal{Q}$, where $G(X)$ denotes the subgraph of G induced by a set $X \subseteq V(G)$.
A class of graphs $\mathcal{C}$ is called *α_w-polynomial* if there exists a polynomial-time algorithm to calcule the weighted stability number $\alpha_w(G)$ for all graphs $G \in \mathcal{C}$. A class of graphs $\mathcal{C}$ is called *α_w-complete* if the corresponding decision problem is NP-complete for graphs in $\mathcal{C}$.
Our main results are the following theorems.

Theorem. *If $(\mathcal{P}, \mathcal{Q})$ is a Ramseian pair, then the class $\mathcal{P} * \mathcal{Q}$ is polynomially recognizible. Moreover, given a graph $G \in \mathcal{P} * \mathcal{Q}$, it is possible to construct a Ramseian $(\mathcal{P}, \mathcal{Q})$-partition for G in polynomial time.*

Theorem. *Let $(\mathcal{P}, \mathcal{Q})$ be a Ramseian pair.*
(i) *If $\mathcal{Q}$ is an α_w-polynomial class, then the class $\mathcal{P} * \mathcal{Q}$ is also α_w-polynomial.*
(ii) *If $\mathcal{Q}$ is an α_w-complete class, then the class $\mathcal{P} * \mathcal{Q}$ is also α_w-complete.*

Similar results for ω_w-polynomial classes and ω_w-complete classes easily follow, where $\omega_w(G)$ is the weighted clique number of a graph G. Finally, a recent result of Alekseev and Lozin (2002) is a particular case of our theorem.
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Let $G = (V, E)$ be a graph. A set $S \subseteq V$ is called a *stable set* if it induces an edgeless subgraph. The largest cardinality of a stable set in G is the *stability number*, $\alpha(G)$, of G. A set $K \subseteq V$ is called a *clique* if it induces

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a complete subgraph. The largest cardinality of a clique in G is the *clique number* of G, denoted by $\omega(G)$.

Definition 1. *A class of graphs $\mathcal{C}$ is α-*bounded *if there exists a constant M such that $\alpha(G) \leq M$ for each graph in $G \in \mathcal{C}$. Similarly, $\mathcal{C}$ is ω-*bounded *if there exists a constant N such that $\omega(G) \leq N$ for each graph in $G \in \mathcal{C}$.*

A class of graphs $\mathcal{C}$ is *polynomially recognizible* if there exists a polynomial-time algorithm for recognizing graphs in $\mathcal{C}$. Let $\mathrm{ISub}(G)$ be the set of all induced subgraphs of a graph G considered up to isomorphism. A class of graphs $\mathcal{C}$ is *hereditary* if $\mathrm{ISub}(G) \subseteq \mathcal{P}$ for each graph $G \in \mathcal{P}$.

Definition 2. *Let $\mathcal{P}$ and $\mathcal{Q}$ be hereditary classes of graphs. The ordered pair $(\mathcal{P}, \mathcal{Q})$ is called* Ramseian *if*

(R1): *both $\mathcal{P}$ and $\mathcal{Q}$ are polynomially recognizible, and*
(R2): *$\mathcal{P}$ is α-bounded, and $\mathcal{Q}$ is ω-bounded.*

Let $G(X)$ denote subgraph of a graph G induced by a set $X \subseteq V(G)$.

Definition 3. *Let $(\mathcal{P}, \mathcal{Q})$ be an ordered pair of graph classes. We denote by $\mathcal{P} * \mathcal{Q}$ the class of all graphs G such that there exists a partition $A \cup B = V(G)$ with*

- *$G(A) \in \mathcal{P}$, and*
- *$G(B) \in \mathcal{Q}$.*

If $(\mathcal{P}, \mathcal{Q})$ is a Ramseian pair then the ordered pair (A, B) above is called a Ramseian $(\mathcal{P}, \mathcal{Q})$-partition.

The most general result on Ramseian partitions was obtained by Zverovich [2 as a consequence of a general theorem on bipartite bihypergraphs. A hereditary class $\mathcal{C}$ of graphs is called *finitely generated* if the number of minimal forbidden induced subgraphs for $\mathcal{C}$ is finite.

Theorem 1 (Zverovich [2]). *Let $(\mathcal{P}, \mathcal{Q})$ be a Ramseian pair such that both $\mathcal{P}$ and $\mathcal{Q}$ are finitely generated hereditary classes. Then the hereditary class $\mathcal{P} * \mathcal{Q}$ is also finitely generated.*

Obviously, if a hereditary class is finitely generated, then it is polynomially recognizible. The converse is not true. However, we can prove an analogue of Theorem 1 for all Ramseian pairs. We do not require the classes $\mathcal{P}$ and $\mathcal{Q}$ to be finitely generated, i.e., we consider a more general situation. But the conclusion that the class $\mathcal{P} * \mathcal{P}$ is polynomially recognizible is weaker than in Theorem 1.

Theorem 2. *If $(\mathcal{P}, \mathcal{Q})$ is a Ramseian pair, then the class $\mathcal{P} * \mathcal{Q}$ is polynomially recognizible. Moreover, given a graph $G \in \mathcal{P} * \mathcal{Q}$, it is possible to find a Ramseian $(\mathcal{P}, \mathcal{Q})$-partition for G in polynomial time.*

Proof. Let G be a graph. Suppose that we have disjoint sets $A_k, B_k \subseteq V(G)$, $k \in \{0, 1, \ldots, |V(G)|\}$, such that $G(A_k) \in \mathcal{P}$ and $G(B_k) \in \mathcal{Q}$. Initially we may put $k = 0$ and $A_0 = B_0 = \emptyset$. Recall that, by (R1), membership in $\mathcal{P}$ and $\mathcal{Q}$ can be tested in polynomial time. Also, it follows from (R2), that there exist constants M and N such that

- $\alpha(H) < M$ for each $H \in \mathcal{P}$, and
- $\omega(H) < N$ for each $H \in \mathcal{Q}$.

If $A_k \cup B_k = V(G)$ then (A_k, B_k) is a Ramseian $(\mathcal{P}, \mathcal{Q})$-partition for G, and therefore $G \in \mathcal{P} * \mathcal{Q}$. Thus, we may assume that $A_k \cup B_k \neq V(G)$. We shall extend (A_k, B_k) to a pair (A_{k+1}, B_{k+1}) that covers more vertices, or discover that there are no such pairs. The latter situation will imply that G has no a Ramseian $(\mathcal{P}, \mathcal{Q})$-partition.

If there exists a Ramseian $(\mathcal{P}, \mathcal{Q})$-partition (A, B) for G, then

- $G(A \cap B_k) \in \mathcal{P}$ [since $A \cap B_k \subseteq A$, $G(A) \in \mathcal{P}$, and $\mathcal{P}$ is a hereditary class], and
- $G(A \cap B_k) \in \mathcal{Q}$ [since $A \cap B_k \subseteq B_k$, $G(B_k) \in \mathcal{Q}$, and $\mathcal{Q}$ is a hereditary class].

It follows that, $G(A \cap B_k) \in \mathcal{P} \cap \mathcal{Q}$. Similarly, $G(B \cap A_k) \in \mathcal{P} \cap \mathcal{Q}$. By Ramsey's theorem, $|A \cap B_k| \leq r(M, N)$ and $|B \cap A_k| \leq r(M, N)$, where $r(M, N)$ is the Ramsey number.

Now we perform the following total search.

Algorithm 1 (Total Search for (A_k, B_k)).

Step 1. *Generate all pairs of sets $X \subseteq A_k$ and $Y \subseteq B_k$ with $|X| \leq r(M, N)$ and $|Y| \leq r(M, N)$.*

Step 2. *For each pair (X, Y), set $A'_k = (A_k \setminus X) \cup Y$ and $B'_k = (B_k \setminus Y) \cup X$.*

Step 3. *If both $G(A'_k) \in \mathcal{P}$ and $G(B'_k) \in \mathcal{Q}$ hold, then choose an arbitrary vertex $u \notin A_k \cup B_k$, and*

- *check*
$$G(A'_k \cup \{u\}) \in \mathcal{P}; \tag{1}$$
- *if (1) holds, then we put $A_{k+1} = A'_k \cup \{u\}$ and $B_{k+1} = B'_k$, and* stop*;*
- *if (1) does not hold, then we check*
$$G(B'_k \cup \{u\}) \in \mathcal{Q}; \tag{2}$$
- *if (2) holds, then we put $A_{k+1} = A'_k$ and $B_{k+1} = B'_k \cup \{u\}$, and* stop*;*
- *if (2) does not hold, then we consider the next pair (X, Y), if any, or* stop.

Step 4. *If either $G(A'_k) \notin \mathcal{P}$ or $G(B'_k) \notin \mathcal{Q}$, then we consider the next pair (X, Y), if any, or* stop.

The algorithm either produces in polynomial time disjoint sets A_{k+1} and B_{k+1} with $G(A_{k+1}) \in \mathcal{P}$, $G(B_{k+1}) \in \mathcal{Q}$, and

$$|A_{k+1} \cup B_{k+1}| = |A_k \cup B_k| + 1,$$

or concludes that there are no such sets. In the former case we have either

- $A_{k+1} \cup B_{k+1} = V(G)$ and hence $G \in \mathcal{P} * \mathcal{Q}$, or
- $A_{k+1} \cup B_{k+1} \neq V(G)$ and we continue the search, i.e., we run Algorithm 1 for the pair (A_{k+1}, B_{k+1}).

The latter case implies that $G \notin \mathcal{P} * \mathcal{Q}$.

Since the search runs in polynomial time, and the maximum possible number of searches is $|V(G)|$, our algorithm for recognizing $\mathcal{P} * \mathcal{Q}$ [and constructing the corresponding partition] also runs in polynomial time. □

Suppose that G is a graph with non-negative weight function $w : V(G) \to [0, +\infty)$. As usual, the *weight of a subset* $X \subseteq V(G)$ is the sum of $w(x)$ taken over all $x \in X$. The *weighted stability number* $\alpha_w(G)$ of G is the maximum weight of a stable set in G.

Definition 4. *A class of graphs $\mathcal{C}$ is called α_w-*polynomial *if there exists a polynomial-time algorithm for calculation $\alpha_w(G)$ for all graphs $G \in \mathcal{C}$.*

*A class of graphs $\mathcal{C}$ is called α_w-*complete *if the corresponding decision problem is* NP*-complete for graphs in $\mathcal{C}$.*

The following theorem is our main result.

Theorem 3. *Let $(\mathcal{P}, \mathcal{Q})$ be a Ramseian pair.*

(i) *If $\mathcal{Q}$ is an α_w-polynomial class, then the class $\mathcal{P}*\mathcal{Q}$ is also α_w-polynomial.*

(ii) *If $\mathcal{Q}$ is an α_w-complete class, then the class $\mathcal{P} * \mathcal{Q}$ is also α_w-complete.*

Proof. (i) Let G be a graph in $\mathcal{P} * \mathcal{Q}$. By Theorem 2, we can construct a Ramseian partition $A \cup B = V(G)$ in polynomial time. Let us fix such a partition. Since $\mathcal{P}$ is an α-bounded class, we can generate in polynomial time the list L of all stable sets in $G(A)$, including the empty set.

For each stable set $S \in L$, we consider the subgraph $H = G(B) - N(S)$ [i.e., H is obtained from $G(B)$ by deleting all vertices that are adjacent to at least one vertex of S]. By definition, H is an induced subgraph of $G(B)$. Since $G(B) \in \mathcal{Q}$ and $\mathcal{Q}$ is a hereditary class, we have $H \in \mathcal{Q}$. The class $\mathcal{Q}$ is an α_w-polynomial, therefore we can find $\alpha_w(H)$ in polynomial time.

Now we define $a(S) = w(S) + \alpha_w(H)$. We put $a(L) = \max\{a(S) : S \in L\}$. Note that $a(L)$ can be found in polynomial time.

It remains to check that $a(L) = \alpha_w(G)$. Since $a(L)$ is the weight of a stable set in G, we have $a(L) \leq \alpha_w(G)$. Now let I be a maximum weight stable set in G. Clearly, the set $S = I \cap A$ belongs to L. The set $S' = I \setminus S$ is a stable set disjoint from A, i.e., $S' \subseteq B$. Moreover, no vertex of S' is adjacent to a vertex of S, since $I = S \cup S'$ is a stable set. Therefore S' is a stable set in $H = G(B) - N(S)$. It follows that

$$\alpha_w(G) = |I| = w(S) + w(S') \leq w(S) + \alpha_w(H) = a(L).$$

We have proved that $a(L) = \alpha_w(G)$.

(ii) The statement follows from the inclusion $\mathcal{Q} \subseteq \mathcal{P} * \mathcal{Q}$ and the condition that $\mathcal{Q}$ is an α_w-complete class. □

Let G be a graph and let w be a non-negative weight function. The *weighted clique number* $\omega_w(G)$ of G is the maximum weight of a clique in G.

Remark 1. a) *The condition that $\mathcal{P}$ and $\mathcal{Q}$ are polynomially recognizible classes is inessential in the statement* (ii) *of Theorem 3. However, if we omit this condition, the class $\mathcal{P} * \mathcal{Q}$ may not be polynomially recognizible.*

b) *A similar situation arises if we omit the condition that $\mathcal{P}$ is a polynomially recognizible class in the statement* (i) *of Theorem 3. In this case an easy modification of the proof is required.*

Definition 5. *A class of graphs $\mathcal{C}$ is called* ω_w-polynomial *if there exists a polynomial-time algorithm for calculation $\omega_w(G)$ for all graphs $G \in \mathcal{C}$.*

A class of graphs $\mathcal{C}$ is called ω_w-complete *if the corresponding decision problem is* NP*-complete for graphs in $\mathcal{C}$.*

Now we can state a "complementary" form of Theorem 3. For a class of graphs $\mathcal{C}$, its complement is defined as $\overline{\mathcal{C}} = \{\overline{G} : G \in \mathcal{C}\}$.

Corollary 1. *Let $(\mathcal{P}, \mathcal{Q})$ be a Ramseian pair.*

(i) *If $\mathcal{P}$ is an ω_w-polynomial class, then the class $\mathcal{P}*\mathcal{Q}$ is also ω_w-polynomial.*

(ii) *If $\mathcal{P}$ is an ω_w-complete class, then the class $\mathcal{P} * \mathcal{Q}$ is also ω_w-complete.*

Proof. It is clear that the complement of an α-bounded class (respectively, an ω-bounded class) is an ω-bounded class (respectively, an α-bounded class). Also, a class $\mathcal{C}$ is polynomially recognizible if and only if $\overline{\mathcal{C}}$ is polynomially recognizible. Thus, a pair $(\mathcal{P}, \mathcal{Q})$ is Ramseian if and only if the pair $(\overline{\mathcal{Q}}, \overline{\mathcal{P}})$ is Ramseian.

Finally, a class is α_w-polynomial (respectively, α_w-complete) if and only if its complement is ω_w-polynomial (respectively, ω_w-complete). Now the result follows from Theorem 3. □

Definition 6. *A graph G is called* (p,q)-colorable *if $V(G)$ can be partitioned into $p+q$ subsets*

$$(X_1 \cup X_2 \cup \cdots \cup X_p) \cup (Y_1 \cup Y_2 \cup \cdots \cup Y_q),$$

some of them may be empty, such that

- *each X_i induces a complete subgraph, and*
- *each Y_j is a stable set.*

We denote by $\mathcal{C}(p,q)$ be the class of all (p,q)-colorable graphs.

In other words, $\mathcal{C}(p,q) = \overline{\mathcal{C}}(p) * \mathcal{C}(q)$, where $\mathcal{C}(r)$ is the class of all r-colorable graphs, and $\overline{\mathcal{C}}(r)$ is its complement.

Corollary 2 (Alekseev and Lozin [1]).

(i) *If $q \leq 2$ then $\mathcal{C}(p,q)$ is an α_w-polynomial class.*

(ii) *If $q \geq 3$ then $\mathcal{C}(p,q)$ is an α_w-complete class.*

We can strengthen the statement (i) of Corollary 2. In fact, a more general class is α_w-polynomial, namely, $\mathcal{P}_p * \mathcal{C}(2)$, where $\mathcal{P}_p = \{G : \alpha(G) \leq p\}$.

Corollary 3. *The class $\mathcal{P}_p * \mathcal{C}(2)$ is α_w-polynomial.*

Proof. The class $\mathcal{C}(2)$ consists of all bipartite graphs, it is polynomially recognizible, ω-bounded and α_w-polynomial. The $\overline{\mathcal{C}}(p)$ class is contained in the polynomially recognizible α-bounded class $\mathcal{P}_p = \{G : \alpha(G) \leq p\}$. By Theorem 3(i), the class $\mathcal{P}_p * \mathcal{C}(2)$ is α_w-polynomial. It contains all the classes $\mathcal{C}(p,q)$ with $q \leq 2$. □

The statement (ii) of Corollary 2 is a particular case of Theorem 3(ii) [see also Remark 1], since the class $\mathcal{C}(q)$ for every $q \geq 3$ contains an α_w-complete class of all 3-colorable graphs.

References

[1] V. E. Alekseev and V. V. Lozin, Independent sets of maximum weight in (p,q)-colorable graphs, Discrete Math. **265** (1-3) (2003) 351–356

[2] I. E. Zverovich, r-Bounded k-complete bihypergraphs and generalized split graphs, Discrete Math. **247** (1-3) (2002) 225–244

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International Conference
TRENDS IN GEOMETRY - IN MEMORY OF BENIAMINO SEGRE
Roma, 7–9 June 2004

`http://www.mat.uniroma1.it/segre2004`
`http://www.lincei.it`; link: Centro Linceo Interdisciplinare

Last June, in the wonderful setting of Accademia dei Lincei's Palazzo Corsini and of the building of the Department of Mathematics of the Università di Roma "La Sapienza", a conference about the present panorama and future tendencies of research in Geometry took place with, as its leitmotiv, the remembrance of the great Italian mathematician Beniamino Segre.

The conference was jointly organised by the Accademia Nazionale dei Lincei, the Centro Linceo Interdisciplinare "B. Segre", the Gruppo Nazionale Strutture Algebriche, Geometriche e loro Applicazioni of the Istituto Nazionale di Alta Matematica "F. Severi", and the University of Rome "La Sapienza" to celebrate the centenary of the birth of Beniamino Segre (1903–1977). The conference was funded by the organising bodies, as well as by the Department of Mathematics "G. Castelnuovo" and the Department of Mathematical Methods and Models of the University of Rome "La Sapienza", by the Cofin [national research groups] "Geometria delle varietà differenziabili", "Gruppi, grafi e geometrie", "Spazi di moduli e teorie di Lie", and by the Banca Monte dei Paschi di Siena - Gruppo MPS.

The Scientific Committee consisted of Enrico Arbarello, Pier Vittorio Ceccherini, Corrado De Concini, Dina Ghinelli, Stefano Marchiafava, Paolo Maroscia, Claudio Procesi, and Edoardo Vesentini. The Organising Committee consisted of P.V. Ceccherini, Daniele A. Gewurz, D. Ghinelli, S. Marchiafava, and Francesca Merola.

More than 160 mathematicians from over ten countries attended the conference.

The conference began in the Sala delle Adunanze of the Accademia dei Lincei in Palazzo Corsini, with a solemn opening session in which Maurizio Brunori, Director of the Centro Linceo Interdisciplinare, described the activities and scientific relevance of the Centro, founded by Beniamino Segre and which now bears his name. A talk by Edoardo Vesentini followed, emphasising the human and scientific character of Beniamino Segre, the diversity of geometrical themes he developed, and the importance of his results for the evolution of geometry and mathematics in Italy. The conference continued at the Accademia all day long, with talks by Marcel Berger, Dieter Jungnickel, Phillip Griffiths, Joseph A. Thas, and Edoardo Sernesi.

The social programme consisted of daily lunchtime buffets and evening guided tours of Rome. In particular, on the first evening the participants were shown the "Villa la Farnesina", part of the Lincei buildings, which boasts frescoes by Raffaello and Giulio Romano.

Four sessions followed on 8 and 9 June at the Department of Mathematics "G. Castelnuovo" of the University of Roma "La Sapienza": after the address by Lucio Boccardo, Head of Department, we had talks by John H. Conway, Joseph Zaks, Gudlaugur Thorbergsson, Nicholas I. Shepherd-Barron, Giuseppe Tomassini, Mikhael Gromov, Peter J. Cameron, Gábor Korchmáros, Yuri I. Manin, James W.P. Hirschfeld.

Concluding the scientific proceedings, several speakers introduced by Giacomo Saban gave a series of personal recollections about Beniamino Segre as a scientist and as a man.

The speakers, distinguished mathematicians of world-wide reknown, have shown the state of the art of some of the most advanced research in Geometry, with special attention to its main fields, those to whose development Beniamino Segre gave contributions of the utmost importance. More precisely, recent developments in the three main branches of Geometry–Algebraic Geometry, Differential Geometry, and Combinatorial Geometry–have been covered in a unitary vision of Mathematics and its applications. In particular, the talks by many speakers have explicitly and directly referred to important results by B. Segre, seen as anticipating later developments in the fields of algebraic, differential, and combinatorial geometry; in the latter field Segre was a true pioneer.

A .pdf booklet, available online, contains the abstracts of the talks given at the conference, whose proceedings are forthcoming.

The chance of discussing, in a single session, subjects of apparently different and far away areas was a characteristic feature of this conference and has been one of the reasons of its remarkable success.

[Report by the organising committee.]

Here is the complete list of the speakers and their talks.

- Marcel Berger (IHÉS, Bures-sur-Yvette, France), *Introducing dynamics in elementary geometry: introduction to some work of Richard Schwartz*
- Peter J. Cameron (Queen Mary, University of London, London, UK), *Finite geometry and permutation groups: some polynomial links*
- John H. Conway (Princeton University, USA), *Some things you can't hear the shape of*
- Phillip Griffiths (IAS, Princeton, USA), *Algebraic cycles and singularities of normal functions*
- Mikhael Gromov (IHÉS, Bures-sur-Yvette, France), *Geometry of infinite Cartesian powers and related spaces*

- James W.P. Hirschfeld (University of Sussex, Brighton, UK), *The number of points on a curve, and applications*

- Dieter Jungnickel (Universität Augsburg, Augsburg, Germany), *Some geometric aspects of Abelian groups*

- Gábor Korchmáros (Università della Basilicata, Potenza, Italy), *Segre-type theorems in finite geometry*

- Yuri I. Manin (Max Planck Institute of Mathematics, Bonn, Germany), *Manifolds with multiplication in tangent bundle*

- Edoardo Sernesi (Università degli Studi Roma Tre, Roma, Italy), *Segre's work on curves and their moduli*

- Nicholas I. Shepherd-Barron (University of Cambridge, Cambridge, UK), *Cubic surfaces and rationality*

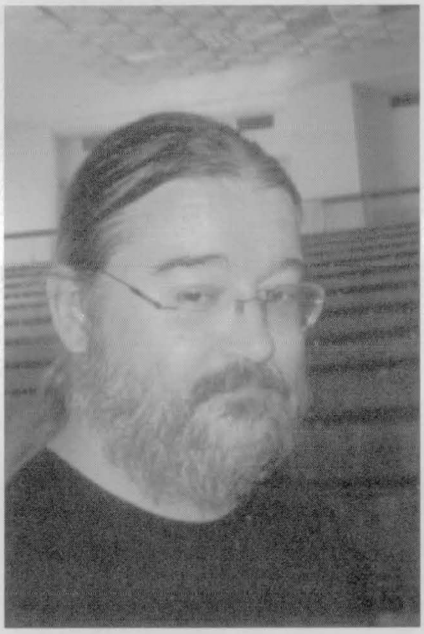
- Joseph A. Thas (Universiteit Gent, Gent, Belgium), *Finite geometries: classical problems and recent developments*

- Gudlaugur Thorbergsson (Universität zu Köln, Köln, Germany), *Transformation groups and submanifold geometry*

- Giuseppe Tomassini (Scuola Normale Superiore, Pisa, Italy), *Extension problems in complex geometry*

- Edoardo Vesentini (Accademia Nazionale dei Lincei, Italy), *Beniamino Segre and Italian geometry*

- Joseph Zaks (University of Haifa, Haifa, Israel), *Geometric graphs and the Beckman-Quarles Theorem*



upper left: Joseph Zaks upper right: Edoardo Sernesi
lower left: Tim Penttila lower right: Marcel Berger &Mikhael Gromov
(pictures by the organising committee)

Internationl Conference
Combinatorics 04
Capomulini, Catania, Italy *

The international conference *"Combinatorics 04"* was held in Capomulini (Catania, Sicily) the week of September 13-18, 2004. It followed the previous combinatorial conferences that are held in Italy from 1981 in Rome (as forerunner) and, after 1982, in every even numbered years [Passo della Mendola 82, Bari 84, Passo della Medola 86, Ravello 88, Gaeta 90, Acireale 92, Pescara 94, Assisi 96, Palermo 98, Gaeta 00, Maratea 02]. The conference was organized by the combinatorial group of Catania University, namely Mario Gionfriddo, Angelo Lizzio, Salvatore Milici, Lorenzo Milazzo, Lucia Gionfriddo, and Alberto Amato. Observe that, from few months, this group have formed an official combinatorial association called "Catania Combinatorics", which have officially organized the conference. The conference was run under the auspices of the Italian national research project *"Strutture geometriche, Combinatorica e loro Applicazioni"*, directed by Guglielmo Lunardon.

The conference was attended by approximately 230 participants, with 35 accompanying persons. The talks dealt with many different areas of combinatorics. There were 19 invited speakers at the conference. They are:

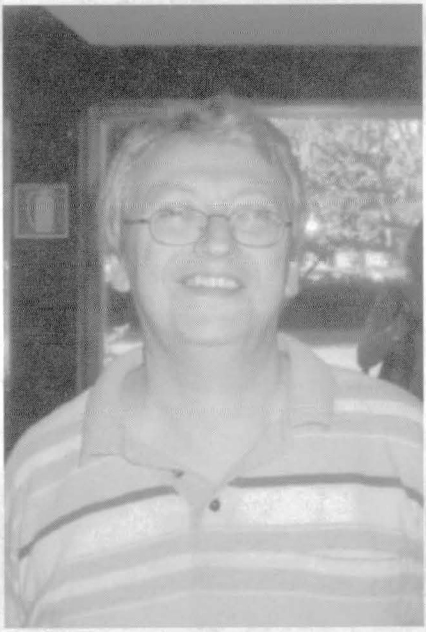
Brian Alspach (Regina, Canada)
Simeon Ball (Barcelona, Spain)
Liz Billington (Queensland, Australia)
Aart Blokhuis (Eindhoven, Holland)
Matthew Brown (Adelaide, Australia)
Marco Buratti (Perugia, Italy)
Domenico Cantone (Catania, Italy)
Charles Colbourn (Arizona, USA)
Antonio Cossidente (Potenza, Italy)
Anthony Hilton (Reading, England)
Masakazu Jimbo (Keio, Japan)
Dieter Jungnickel (Augsburg, Germany)
Alexander Kreuzer (Hamburg, Germany)
Udo Ott (Braunschweig, Germany)
Antonio Pasini (Siena, Italy)
Chris Rodger (Alabama, USA)
Alex Rosa (Hamilton, Canada)
S. Simic (Beograd, Serbia-Montenegro)
Leo Storme (Gent, Belgium)
Vitaly Voloshin (Alabama, USA)

All the invited lectures will be published in a special volume, edited by *"Catania Combinatorics"*.

You can see other news about the conference, including many photographs, on the website: www.dmi.unict.it/combinatorics04.

The Italian group of Combinatorics has announced that the next conference of the series will be held in the Isle of Ischia, near Naples, in 2006. See you at *"Combinatoics 06"*.

* Pictures and report by Mario Gionfriddo

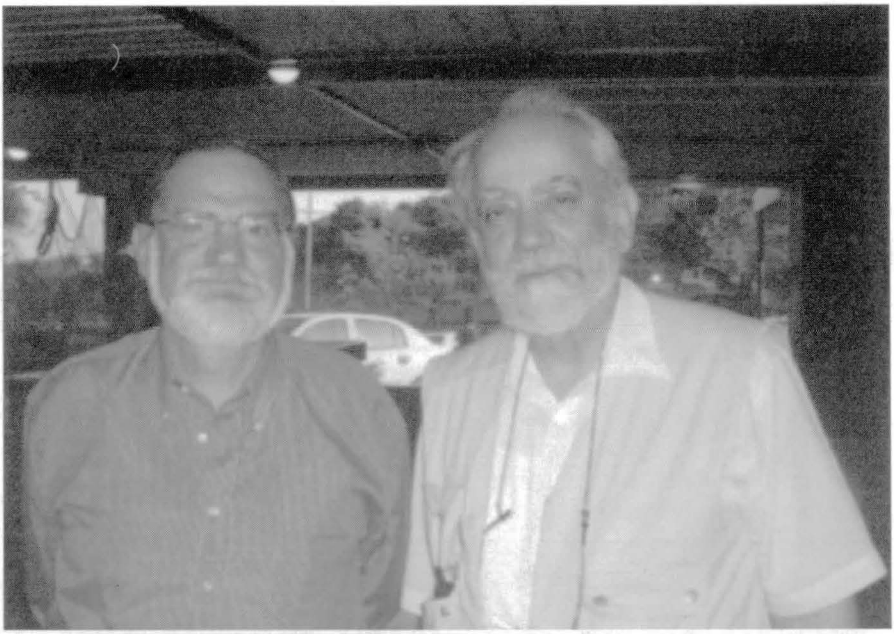


upper left: Salvatore Milici
lower left: Curt Lindner

upper right: Elizabeth Billington
lower right: Joe Lauri

upper: Lucia Gionfriddo and Angelo Lizzio
lower: Mario Gionfriddo and Guglielmo Lunardon

upper: Chris Rodger and Vitaly Voloshin
lower: Arrigio Bonisoli and Gaetano Quattrocchi



upper: Mario Gionfriddo and PierVittorio Ceccherini
lower: Gaetano Quattrocchi, Yanxun Chang and Gianfranco Lo Faro

The 48th Annual Meeting of the Australian Mathematical Society *

The 48th Annual Meeting of the Australian Mathematical Society took place at the Royal Melbourne Institute of Technology University, September 28 to October 1, 2004. Professor Kathy Horadam was the Director and Chair; she also served on the local committee along with Andrew Eberhard and Asha Rao. There were special sessions on Combinatorics (co-sponsored by the Combinatorial Mathematics Society of Australia and organised by Brendan McKay and Asha Rao) and on Cryptography and Coding, organized by Lynn Batten and Ed Dawson. We list the talks from these special sessions, along with a few others.

Registration took place on Monday afternoon and Tuesday morning. The hospitality at RMIT was outstanding; the morning and afternoon tea/coffee sessions provided delicious munchies, along with fresh fruit and, more importantly, the chance to mingle with friends and colleagues. The lunch period was an opportunity to sample the offerings of local cafés; there must be over a hundred in the immediate vicinity.

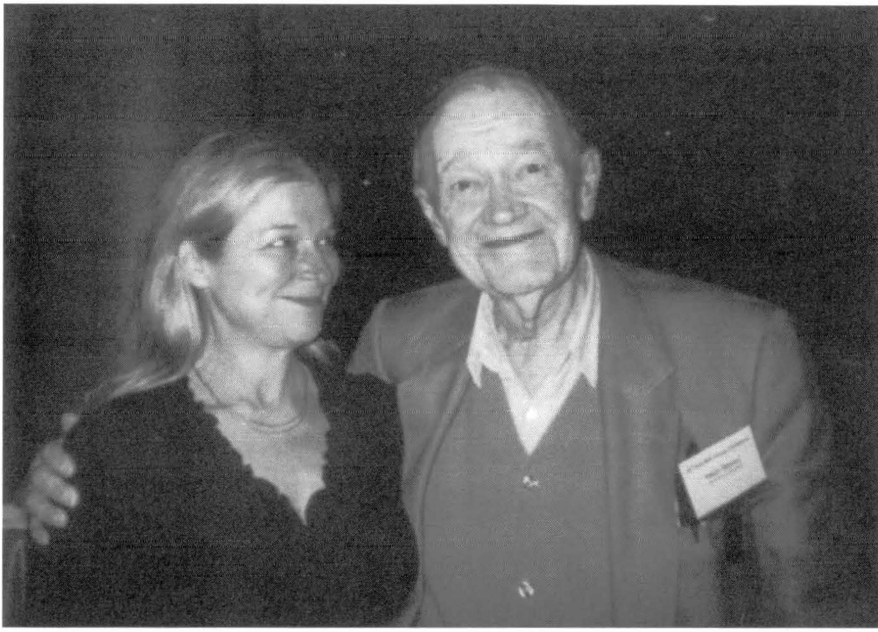
The conference banquet was held in the Melbourne Aquarium on Thursday evening. Food, drink, and fellowship were present in abundance.

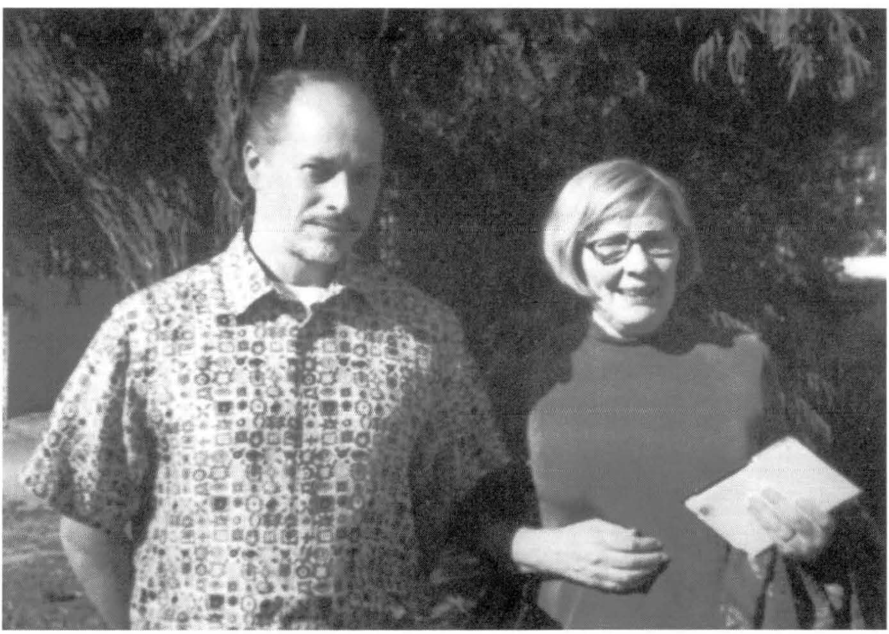
Over-all, this was a superbly organized conference and we owe a hearty vote of thanks to Kathy Horadam, Andrew Eberhard and Asha Rao.

* reported by E.A. Ruet d'Auteuil

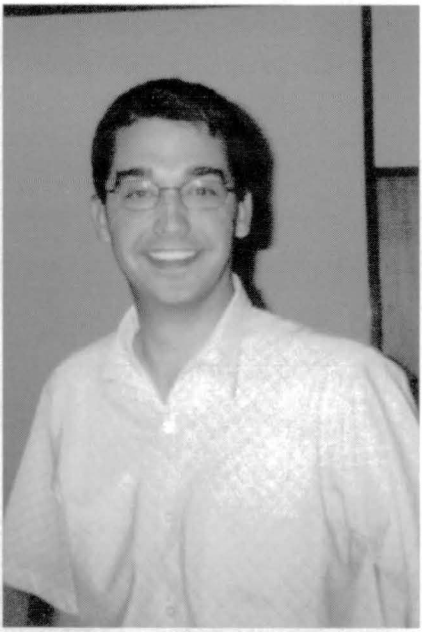
Yousef Bani-Hammad *ALI primality testing*
Juanma Gonzalez-Nieto *Provable security for protocols in the Canetti-Krawczyk model*
Liam Wagner *The design and maintenance of secure communication networks*
Amitabh Saxena *An authentication method for mobile agents with signature chaining using non-commutative associative one-way functions*
Neil Sloane *The on-line encyclopedia of integer sequences or, confessions of a sequence addict*
Rei Safavi-Naini *Deletion correction codes*
Kristine Lally *Algebraic lower bounds on the free distance of a convolutional code*
Ed Dawson *The LILI stream cipher - is it still secure?*
Lynn Batten *Algebraic attacks on stream ciphers*
Jennie Seberry *Space-time block codes*
James Parkinson *Averaging operators on affine buildings*
Mark Fackrell *Minimum and maximum algebraic degree of phase-type distributions*

Jonathan A. Cohen *Unlocking the structure of large base groups*
Rosemary Bailey *Association schemes and their products*
K.A. Sugeng *On adjacency matrices of edge-antimagic-vertex graphs*
Nimalsiri Pinnawala *Butson Hadamard matrices and cocyclic codes over* Z_n
Owen Jones *Self-similar processes via the crossing tree*
Cheryl E. Praeger *Limits of finite vertex-primitive graphs*
Stephen Lack *Pure-braided monoidal categories*
Simon McNichol *Digital fingerprinting, a wide attack*
David van Golstein Brouwers *Totally Goldbach numbers*
Geoffrey Pearce *Transitive factorizations of graphs*
Geoff Prince *Congruence design for second order ODE's*
Ralph G. Stanton *A class of minimal pairwise balanced designs*
John Bamberg *Transitive m-systems*
Michael Giudici *Locally 2-arc transitive graphs, homogenous factorisations and partial linear spaces*
Catherine Greenhill *Proving results in Ramsey theory by flipping a coin*
Neil Sloane *The grand tour in four dimensions and other other mysteries*
David Glynn *Quantum codes: connections with graphs and geometry*
Stephen Howe *Recognising the finite symmetric groups acting as matrix groups*
Bolis Basit *Asymptotic behaviour of orbits of* C_0*-semi-groups and solutions of the abstract Cauchy problem*
Peter Taylor *AMSI and ICE-EM - Opportunities for teachers*
Marcel Nicolau *Complex structures on compact Lie groups*
Stephen Hyde *Tilings of the hyperbolic plane and networks*
Don Taylor *Complex reflection groups and root systems*
Lou Caccetta *Optimization using branch and cut method*
Adil M. Bagirov *A derivative-free algorithm for large nonsmooth optimization*
Lou Caccetta *Industrial modelling and optimization*
M.A. Nyblom *An example of a non-homogenous spectrum*
Ian T. Roberts *On the finiteness of antichains in product posets*
Andrew Eberhard *Some general sufficiency conditions for optimality in nonsmooth analysis obtained by via double envelope approximations*





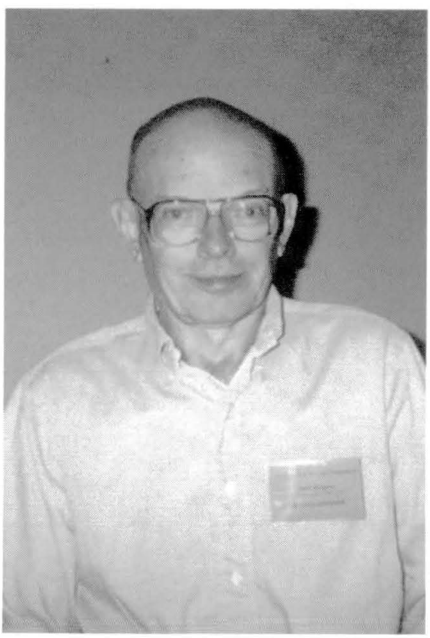
upper: Kathy Horadam and Ralph Stanton
lower: Ernie Ruet d'Auteuil and Anne Street



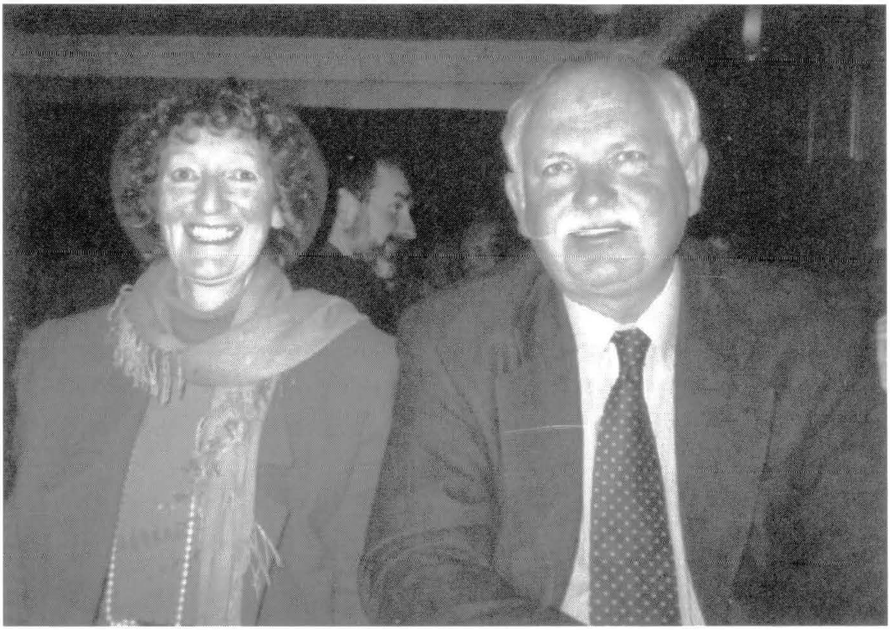
upper left: Cheryl Praeger
lower left: John Bamberg

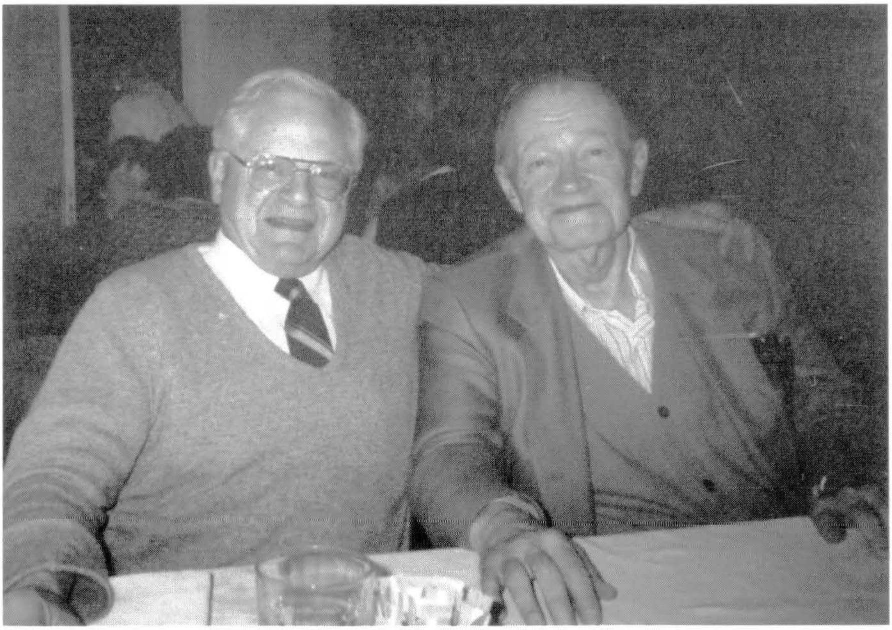
upper right: Michael Giudici
lower right: Asha Rao

upper left: Rei Safavi-Naini   upper right: Lynn Batten
lower left: Ed Dawson   lower right: Kiki Sugeng



upper left:	Nimalsiri Pinnawala	upper right:	Rosemary Bailey
lower left:	Neil Sloane	lower right:	David Glynn





upper: Lois and Peter Taylor
lower: Joe Gani and Ralph Stanton

Graph Theory Day 48*

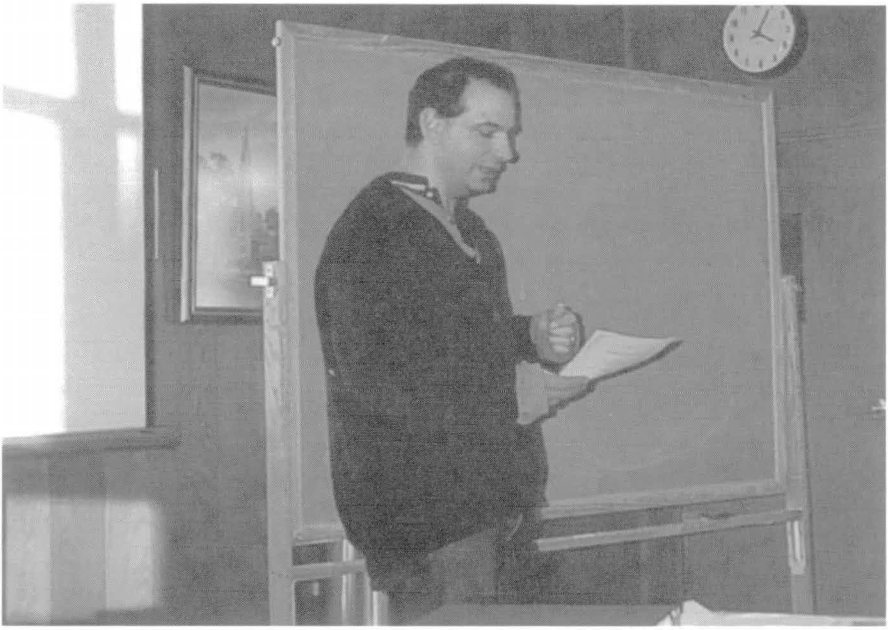
This conference was held on November 13, 2004 at Mount Saint Mary College in Newburgh, New York. The organizers were Mike Daven and Lee Fothergill (both of Mount Saint Mary College), Naomi Daven (Stevens Institute of Technology), Lou Quintas (Pace University), and John Kennedy (Queens College).

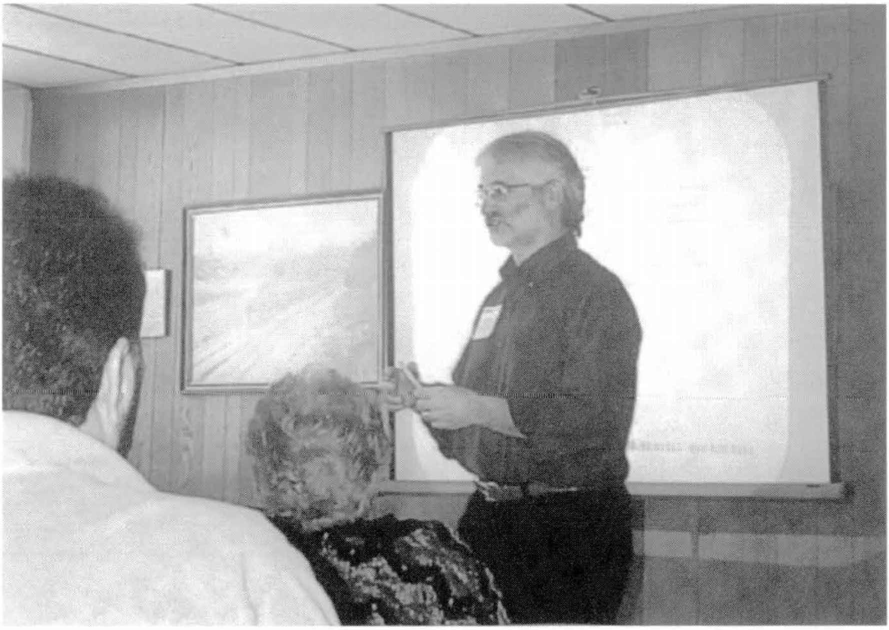
The invited speakers were:
- Jeff Dinitz (University of Vermont), Scheduling Leagues and Tournaments.
- Steven B. Horton (United States Military Academy), On Some Problems Related to Dominating Sets.

After the main speakers, a Graph Theory Notes session was held. The speakers during this session were:
- Brett Harrison (junior, Half Hollow Hills High School West)
- Pranava Jha (St. Cloud State University)
- Heather Axelrod (student, SUNY Purchase)
- Eric Choi (student, SUNY Purchase)
- Dan Gagliardi (St. Lawrence University)

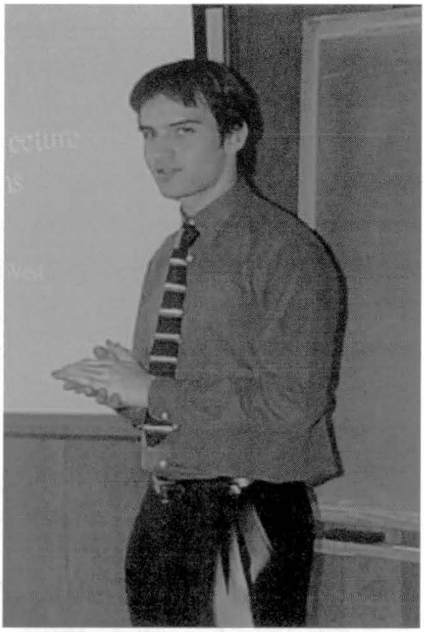
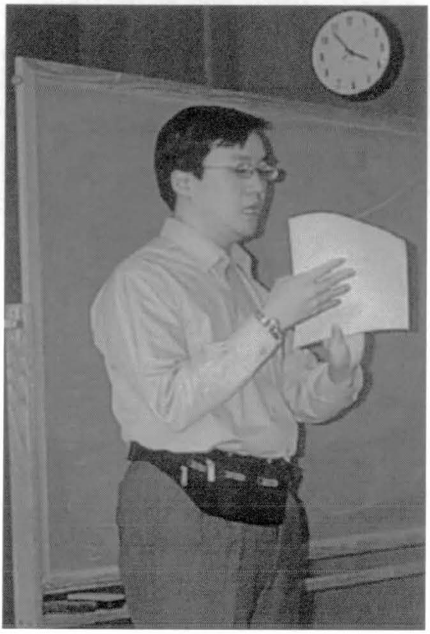
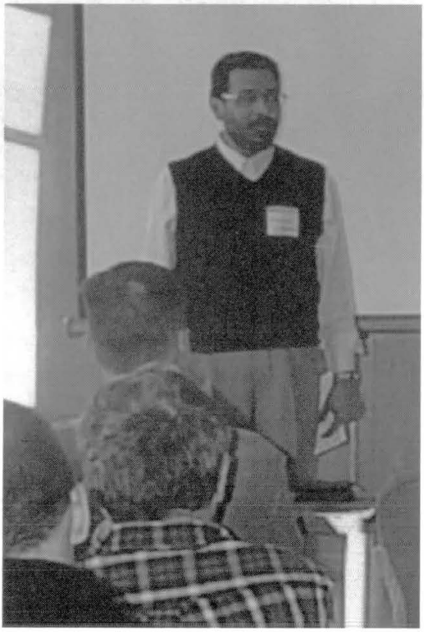
* Pictures and report by Mike Daven

Steven Horton





upper: Dan Gagliardi
lower: Jeff Dinitz

upper left: Heather Axelrod   upper right: Brett Harrison
lower left: Eric Choi   lower right: Pranava Jha

Combinatorial Potlatch 2004
Simon Fraser University
November 20, 2004

The n-th annual Combinatorial Potlatch was held at the Harbour Centre Campus of Simon Fraser University in Vancouver, British Columbia, Canada on November 20, 2004. The early history of the Potlatches is sketchy, but this was at least the twentieth such gathering since 1983.

Approximately 45 combinatorialists, primarily from Oregon, Washington and British Columbia, enjoyed the following invited talks:

Xuding Zhu, *The game chromatic number of a graph*

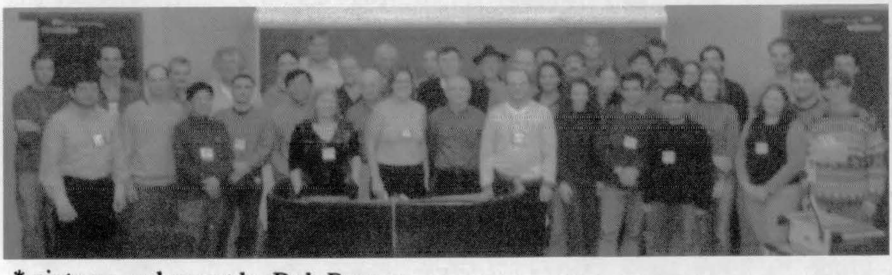
John Gimbel, *The traveling sales rep gets into abelian groups*

Jozsef Solymosi, *Bounds on incidences and problems from additive number theory*

Lunch, happy hour and dinner at local restaurants and brewpubs, and ample breaks between talks allowed for significant socializing and informal discussion. More details about the Combinatorial Potlatch can be found at `http://buzzard.ups.edu/potlatch/`.

Xuding Zhu, Peter Horak

John Gimbel, Jozsef Solymosi, Xuding Zhu



* pictures and report by Rob Beezer